STAT 110 A, Probability & Statistics for Engineers I

UCLA Statistics, Spring 2004

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SOLUTION HOMEWORK 2

Due Date: Friday, Apr. 30, 2004

http://www.stat.ucla.edu/%7Edinov/courses_students.dir/04/Spring/Stat110A.dir/HWs.dir/HW2.html

- **(HW_2_1)** [Sec. 2.3, #29]

  a. (5)(4) = 20 (5 choices for president, 4 remain for vice president)

  b. (5)(4)(3) = 60

  c. $\binom{5}{2} = \frac{5!}{2!3!} = 10$ (No ordering is implied in the choice)

- **(HW_2_2)** [Sec. 2.3, #33]

  a. $\binom{20}{5} = \frac{20!}{5!15!} = 15{,}504$

  b. $\binom{8}{4} \bullet \binom{12}{1} = 840$

  c. P(exactly 4 have cracks) = $\frac{\binom{8}{4}\binom{12}{1}}{\binom{20}{5}} = \frac{840}{15{,}504} = .0542$

  d. P(at least 4) = P(exactly 4) + P(exactly 5)

$$= \frac{\binom{8}{4}\binom{12}{1}}{\binom{20}{5}} + \frac{\binom{8}{5}\binom{12}{0}}{\binom{20}{5}} = .0542 + .0036 = .0578$$

- (**HW_2_3**) [Sec. 2.3, #40]

a. If the A's are distinguishable from one another, and similarly for the B's, C's and D's, then there are 12! Possible chain molecules. Six of these are:
$A_1A_2A_3B_2C_3C_1D_3C_2D_1D_2B_3B_1$, $A_1A_3A_2B_2C_3C_1D_3C_2D_1D_2B_3B_1$
$A_2A_1A_3B_2C_3C_1D_3C_2D_1D_2B_3B_1$, $A_2A_3A_1B_2C_3C_1D_3C_2D_1D_2B_3B_1$
$A_3A_1A_2B_2C_3C_1D_3C_2D_1D_2B_3B_1$, $A_3A_2A_1B_2C_3C_1D_3C_2D_1D_2B_3B_1$
These 6 (=3!) differ only with respect to ordering of the 3 A's. In general, groups of 6 chain molecules can be created such that within each group only the ordering of the A's is different. When the A subscripts are suppressed, each group of 6 "collapses" into a single molecule (B's, C's and D's are still distinguishable). At this point there are $\frac{12!}{3!}$ molecules. Now suppressing subscripts on the B's, C's and D's in turn gives ultimately $\frac{12!}{(3!)^4} = 369{,}600$ chain molecules.

b. Think of the group of 3 A's as a single entity, and similarly for the B's, C's, and D's. Then there are 4! Ways to order these entities, and thus 4! Molecules in which the A's are contiguous, the B's, C's, and D's are also. Thus, P(all together) = $\frac{4!}{369.600} = .00006494$.

- (**HW_2_4**) [Sec. 2.4, #45]

a. P(A) = 0.15+0.10+0.10+0.10=0.45, P(B) =0.10+0.15 = 0.25, $P(A \cap B) = 0.10$

b. P(A|B) = $\frac{P(A\cap B)}{P(B)} = \frac{0.10}{0.25} = 0.4$

P(B|A) = $\frac{P(A\cap B)}{P(A)} = \frac{0.10}{0.45} \approx 0.22$

c. P(A|C) = $\frac{P(A\cap C)}{P(C)} = \frac{0.15}{0.30} \approx 0.5$

P(A|C') = $\frac{P(A\cap C')}{P(C')} = \frac{0.30}{0.70} \approx 0.4285$

- (**HW_2_5**) [Sec. 2.5, #72]

Using subscripts to differentiate between the selected individuals,
$P(O_1 \cap O_2) = P(O_1) \bullet P(O_2) = (.44)(.44) = .1936$
$P(\text{two individuals match}) = P(A_1 \cap A_2) + P(B_1 \cap B_2) + P(AB_1 \cap AB_2) + P(O_1 \cap O_2)$
$= .42^2 + .10^2 + .04^2 + .44^2 = .3816$

- (**HW_2_6**) [Sec. 2.5, #77]

Let A_1 = older pump fails, A_2 = newer pump fails, and $x = P(A_1 \cap A_2)$. Then $P(A_1) = .10 + x$, $P(A_2) = .05$, and $x = P(A_1 \cap A_2) = P(A_1) \bullet P(A_2) = (.10 + x)(.05)$. The resulting quadratic equation, $.05x + .005 = x$, and so $0.95\,x = 0.005$. Then $x=0.00526$.

- (**HW_2_7**) [Sec. 2.5, #75]

Let q denote the probability that a rivet is defective.

a. P(seam need rework) = .14 = 1 – P(seam doesn't need rework)
= 1 – P(no rivets are defective)
= 1 – P(1^{st} isn't def $\cap \ldots \cap$ 25^{th} isn't def)
$= 1 - (1 - q)^{25}$, so $.86 = (1 - q)^{25}$, $1 - q = (.86)^{1/25}$, and thus $q = 1 - .993985 = .006$.

b. The desired condition is $.10 = 1 - (1 - q)^{25}$, i.e. $(1 - q)^{25} = .90$, from which $q = 1 - .99579 = .00421$.