

STAT 110 A, Probability & Statistics for Engineers I

UCLA Statistics, Spring 2004

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SOLUTION HOMEWORK 4

Due Date: Friday, May 21, 2004

http://www.stat.ucla.edu/%7Edinov/courses_students.dir/04/Spring/Stat110A.dir/HWs.dir/HW4.html

(HW 4 1) [Sec. 3.5, #69]

- a. With S = a female child and F = a male child, let X = the number of F's before the 2nd S. Then $P(X = x) = nb(x; 2, .5)$
- b. $P(\text{exactly 4 children}) = P(\text{exactly 2 males})$
 $= nb(2; 2, .5) = (3)(.0625) = .188$
- c. $P(\text{at most 4 children}) = P(X \leq 2)$
 $= \sum_{x=0}^2 nb(x; 2, .5) = .25 + 2(.25)(.5) + 3(.0625) = .688$
- d. $E(X) = \frac{(2)(.5)}{.5} = 2$, so the expected number of children = $E(X + 2)$
 $= E(X) + 2 = 4$

(HW 4 2) [Sec. 3.6, #76]

- a. $P(X = 1) = F(1; 2) - F(0; 2) = .982 - .819 = .163$
- b. $P(X \geq 2) = 1 - P(X \leq 1) = 1 - F(1; 2) = 1 - .982 = .018$
- c. $P(1^{\text{st}} \text{ doesn't} \cap 2^{\text{nd}} \text{ doesn't}) = P(1^{\text{st}} \text{ doesn't}) \cdot P(2^{\text{nd}} \text{ doesn't})$
 $= (.819)(.819) = .671$

(HW 4 3) [Sec. 4.1, #2]

$F(x) = \frac{1}{10}$ for $-5 \leq x \leq 5$, and = 0 otherwise

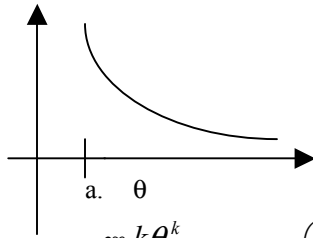
- a. $P(X < 0) = \int_{-5}^0 \frac{1}{10} dx = .5$

b. $P(-2.5 < X < 2.5) = \int_{-2.5}^{2.5} \frac{1}{10} dx = .5$

c. $P(-2 \leq X \leq 3) = \int_{-2}^3 \frac{1}{10} dx = .5$

d. $P(k < X < k + 4) = \int_k^{k+4} \frac{1}{10} dx = \frac{x}{10} \Big|_k^{k+4} = \frac{1}{10} [(k + 4) - k] = .4$

a)



b) $= \int_{-\infty}^{\infty} f(x; k, \theta) dx = \int_{\theta}^{\infty} \frac{k\theta^k}{x^{k+1}} dx = \theta^k \cdot \left(-\frac{1}{x^k} \right) \Big|_{\theta}^{\infty} = \frac{\theta^k}{\theta^k} = 1$

c) $P(X \leq b) = \int_{\theta}^b \frac{k\theta^k}{x^{k+1}} dx = \theta^k \cdot \left(-\frac{1}{x^k} \right) \Big|_{\theta}^b = 1 - \left(\frac{\theta}{b} \right)^k$

d) $P(a \leq X \leq b) = \int_a^b \frac{k\theta^k}{x^{k+1}} dx = \theta^k \cdot \left(-\frac{1}{x^k} \right) \Big|_a^b = \left(\frac{\theta}{a} \right)^k - \left(\frac{\theta}{b} \right)^k$

e) $E(X) = \int_{\theta}^{\infty} x \cdot \frac{k\theta^k}{x^{k+1}} dx = k\theta^k \int_{\theta}^{\infty} \frac{1}{x^k} dx = \frac{k\theta^k x^{-k+1}}{-k+1} \Big|_{\theta}^{\infty} = \frac{k\theta}{k-1}$

f) $E(X) = \infty$

g) $E(X^2) = k\theta^k \int_{\theta}^{\infty} \frac{1}{x^{k-1}} dx = \frac{k\theta^2}{k-2}$, so

$$\text{Var}(X) = \left(\frac{k\theta^2}{k-2} \right) - \left(\frac{k\theta}{k-1} \right)^2 = \frac{k\theta^2}{(k-2)(k-1)^2}$$

h) $\text{Var}(x) = \infty$, since $E(X^2) = \infty$.

(HW 4 5) [Sec. 4.3, #33]

- a. $P(X \geq 10) = P(Z \geq .43) = 1 - \Phi(.43) = 1 - .6664 = .3336$.
 $P(X > 10) = P(X \geq 10) = .3336$, since for any continuous distribution, $P(x = a) = 0$.
- b. $P(X > 20) = P(Z > 4) \approx 0$
- c. $P(5 \leq X \leq 10) = P(-1.36 \leq Z \leq .43) = \Phi(.43) - \Phi(-1.36) = .6664 - .0869 = .5795$
- d. $P(8.8 - c \leq X \leq 8.8 + c) = .98$, so $8.8 - c$ and $8.8 + c$ are at the 1st and the 99th percentile of the given distribution, respectively. The 1st percentile of the standard normal distribution has the value -2.33 , so
 $8.8 - c = \mu + (-2.33)\sigma = 8.8 - 2.33(2.8) \Rightarrow c = 2.33(2.8) = 6.524$.

(HW 4 6) [Sec. 4.3, #42]

- a. $P(67 \leq X \leq 75) = P(-1.00 \leq Z \leq 1.67) = .7938$
- b. $P(70 - c \leq X \leq 70 + c) = P\left(\frac{-c}{3} \leq Z \leq \frac{c}{3}\right) = 2\Phi\left(\frac{c}{3}\right) - 1 = .95 \Rightarrow \Phi\left(\frac{c}{3}\right) = .9750$
 $\frac{c}{3} = 1.96 \Rightarrow c = 5.88$
- c. $10 \cdot P(\text{a single one is acceptable}) = 7.938$
- d. $p = P(X < 73.84) = P(Z < 1.28) = .9$, so $P(Y \leq 8) = B(8; 10, .9) = .264$