Stat13 Homework 3

http://www.stat.ucla.edu/~dinov/courses_students.html

Suggested Solutions

HW_3_1

X = number disk drives which malfunction

(1) This is Binomial distribution with n = 11, p = 0.03.

(2) In the context of this exercise, state the assumptions required for X to have a Binomial distribution.

→ The occurrence of each disk drive's malfunction is independent of another's. Binomial distribution requires each trial to be independent of another.

(3) Are the Binomial assumptions satisfied here?

→ Yes. The malfunction of a disk drive should not be dependent of another disk drive.

(4) Calculate the probability that:

1. No disk drive will malfunction during the warranty period.

$$P(X = 0) = \binom{11}{0} 0.03^0 \; 0.97^{11} = \mathbf{0.7153}$$

2. Exactly one disk drive will malfunction during the warranty period.

$$P(X = 1) = \binom{11}{1} 0.03^1 \; 0.97^{10} = \mathbf{0.2434}$$

3. At least two disk drives will malfunction during the warranty period.

$P(X >= 2) = 1 – P(X = 0 \text{ or } X = 1) = 1 – (P(X = 0) + P(X = 1))$
$= 1 – 0.7153 – 0.2434 =$ **0.04135**

4. Between 2 and 5 (inclusive) disk drives will malfunction during the warranty period.

$P(2 \le X \le 5) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) =$

$$\binom{11}{2} 0.03^2\, 0.97^9 + \binom{11}{3} 0.03^3\, 0.97^8 + \binom{11}{4} 0.03^4\, 0.97^7 + \binom{11}{5} 0.03^5\, 0.97^6$$

$= 0.03763144 + 0.003491577 + 0.0002159738 + 0.00000935 =$ **0.04135**

Note: The answers of part 3 and 4 are only approximately equal. $P(X \geq 5)$ is close to 0.

HW_3_2

Let X = wife's blood type, Y = husband's blood type.

The wife's blood type is indepedent of the husband's, so

P(X = x and Y = y) = P(X = x) * P(Y = y)

(1) What is the probability that both husband and wife have type A blood?

The wife's blood type is indepedent of the husband's.

P(X = A and Y = A) = P(X = A) * P(Y = A) = .25 * .25 = **0.0625**

(2) What is the probability that at least one of them has blood type AB?

P(X = AB or Y = AB) = P(X = AB) + P(Y = AB) **-** P(X = AB and Y = AB)

= .11 + .11 **-** .11 * .11 = **0.2079**

Or P(X = AB or Y = AB)

= P(X = AB and Y = AB) +

P(X = AB and Y = A) + P(X = AB and Y = B) + P(X = AB and Y = O) +

P(X = A and Y = AB) + P(X = B and Y = AB) + P(X = O and Y = AB)

= .11 * .11 + .11 * (.25 + .15 + .49) *2 = **0.2079**

(3) What is the probability that they have the same blood type?

P(X = Y)

= P(X = A and Y = A) + P(X = B and Y = B) +P(X = AB and Y = AB) + P(X = O and Y = O)

= .15 * .15 + .25 * .25 + .49 * .49 + .11 * .11 = **0.3372**

HW_3_3

Gender \ Days of Exercise	A(0-1)	B(2-3)	C(4-5)	D(6-7)	Total
Male	40	53	26	6	125

Female	34	68	37	11	150
Combined M and F	74	121	63	17	Total = 275

(1) is a female?

P (female)

= number of female / (number of male + number of female)

= 150 / 275 = **0.5454**

(2) is a female and exercises 2-5 times a week?

P (female and exercises 2-5 times a week)

= (cell(2, 2) + cell(2, 3)) / total = (68 + 37) / 27 = **0.3818**

(3) is a male or exercises 4 or more times a week?

Let events A = male; B = exercises 4 or more times a week

P (A *or* B) = P(A) + P(B) – P(A *and* B)

P (A) = 125 / 275 = 0.4545

P(B) = (63 + 17) / 275 = 0.2909 (From the last row, columns C and D)

P(A *and* B) = (26 + 6)/275 =0.116

So, P(A *or* B) = **0.629**

(4) is a male given that the person exercises 3 times or less a week?

Let events A = male; B = the person exercises 3 times or less a week

P(A | B) = P(A and B) / P(B)

P(A and B) = (40 + 53) / 275 (columns A and B for male)

P(B) = (74 + 121) / 275 (Last row, columns A and B).

P(A |B) = ((40 + 53) / 275) / ((74 + 121) / 275) = **0.4769**