

Stat13 Homework 3

http://www.stat.ucla.edu/~dinov/courses_students.html

Suggested Solutions

HW 3 1

X = number disk drives which malfunction

(1) This is Binomial distribution with $n = 11$, $p = 0.03$.

(2) In the context of this exercise, state the assumptions required for X to have a Binomial distribution.

→ The occurrence of each disk drive's malfunction is independent of another's. Binomial distribution requires each trial to be independent of another.

(3) Are the Binomial assumptions satisfied here?

→ Yes. The malfunction of a disk drive should not be dependent of another disk drive.

(4) Calculate the probability that:

1. No disk drive will malfunction during the warranty period.

$$P(X = 0) = \binom{11}{0} 0.03^0 0.97^{11} = \mathbf{0.7153}$$

2. Exactly one disk drive will malfunction during the warranty period.

$$P(X = 1) = \binom{11}{1} 0.03^1 0.97^{10} = \mathbf{0.2434}$$

3. At least two disk drives will malfunction during the warranty period.

$$\begin{aligned} P(X \geq 2) &= 1 - P(X = 0 \text{ or } X = 1) = 1 - (P(X = 0) + P(X = 1)) \\ &= 1 - 0.7153 - 0.2434 = \mathbf{0.04135} \end{aligned}$$

4. Between 2 and 5 (inclusive) disk drives will malfunction during the warranty period.

$$P(2 \leq X \leq 5) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) =$$

$$\begin{aligned} &\binom{11}{2} 0.03^2 0.97^9 + \binom{11}{3} 0.03^3 0.97^8 + \binom{11}{4} 0.03^4 0.97^7 + \binom{11}{5} 0.03^5 0.97^6 \\ &= 0.03763144 + 0.003491577 + 0.0002159738 + 0.00000935 = \mathbf{0.04135} \end{aligned}$$

Note: The answers of part 3 and 4 are only approximately equal. $P(X \geq 5)$ is close to 0.

HW 3 2

Let X = wife's blood type, Y = husband's blood type.

The wife's blood type is independent of the husband's, so

$$P(X = x \text{ and } Y = y) = P(X = x) * P(Y = y)$$

(1) What is the probability that both husband and wife have type A blood?

The wife's blood type is independent of the husband's.

$$P(X = A \text{ and } Y = A) = P(X = A) * P(Y = A) = .25 * .25 = \mathbf{0.0625}$$

(2) What is the probability that at least one of them has blood type AB?

$$P(X = AB \text{ or } Y = AB) = P(X = AB) + P(Y = AB) - P(X = AB \text{ and } Y = AB)$$

$$= .11 + .11 - .11 * .11 = \mathbf{0.2079}$$

$$\text{Or } P(X = AB \text{ or } Y = AB)$$

$$= P(X = AB \text{ and } Y = AB) +$$

$$P(X = AB \text{ and } Y = A) + P(X = AB \text{ and } Y = B) + P(X = AB \text{ and } Y = O) +$$

$$P(X = A \text{ and } Y = AB) + P(X = B \text{ and } Y = AB) + P(X = O \text{ and } Y = AB)$$

$$= .11 * .11 + .11 * (.25 + .15 + .49) * 2 = \mathbf{0.2079}$$

(3) What is the probability that they have the same blood type?

$$P(X = Y)$$

$$= P(X = A \text{ and } Y = A) + P(X = B \text{ and } Y = B) + P(X = AB \text{ and } Y = AB) + P(X = O \text{ and } Y = O)$$

$$= .15 * .15 + .25 * .25 + .49 * .49 + .11 * .11 = \mathbf{0.3372}$$

HW 3 3

Gender \ Days of Exercise	A(0-1)	B(2-3)	C(4-5)	D(6-7)	Total
Male	40	53	26	6	125

Female	34	68	37	11	150
Combined M and F	74	121	63	17	Total = 275

(1) is a female?

P (female)

= number of female / (number of male + number of female)

$$= 150 / 275 = \mathbf{0.5454}$$

(2) is a female and exercises 2-5 times a week?

P (female and exercises 2-5 times a week)

$$= (\text{cell}(2, 2) + \text{cell}(2, 3)) / \text{total} = (68 + 37) / 27 = \mathbf{0.3818}$$

(3) is a male or exercises 4 or more times a week?

Let events A = male; B = exercises 4 or more times a week

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A) = 125 / 275 = 0.4545$$

$$P(B) = (63 + 17) / 275 = 0.2909 \text{ (From the last row, columns C and D)}$$

$$P(A \text{ and } B) = (26 + 6) / 275 = 0.116$$

$$\text{So, } P(A \text{ or } B) = \mathbf{0.629}$$

(4) is a male given that the person exercises 3 times or less a week?

Let events A = male; B = the person exercises 3 times or less a week

$$P(A | B) = P(A \text{ and } B) / P(B)$$

$$P(A \text{ and } B) = (40 + 53) / 275 \text{ (columns A and B for male)}$$

$$P(B) = (74 + 121) / 275 \text{ (Last row, columns A and B).}$$

$$P(A | B) = ((40 + 53) / 275) / ((74 + 121) / 275) = \mathbf{0.4769}$$