

Stat13 Homework 5

http://www.stat.ucla.edu/~dinov/courses_students.html

(50 points, student scores will be converted to scores out of 100)

Suggested Solutions

HW_5_1 <20 points>

$$E(x) = -0.165$$

$$SD(x) = 6.84 \text{ or } 6.97$$

$$Pr = 0.411 \text{ for } 0.431$$

For $n=100$

$$E(\bar{x}) = -0.165$$

$$SD(\bar{x}) = 0.684 \text{ or } 0.697$$

$$Pr = 0.405 \text{ or } 0.406$$

For $n=1000$

$$E(\bar{x}) = -0.165$$

$$SD(\bar{x}) = 0.216 \text{ or } 0.22$$

$$Pr = 0.222 \text{ or } 0.227$$

For $n=5000$

$$E(\bar{x}) = -0.165$$

$$SD(\bar{x}) = 0.097 \text{ or } 0.099$$

$$Pr = 0.044 \text{ or } 0.047$$

For $n=10000$

$$E(\bar{x}) = -0.165$$

$$SD(\bar{x}) = 0.0684 \text{ or } 0.0697$$

$$Pr = 0.0079 \text{ or } 0.009.$$

(1) <4 points>

$$E(X) = -5 \cdot 0.589 + 5 \cdot 0.346 + 10 \cdot 0.06 + 60 \cdot 0.005 = -0.315 \text{ <1 point>}$$

$$SD(X) = \sqrt{(-5 + 0.315)^2 \cdot 0.589 + (5 + 0.315)^2 \cdot 0.346 + (10 + 0.315)^2 \cdot 0.06 + (60 + 0.315)^2 \cdot 0.005} = 6.875738 \text{ <2 points>}$$

$$\text{Prob(positive return in a single bet)} = 1 - 0.589 = 0.411 \text{ <1 point>}$$

(2) <4 points>

$$E(\bar{X}) = E(X) = -0.315 \text{ <1 point>}$$

$$SE(\bar{X}) = SD(X) / \sqrt{100} = 0.688 \text{ <1 point>}$$

$\bar{X}$ is normally distributed, based on Central Limit Theorem. <1 point>

$$P(\bar{X} > 0) = P(z > (0 + 0.315) / 0.688) = P(z > 0.459) = 1 - 0.6769 = 0.3231 \text{ <1 point>}$$

(3) <9 points>

For 1000 bets:

$$E(\bar{X}) = -0.315$$

$$SE(\bar{X}) = 0.218$$

$P(\bar{X} > 0) = P(z > 1.44) = 0.074$ <3 points>

For 5000 bets:

$E(\bar{X}) = -0.315$

$SE(\bar{X}) = 0.097$

$P(\bar{X} > 0) = P(z > 3.25) = 0.00058$ <3 points>

For 10000 bets:

$E(\bar{X}) = -0.315$

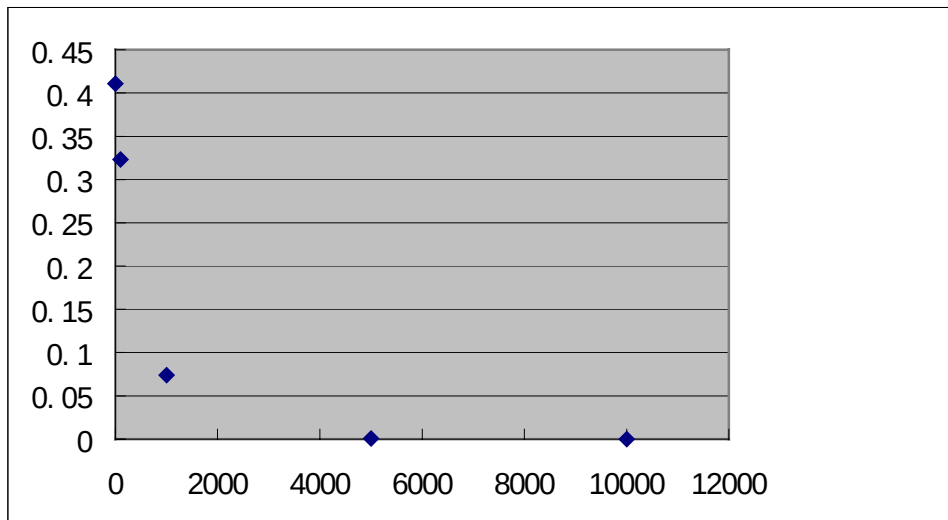
$SE(\bar{X}) = 0.0688$

$P(\bar{X} > 0) = P(z > 4.58) = 0.00000234$ <3 points>

(4) <3 points>

<2 points for the scatter plot>

The higher the number of independent bets, the less likely to make a net gain. <1 point>



HW_5_2 <8 points>

(1) <2 points>

$SE(\bar{X}_1 - \bar{X}_2) =$

$\sqrt{sd^2(X_1)/n_1 + sd^2(X_2)/n_2} = \sqrt{1.91*1.91/40 + 1.43*1.43/58} = 0.356$

(2) <2 points>

$E(\bar{X}_1 - \bar{X}_2) = m_1 - m_2 = 3.1$ <1 point>

Two standard error interval: $[3.1 - 2*0.356, 3.1 + 2*0.356] = [2.388, 3.812]$ <1 point>

(3) <4 points>

According to the study:

The difference between the mean of the two groups is normally distributed with mean of 3.1 and standard error of 0.356. The two standard error interval is totally above 0. It is significant that group one (sexual content) has a higher mean of remembering than group two (general audience content). The result of the study can be summarized as: Sexual content ads are more likely to be remembered, at least in the student-aged population.

HW_5_3 <12 points>

(1) <3 points, 1 point each>

one sample:

$$\text{mean} = E(X) = 6.75$$

$$\text{s.d.} = \text{sd}(X) = 1.1$$

four sample:

$$\text{mean} = E(X) = 6.75$$

$$\text{s.d.} = \text{sd}(X)/2 = 0.55$$

16 sample:

$$\text{mean} = E(X) = 6.75$$

$$\text{s.d.} = \text{sd}(X)/4 = 0.275$$

(2) <2 point>

They differ in standard deviation. Because with larger sample, variance of single units tend to be averaged out. The bigger the sample size, the smaller the standard error.

(3) <7 points>

$$Y = 1.3X - 1.3 \text{ <1 point>}$$

$$E(Y) = 1.3 * E(X) - 1.3 = 7.475 \text{ <1 point>}$$

$$\text{SD}(Y) = 1.3 * \text{SD}(X) = 1.43 \text{ <1 point>}$$

$$E(X_1 + X_2 + X_3 + Y_1 + Y_2 + Y_3 + Y_4 + Y_5)$$

$$= E(X_1) + E(X_2) + E(X_3) + E(Y_1) + E(Y_2) + E(Y_3) + E(Y_4) + E(Y_5)$$

$$= 3 * 6.75 + 5 * 7.475$$

$$= 57.625 \text{ <2 points>}$$

$$\text{SD}(X_1 + X_2 + X_3 + Y_1 + Y_2 + Y_3 + Y_4 + Y_5)$$

$$= \sqrt{\text{sd}(X_1)^2 + \text{sd}(X_2)^2 + \text{sd}(X_3)^2 + \text{sd}(Y_1)^2 + \text{sd}(Y_2)^2 + \text{sd}(Y_3)^2 + \text{sd}(Y_4)^2 + \text{sd}(Y_5)^2}$$

$$= \sqrt{3 * 1.1^2 + 5 * 1.43^2}$$

$$= 3.72 \text{ <2 points>}$$

HW_5_4 <10 points>

Omitted here.