[Stat13 Homework 6](http://www.stat.ucla.edu/~dinov/courses_students.html)

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Suggested Solutions

7.8:

$SE_1 = \frac{0.400}{\sqrt{9}} = 0.133$, $SE_2 = \frac{0.220}{\sqrt{6}} = 0.09$

$SE = \sqrt{SE_1^2 + SE_2^2} = 0.16$

7.9

$SE = \sqrt{SE_1^2 + SE_2^2} = 10.2$

7.11

$SE = \sqrt{SE_1^2 + SE_2^2} = 9.192$, $df = \frac{(SE_1^2 + SE_2^2)^2}{SE_1^4/(n_1 - 1) + SE_2^4/(n_2 - 1)} = 6$

95% CI: $\bar{y}_1 - \bar{y}_2 \pm t_{0.025} \cdot SE(\bar{y}_1 - \bar{y}_2) = (-45.5, -0.5)$

90% CI: $\bar{y}_1 - \bar{y}_2 \pm t_{0.5} \cdot SE(\bar{y}_1 - \bar{y}_2) = (-40.9, -5.1)$

7.12

a) $SE = \sqrt{SE_1^2 + SE_2^2} = 1.867$, $df = \frac{(SE_1^2 + SE_2^2)^2}{SE_1^4/(n_1 - 1) + SE_2^4/(n_2 - 1)} = 190$

95% CI: $\bar{y}_1 - \bar{y}_2 \pm t_{0.025} \cdot SE(\bar{y}_1 - \bar{y}_2) = (6.1, 13.5)$, (use df = 140)

b) We are 95% confident that the population mean reduction in systolic blood pressure for those who receive training for eight weeks is larger than that for others by an amount that might be as small as 6.1 mmHg or as large as 13.5 mmHg.

7.13

No. The CI found in 7.11 is valid even if the distribution is not normal, because the sample sizes are large.

7.14

a) $SE = \sqrt{SE_1^2 + SE_2^2} = 4.050$, $df = \frac{(SE_1^2 + SE_2^2)^2}{SE_1^4/(n_1 - 1) + SE_2^4/(n_2 - 1)} = 17.2$

90% CI: $\bar{y}_1 - \bar{y}_2 \pm t_{0.05} \cdot SE(\bar{y}_1 - \bar{y}_2) = (-5.0, 9.0)$, (use df = 17)

b) We are 90% confident that the population mean prothrombin time for rats treated with an antibiotic is smaller than that for control rats by an amount that might be as much as 5 seconds or is larger than that for control rats by an amount that might be as much as 9 seconds.

7.16

a) $SE = \sqrt{SE_1^2 + SE_2^2} = 0.08763$, $df = \frac{(SE_1^2 + SE_2^2)^2}{SE_1^4/(n_1 - 1) + SE_2^4/(n_2 - 1)} = 35.7$

95% CI: $\bar{y}_1 - \bar{y}_2 \pm t_{0.025} \cdot SE(\bar{y}_1 - \bar{y}_2) = (-0.12, 0.24)$, (use df = 30)

b) We are 95% confident that the population mean head width of all females who mate successfully is smaller than that for rejected females by an amount that might be as much as 0.12 mm or is larger than that for rejected females by an amount that might be as much as 0.24 mm.

7.17

We are 97.5% confident that the population mean drop in systolic blood pressure of adults placed on a diet rich in fruits and vegetables for eight weeks is larger than that for adults placed on a standard diet by an amount that might be as small as 0.9 mmHg or as large as 4.7 mmHg.

7.19

$$SE = \sqrt{SE_1^2 + SE_2^2} = 5.02,\ df = \frac{(SE_1^2 + SE_2^2)^2}{SE_1^4/(n_1-1) + SE_2^4/(n_2-1)} = 17.3$$

90% CI: $\bar{y}_1 - \bar{y}_2 \pm t_{0.05} \cdot SE(\bar{y}_1 - \bar{y}_2) = (-7.33, 10.13)$, (use df = 17)

7.21

$$SE = \sqrt{SE_1^2 + SE_2^2} = 0.509,\ df = \frac{(SE_1^2 + SE_2^2)^2}{SE_1^4/(n_1-1) + SE_2^4/(n_2-1)} = 38$$

95% CI: $\bar{y}_1 - \bar{y}_2 \pm t_{0.025} \cdot SE(\bar{y}_1 - \bar{y}_2) = (-1.62, 0.46)$, (use df = 30)

7.27

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$$SE = \sqrt{SE_1^2 + SE_2^2} = 4.09,\ t_{stat} = \frac{\hat{\theta} - \theta_0}{SE(\hat{\theta})} = 1.07,\ df = \frac{(SE_1^2 + SE_2^2)^2}{SE_1^4/(n_1-1) + SE_2^4/(n_2-1)} = 71.9$$

$0.2 < p - value < 0.4$. Because $p - value > \alpha$, we do not reject H_0.

7.30

a) $H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$$SE = \sqrt{SE_1^2 + SE_2^2} = 0.62056,\ t_{stat} = \frac{\hat{\theta} - \theta_0}{SE(\hat{\theta})} = -3.26,$$

$$df = \frac{(SE_1^2 + SE_2^2)^2}{SE_1^4/(n_1-1) + SE_2^4/(n_2-1)} = 94.3$$

$0.001 < p - value < 0.01$. Because $p - value < \alpha$, we reject H_0.

b) There is sufficient evidence ($0.001 < p - value < 0.01$) to conclude that mean tibia length is larger in females than in males.

c) Judging from the means and SDs, the two distributions overlap substantially, so tibia length would be a poor predictor of sex.

d) $H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$$SE = \sqrt{SE_1^2 + SE_2^2} = 1.962,\ t_{stat} = \frac{\hat{\theta} - \theta_0}{SE(\hat{\theta})} = -1.03,$$

$$df = \frac{(SE_1^2 + SE_2^2)^2}{SE_1^4/(n_1-1) + SE_2^4/(n_2-1)} = 7.8$$

$0.2 < p - value < 0.4$. Because $p - value > \alpha$, we do not reject H_0.

7.31

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$SE = \sqrt{SE_1^2 + SE_2^2} = 5.06$, $t_{stat} = \dfrac{\hat{\theta} - \theta_0}{SE(\hat{\theta})} = 0.49$, $df = \dfrac{(SE_1^2 + SE_2^2)^2}{SE_1^4/(n_1 - 1) + SE_2^4/(n_2 - 1)} = 7.7$

$p - value > 0.4$. Because $p - value > \alpha$, we do not reject H_0.

b) According to the p-value found in part a), the fact that μ_1 is larger than μ_2 could easily be attributed to chance.

7.33

a) $H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 \neq \mu_2$

$SE = \sqrt{SE_1^2 + SE_2^2} = 67.08$, $t_{stat} = \dfrac{\hat{\theta} - \theta_0}{SE(\hat{\theta})} = 3.73$,

$$df = \frac{(SE_1^2 + SE_2^2)^2}{SE_1^4/(n_1 - 1) + SE_2^4/(n_2 - 1)} = 33.6$$

$p - value < 0.001$. Because $p - value < \alpha$, we reject H_0.

b) There is sufficient evidence ($p - value < 0.001$) to conclude that albumin is more effective than polygelatin as a plasma expander.

7.34

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$SE = \sqrt{SE_1^2 + SE_2^2} = 13.53$, $t_{stat} = \dfrac{\hat{\theta} - \theta_0}{SE(\hat{\theta})} = -0.14$,

$$df = \frac{(SE_1^2 + SE_2^2)^2}{SE_1^4/(n_1 - 1) + SE_2^4/(n_2 - 1)} = 17.9$$

$p - value > 0.4$. Because $p - value > \alpha$, we do not reject H_0. There is insufficient evidence ($p - value > 0.4$) to conclude that two diets differ in their effects on cholesterol.

7.35

a) True. We would reject H_0 because p-value is less than α.

b) True. We would reject H_0 because p-value is less than α.

c) True. This follows directly from the definition of a p-value.

7.37

a) $H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 \neq \mu_2$

$SE = \sqrt{SE_1^2 + SE_2^2} = 10.21$, $t_{stat} = \dfrac{\hat{\theta} - \theta_0}{SE(\hat{\theta})} = 0.92$,

$$df = \frac{(SE_1^2 + SE_2^2)^2}{SE_1^4/(n_1 - 1) + SE_2^4/(n_2 - 1)} = 10.4$$

$0.2 < p - value < 0.4$. Because $p - value > \alpha$, we do not reject H_0.

b) There is insufficient evidence ($0.2 < p - value < 0.4$) to conclude that the mean number of colonies differs for control and soap.

7.43

A type II error may be made.

7.45.

Yes. Because 0 is outside of the CI, we know that the p-value is less than 0.05, so the p-value is less than 0.1. Thus we reject the null hypothesis: $\mu_1 - \mu_2 = 0$.