STAT 35, Interactive and Computational Probability
UCLA Statistics

http://www.stat.ucla.edu/~dinov/courses_students.html

SOLUTIONS TO HOMEWORK 2

Solutions

Question 2_1

The given information is as follows:

$P(A_1) = 0.22$
$P(A_2) = 0.25$
$P(A_3) = 0.28$
$P(A_1 \cap A_2) = 0.11$
$P(A_1 \cap A_3) = 0.05$
$P(A_2 \cap A_3) = 0.07$
$P(A_1 \cap A_2 \cap A_3) = 0.01$

And we consider the following events:

(a) $A_1 \cup A_2$

This is the event where the firm is awarded projects #1 and #2.

When calculating probabilities of events, we must ensure that each outcome in the event is counted, but that no outcome is counted twice.

$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = 0.22 + 0.25 - 0.11 = \mathbf{0.36}$

(b) $A'_1 \cap A'_2$

This means that the firm is awarded neither project #1 nor project #2.

$P(A'_1 \cap A'_2) = 1 - P(A_1 \cup A_2) = 1 - 0.36 = \mathbf{0.64}$

(c) $A_1 \cup A_2 \cup A_3$

This means that the firm was awarded at least one of the projects.

$$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3)$$
$$+$$
$$P(A_1 \cap A_2 \cap A_3)$$
$$= 0.22 + 0.25 + 0.28 – 0.11 -0.05 – 0.07 + 0.01$$
$$= \mathbf{0.53}$$

(d) $A'_1 \cap A'_2 \cap A'_3$

This means that the firm was not awarded any of the projects. Notice that this event is the complement of the previous one where the firm receives at least one of the projects.

$P(A'_1 \cap A'_2 \cap A'_3) = 1 - P(A_1 \cup A_2 \cup A_3) = 1 – 0.53 =$ **0.47**

(e) $A'_1 \cap A'_2 \cap A_3$

Here the firm does not receive projects #1 and #2 but does receive project #3. Notice the following:

$P((A'_1 \cap A'_2) \cap A_3) + P((A'_1 \cap A'_2) \cap A'_3) = P(A'_1 \cap A'_2)$

And we have already computed two of these probabilities in parts (d) and (b).

Thus $P((A'_1 \cap A'_2) \cap A_3) + 0.47 = 0.64$

And we conclude that $P(A'_1 \cap A'_2 \cap A_3) =$ **0.17**

(f) $(A'_1 \cap A'_2) \cup A_3$

In this case, either the firm gets neither of projects #1 and #2, or the firm gets project #3.

$$P((A'_1 \cap A'_2) \cup A_3) = P(A'_1 \cap A'_2) + P(A_3) - P((A'_1 \cap A'_2) \cap A_3)$$
$$= 064. + 0.28 – 0.17 = \mathbf{0.75}$$

Question 2_2

Auto	N	L	M	H
L	0.04	0.06	0.05	0.03
M	0.07	0.10	0.20	0.10
H	0.02	0.03	0.15	0.15

This table gives the population proportions of the various combinations of policy choices, and these will be the probabilities of these combinations appearing in a random sample from the population. Some of the probabilities that we need may simply be read from the table of proportions, and others are merely the sum of a few of the entries in the table.

(a) The probability of an individual having a medium auto deductible and a high homeowner's deductible is **0.10**

(b) The probability that the individual has a low auto deductible is called a marginal probability, because we are only considering the auto deductible and not the homeowner's deductible. To find this probability we add all of the proportions of combinations that include a low auto deductible. These are the numbers in the first row of the table.
P(low auto deductible) = 0.04 + 0.06 + 0.05 + 0.03 = **0.18**

Likewise, the probability of a low homeowner's deductible is also a marginal probability, found by adding all of the joint probabilities in column #2:
P(low homeowner's deductible) = 0.06 + 0.10 + 0.03 = **0.19**

(c) There are three entries in the table where the individual is in the same category for both auto and homeowner's deductibles. Adding these, we get:
P(same for both) = 0.06 + 0.20 + 0.15 = **0.41**

(d) Observe that the two categories being different is the complement of their being the same. Thus we may compute the probability of this event by using the rule for complements:
P(deductibles are different) = 1 – (same for both) = 1- 0.41 = **0.59**

(e) To find the probability that the individual has at least one low deductible, we may add all of the entries in the table that fit this description – and these are the entries in the first row and those in the second column. We must be careful to avoid double-counting.
P(at least one is low) = 0.04 + 0.06 + 0.05 + 0.03 + 0.10 + 0.03 = **0.31**

(f) We may use the rule of complements to find the probability that neither deductible is low – we must refer to our result in part (e):
P(neither is low) = 1 – P(at least one is low) = 1 – 0.31 = **0.69**

Question 2_3

The theoretical probability of the event A = {the drawn card is a king or a club} may be determined by counting, since there are 52 cards in the deck, all of which are equally likely to be drawn.

Since there are 4 kings in the deck, P(king) = 4/52

Likewise, there are 13 clubs in the deck, and P(club) = 13/52

Now to find P(A), we must be careful to avoid double-counting – since there is a card that is both a king and a club.

P(A) = P(king or club)
= P(king) + P(club) – P(king and club)
= 4/52 + 13/52 – 1/52
= 16/52
= **4/13**

The empirical probability that you observed in your experiments will differ from this to some degree, and from one person to the next – especially since 20 experiments is not very many. But the discrepancy should decrease as the number of hands increases, due to the law of large numbers.

Question 2_4

If we toss a fair coin six times, and count the number of heads we observe, then this should be random and follow the binomial distribution with n=6 and p=0.5.

P(only one heads) = ${}_6C_1\ (0.5)^6 = 0.09375$

P(zero heads) = ${}_6C_0\ (0.5)^6 = 0.0156$

The frequencies that you observe in your experiments will differ to some degree from these theoretical probabilities, due to the limited number of tosses you are doing.