STAT 35, Interactive and Computational Probability
UCLA Statistics

http://www.stat.ucla.edu/~dinov/courses_students.html

SOLUTIONS TO HOMEWORK 3

Solutions

Question 3_1

If we are sampling 5 buses from 20, we may count the number of ways to do this using the 'choose' notation.

$C_{5,20}$ = 20! / (5!15!) = 15504

For the more complex problems posed in this question, we should recall the product rule of counting. It is discussed in Chapter 2 of the lecture notes, starting on slide 18. One approach is to choose the given number of buses with visible cracks from the 8 buses there are of that description, and choose the rest of the buses from the other 12 buses.

Thus for example, if there are to be exactly 4 buses with visible cracks, these are chosen out of 8 such buses: $C_{4,8}$ = 8!/(4!4!) = 70. The other bus is free of cracks, and this is chosen out of 12 such buses – $C_{1,12}$ = 12. Since the second choice is made without regard to the results of the first choice, we may apply the product rule: the number of ways to pick 5 buses out of the 20 such that exactly 4 have visible cracks is 70 x 12 = 840.

If buses are being chosen at random, this means that each of the 20 buses has an equal chance of being chosen as one of the 5 in our sample. Thus all of the 15504 choices are equally likely. Of these, 840 of the samples will have exactly 4 buses with visible cracks. Thus the probability of such a sample is 840/15504 which is 0.05418.

It is also possible for our sample to have all 5 buses with visible cracks. Since these will all be from the 8 buses that are like this, the number of ways this can happen is C5,8 = 56, and the probability of it happening is thus 56/15504 = 0.003612.

To find the probability of at least 4 buses in our sample having visible cracks:

P(at least 4) = P(exactly 4) + P(exactly 5) = 0.05418 + 0.003612 = 0.5779.

Question 3_2

A problem similar to this is solved on page 42 of Chapter 2 in the lecture notes. Since there are three molecules of each type (and 12 altogether) the number.of different chain molecules is 12!/(3!3!3!3!) = 369600. Some of these will have all three molecules of each type next to one another, and if we count the different ways that this can happen, we find 4! = 24 permutations of the 4 types. Thus the probability of this happening is 24 / 369600 = 0.0000649. It is thus an extremely rare event.

Question 3_3

We are given a table of joint probabilities, and may calculate from it various marginal and conditional probabilities. The marginal probabilities are the easiest to compute; one merely needs to add the probabilities for the corresponding row or column.

(a) P(A) = 0.15 + 0.10 + 0.10 + 0.10 = **0.45**

P(B) = 0.10 + 0.15 = **0.25**

A joint probability may simply be read off of the table:

$P(A \cap B)$ = **0.10**

(b) P(A|B) = $P(A \cap B)$ / P(B)
= (0.10) / (0.25)
= **0.4**

This is the chance of a black car having an automatic transmission.

P(B|A) = $P(A \cap B)$ / P(A)
= (0.10) / (0.45)
= **0.22**

This means that if we know that the car has an automatic transmission, there is a 22% chance that the car is black.

(c) P(A|C) = $P(A \cap C)$ / P(C)
= (0.15) / (0.15 + 0.15)
= **0.50**

Likewise, half of all white cars have automatic transmissions, while they are slightly less common amongst the cars of other colors, as we see below:

$$
\begin{aligned}
P(A|C') &= P(A \cap C') / P(C') \\
&= (0.1 + 0.1 + 0.1) / (0.1 + 0.1 + 0.1 + 0.05 + 0.15 + 0.2) \\
&= 0.3 / 0.7 \\
&= \mathbf{0.43}
\end{aligned}
$$

Question 3_4

These two formulas for conditional probability count the same thing, and will thus be equivalent, provided that one uses empirical probabilities throughout.