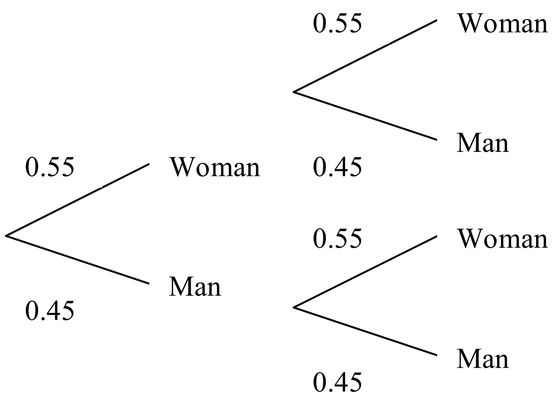
# Homework 2 – Solution Key

## Exercise 3.6

$p = 0.55$, $n = 2$

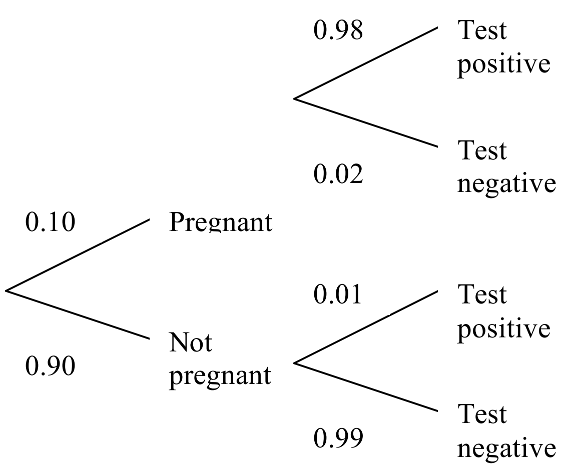



| **Event** | **Probability** |
|---|---|
| Woman, woman | .3025 |
| Woman, man | .2475 |
| Man, woman | .2475 |
| Man, man | .2025 |

a) If we take a sample of two students, the probability that both chosen students are women =0.55*0.55= .3025.

b) If we take a sample of two students, the probability that at least one of the two students is a woman =(0.55*0.55) +(0.55*0.45)+(0.45*0.55)=.7975. This can also be calculated using 1-Pr{none are women} =1-(.45*.45)=.7975

## Exercise 3.9




| **Event** | **Probability** |
|---|---|
| True positive | .098 |
| False negative | .002 |
| False positive | .009 |
| True negative | .891 |

a) Suppose that 1,000 women take early pregnancy tests and 100 of them are really pregnant. The probability that a randomly chosen women from this group will test positive is = 0.1*0.98+0.9*0.01=0.107

**...**

b) Suppose that 1,000 women take early pregnancy tests and 50 of them are really pregnant. The probability that a randomly chosen women from this group will test positive is = (0.05*.98) + (.95*0.1)= .0585

**Exercise 3.12**

a) The probability that someone in this study smokes = $\Pr\{smoke\} = \frac{1213}{6549} = .1852$

b) The conditional probability that someone in this study smokes, given that the person has high income $= \Pr\left\{\frac{smoke}{high\_income}\right\} = \frac{247}{2115} = .1168$

c) No. Being a smoker is not independent of having a high income. The probability from part a) is not equal to the probability in part b) $\Pr\{smoke\} \neq \Pr\left\{\frac{smoke}{high\_income}\right\}$

**Exercise 3.18**

a) If one of the 5,000 broods is chosen at random, the probability that the chosen brood has 3 young in the nest = $\Pr\{Y = 3\} = \frac{610}{5000} = .122$

b) If one of the 5,000 broods is chosen at random, the probability that the chosen brood has more than 7 young in the nest =

$$\Pr\{Y \geq 7\} = \Pr\{Y = 7\} + \Pr\{Y = 8\} + \Pr\{Y = 9\} + \Pr\{Y = 10\} = \frac{130 + 26 + 3 + 1}{5000} = .032$$

c) If one of the 5,000 broods is chosen at random, the probability that the chosen brood has between 4 and 6 young in the nest =

$$\Pr\{4 \leq Y \leq 6\} = \Pr\{Y = 4\} + \Pr\{Y = 5\} + \Pr\{Y = 6\} = \frac{1400 + 1760 + 750}{5000} = .782$$

**Exercise 3.20**

The mean size of the chosen brood, $\mu_Y$, of the random variable $Y$, is $E(Y) =$

$$\sum y_i \Pr(Y = y_i) = (1*90/5000) + (2*230/5000) + \ldots.(10*1/5000) = \frac{22435}{5000} = 4.487$$

**Exercise 3.24**

The mean the number of visits, $\mu_Y$, of the random variable $Y$, is $E(Y) =$

$$\sum y_i \Pr(Y = y_i) = (0*0.15) + (1*0.5) + (2*0.35) = 1.2$$

**Exercise 3.25**

The standard deviation of the number of visits, $\sigma_Y$, of the random variable Y , $= \sqrt{\sigma_Y^2} = \sqrt{.46} = .6782$

$Var(\mathrm{Y}) = \sigma_Y^2 = \sum (y_i - \mu_Y)^2 \Pr(Y = y_i)$

$= [(0 - 1.2)^2 * 0.15] + [(1 - 1.2)^2 * 0.5] + [(2 - 1.2)^2 * 0.35] = .46$

**Exercise 3.29**

$p = 0.60$, $n = 10$

a) The probability that the percentage of streaked-shelled snails in the sample will be 50% is the same as the probability that 5 out of 10 snails are streaked snails.

$\Pr\{j\_successes\} = \Pr\{Y = j\} = {}_nC_j p^j (1-p)^{n-j}$

$\Pr\{Y = 5\} = {}_{10}C_5 (0.6^5)(0.4^5) = 252 * (0.6^5)(0.4^5) = .2007$

b) The probability that the percentage of streaked-shelled snails in the sample will be 60% is

$\Pr\{Y = 6\} = {}_{10}C_6 (0.6^6)(0.4^4) = 210(0.6^6)(0.4^4) = .2508$

c) The probability that the percentage of streaked-shelled snails in the sample will be 70% is

$\Pr\{Y = 7\} = {}_{10}C_7 (0.6^7)(0.4^3) = 120(0.6^7)(0.4^3) = .2150$

**Exercise 3.34**

$p = \frac{1}{8} = .125$, $n = 16$

a) In a randomly chosen group of 16 children from the population, the probability that none has high blood lead $= \Pr\{Y = 0\} = {}_{16}C_0 \left(\frac{1}{8}\right)^0 \left(\frac{7}{8}\right)^{16} = 1 * 1 * \left(\frac{7}{8}\right)^{16} = .1181$

b) In a randomly chosen group of 16 children from the population, the probability that one has high blood lead $= \Pr\{Y = 1\} = {}_{16}C_1 \left(\frac{1}{8}\right)^1 \left(\frac{7}{8}\right)^{15} = 16 * \left(\frac{1}{8}\right)^1 \left(\frac{7}{8}\right)^{15} = .2699$

c) In a randomly chosen group of 16 children from the population, the probability that two has high blood lead $= \Pr\{Y = 2\} = {}_{16}C_2 \left(\frac{1}{8}\right)^2 \left(\frac{7}{8}\right)^{14} = 120 * \left(\frac{1}{8}\right)^2 \left(\frac{7}{8}\right)^{14} = .2891$

c) In a randomly chosen group of 16 children from the population, the probability that three or more have high blood lead $= \Pr\{Y \geq 3\} = 1 - \Pr\{Y \leq 2\} =$

$1 - [\Pr\{Y = 0\} + \Pr\{Y = 1\} + \Pr\{Y = 2\}] = 1 - [0.1181 + .2699 + .2891] = .3229$