# Homework 6

## Question 7.9

$$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = \sqrt{SE_1^2+SE_2^2} = \sqrt{5.5^2+8.6^2} = 10.2$$

## Question 7.10

Let 1 denote males and 2 denote females. Then:

$$\bar{y}_1 = 45.8; SE_1 = 2.8/\sqrt{489} = .127$$
$$\bar{y}_2 = 40.6; SE_2 = 2.9/\sqrt{469} = .134$$

The standard error is thus: $SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{.127^2+.134^2} = .185$. The critical value $t_{.025}$ is determined from Student's $t$ distribution with $df = 950$. Rounding down $df = 140$ [1] we find that $t(140)_{.025} = 1.977$. Therefore, the confidence interval is:

$$\begin{array}{rcl} (\bar{y}_1-\bar{y}_2) & \pm & t_{.025}SE_{(\bar{y}_1-\bar{y}_2)} \\ (45.8-40.6) & \pm & (1.977)(.185) \\ (4.834 & , & 5.566) \end{array}$$

## Question 7.11

Let 1 denote dark and 2 denote photoperiod. Then:

$$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{\frac{13^2}{4}+\frac{13^2}{4}} = 9.192$$

**a)** The critical value $t_{.025}$ is determined from Student's $t$ distribution with $df = 6$. Using $df = 6$ we find that $t(6)_{.025} = 2.447$. Therefore, the confidence interval is:

$$\begin{array}{rcl} (\bar{y}_1-\bar{y}_2) & \pm & t_{.025}SE_{(\bar{y}_1-\bar{y}_2)} \\ (92-115) & \pm & (2.447)(9.192) \\ (-45.5 & , & -0.50) \end{array}$$

**b)** The critical value $t_{.050}$ is determined from Student's $t$ distribution with $df = 6$. Using $df = 6$ we find that $t(6)_{.050} = 1.943$. Therefore, the confidence interval is:

$$\begin{array}{rcl} (\bar{y}_1-\bar{y}_2) & \pm & t_{.050}SE_{(\bar{y}_1-\bar{y}_2)} \\ (92-115) & \pm & (1.943)(9.192) \\ (-40.9 & , & -5.1) \end{array}$$

[1] Students are encouraged to round down in order to be conservative. However, students may also receive full credit if they round up to $df = 1000$. These students will have a confidence interval of $(4.84, 5.56)$

## Question 7.13

No. The confidence interval found in Exercise 7.11 is valid even if the distribution are not normal, because the sample sizes are large.

## Question 7.14

Let 1 denote antibotic and 2 denote control. Then:

$$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{\frac{10^2}{10} + \frac{8^2}{10}} = 4.050$$

**a)** The critical value $t_{.050}$ is determined from Student's $t$ distribution with $df = 17.2$. Using $df = 17$ we find that $t(17)_{.050} = 1.740$. Therefore, the confidence interval is:

$$\begin{array}{rcl} (\bar{y}_1 - \bar{y}_2) & \pm & t_{.050} SE_{(\bar{y}_1-\bar{y}_2)} \\ (25 - 23) & \pm & (1.740)(4.050) \\ (-5.0 & , & 9.0) \end{array}$$

**b)** We are 90% confident that the population mean prothrombin time for rats treated with an antibiotic ($\mu_2$) is smaller than that for control rats ($\mu_2$) by an amount that might be as much as 5 seconds or is larger than that for control rats ($\mu_2$) by an amount that might be as large as 9 seconds.

## Question 7.15

Let 1 denote control and 2 denote Pargyline. Then:

$$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{\frac{5.4^2}{900} + \frac{11.7^2}{905}} = .4286$$

**a)** The critical value $t_{.025}$ is determined from Student's $t$ distribution with $df = 1000$. Using $df = 1000$ [2] we find that $t(1000)_{.025} = 1.962$. Therefore, the confidence interval is:

$$\begin{array}{rcl} (\bar{y}_1 - \bar{y}_2) & \pm & t_{.025} SE_{(\bar{y}_1-\bar{y}_2)} \\ (14.9 - 46.5) & \pm & (1.962)(.4286) \\ (-32.441 & , & -30.759) \end{array}$$

**b)** The critical value $t_{.005}$ is determined from Student's $t$ distribution with $df = 1000$. Using $df = 1000$ [3] we find that $t(1000)_{.005} = 2.581$. Therefore, the confidence interval is:

$$\begin{array}{rcl} (\bar{y}_1 - \bar{y}_2) & \pm & t_{.005} SE_{(\bar{y}_1-\bar{y}_2)} \\ (14.9 - 46.5) & \pm & (2.581)(.4286) \\ (-32.706 & , & -30.494) \end{array}$$

[2] Students are encouraged to round down in order to be conservative. However, students may also receive full credit if they round up to $df = \infty$. These students will have a confidence interval of $(-32.4, -30.8)$

[3] Students are encouraged to round down in order to be conservative. However, students may also receive full credit if they round up to $df = \infty$. These students will have a confidence interval of $(-32.7, -30.5)$

## Question 7.16

Let 1 denote successful and 2 denote unsuccessful. Then:

$$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{\frac{.283^2}{22} + \frac{.262^2}{17}} = .08763$$

**a)** The critical value $t_{.025}$ is determined from Student's $t$ distribution with $df = 30$. Using $df = 30$ [4] we find that $t(30)_{.025} = 2.042$. Therefore, the confidence interval is:

$$\begin{array}{rcl} (\bar{y}_1 - \bar{y}_2) & \pm & t_{.025}SE_{(\bar{y}_1-\bar{y}_2)} \\ (8.498 - 8.440) & \pm & (2.042)(.08763) \\ (-.121 & , & .237) \end{array}$$

**b)** We are 90% confident that the population mean head width of all femails who mate successfully ($\mu_1$) sis smaller thatn that for rejected femails ($\mu_2$) by an amount that might be as much as .12mm or is larger than that for rejected females ($\mu_2$) by an amount that might be as large as .24mm.

## Question 7.18

We are 97.5% confident that the population mean drop in diastolic blood pressure of adults placed on a diet rich in fruits and vegetables for eight weeks ($\mu_1$) is smaller than that for adults placed on a stadard diet ($\mu_2$) by an amount that might be as much as .3mm Hg or is larger that that for adults placed on a standard diet ($\mu_2$) by an amount that might be as much as 2.4mm Hg.

## Question 7.21

Let 1 denote red and 2 denote green. Then:

$$\bar{y}_1 - \bar{y}_2 = \sqrt{.36^2 + .36^2} = .509$$

**a)** The critical value $t_{.025}$ is determined from Student's $t$ distribution with $df = 30$. Using $df = 30$ [5] we find that $t(30)_{.025} = 2.042$. Therefore, the confidence interval is:

$$\begin{array}{rcl} (\bar{y}_1 - \bar{y}_2) & \pm & t_{.025}SE_{(\bar{y}_1-\bar{y}_2)} \\ (8.36 - 8.94) & \pm & (2.042)(.509) \\ (-.1619 & , & .459) \end{array}$$

## Question 7.28

Let 1 denote diet 1 and 2 denote diet 2. We have $\bar{y}_1 - \bar{y}_2 = 3.49 - 3.05 = .44$
**a)**

$$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{\frac{.4^2}{5} + \frac{.4^2}{5}} = .2530$$

[4] Students are encouraged to round down in order to be conservative. However, students may also receive full credit if they round up to $df = 40$. These students will have a confidence interval of $(-.12, .24)$

[5] Students are encouraged to round down in order to be conservative. However, students may also receive full credit if they round up to $df = 40$. These students will have a confidence interval of $(-1.61, 0.45)$

Therefore $t = .44/.2530 = 1.74$ and Table 4 gives $t(8)_{.10} = 1.397$ and $t(8)_{.05} = 1.860$ so $.10 < P < .20$.
**b)**

$$SE_{(\bar{y}_1 - \bar{y}_2)} = \sqrt{\frac{.4^2}{10} + \frac{.4^2}{10}} = .1789$$

Therefore $t = .44/.1789 = 2.46$ and Table 4 gives $t(18)_{.02} = 2.214$ and $t(18)_{.01} = 2.552$ so $.02 < P < .04$.
**c)**

$$SE_{(\bar{y}_1 - \bar{y}_2)} = \sqrt{\frac{.4^2}{15} + \frac{.4^2}{15}} = .1461$$

Therefore $t = .44/.1461 = 3.01$ and Table 4 gives $t(28)_{.005} = 2.763$ and $t(28)_{.0005} = 3.674$ so $.001 < P < .01$.

## Question 7.30

**a)** The null and alternative hypotheses are $H_o$: Mean tibia length does not depend on gender ($\mu_1 = \mu_2$) and $H_a$: Mean tibia length does depend on gender ($\mu_1 \neq \mu_2$). The standard error of the difference is:

$$SE_{(\bar{y}_1 - \bar{y}_2)} = \sqrt{\frac{2.87^2}{60} + \frac{3.52^2}{50}} = .62056$$

The test statistics is then:

$$t = \frac{\bar{y}_1 - \bar{y}_2}{SE_{(\bar{y}_1 - \bar{y}_2)}} = \frac{78.42 - 80.44}{.62056} = -3.26$$

Therefore $p.value \leq \alpha$ and we reject $H_o$.
**b)** There is sufficient evidence at the $\alpha = .05$ level to conclude that mean tibia length is larger in females than in males.
**c)** Judging from the means and the SDs, the two distributions overlap substantially, so the tibia length would be a poor predictor of sex.
**d)**

$$SE_{(\bar{y}_1 - \bar{y}_2)} = \sqrt{\frac{.2.87^2}{6} + \frac{3.52^2}{5}} = 1.962$$

The test statistics is then:

$$t = \frac{\bar{y}_1 - \bar{y}_2}{SE_{(\bar{y}_1 - \bar{y}_2)}} = \frac{78.42 - 80.44}{1.962} = -1.03$$

Therefore $p.value \geq \alpha$ and we cannot reject $H_o$.

## Question 7.32

**a)** The null and alternative hypothesis are $H_o : \mu_1 = \mu_2$ and $H_a : \mu_1 \neq \mu_2$ where 1 denotes flooded and 2 denotes control. These hypotheses may be stated as $H_o$: Flooding has no effect on ATP and $H_a$: Flooding has some effect on ATP. The standard error of the difference is:

$$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{\frac{.184^2}{4} + \frac{.241^2}{4}} = .1516$$

The test statistics is then:

$$t = \frac{\bar{y}_1 - \bar{y}_2}{SE_{(\bar{y}_1-\bar{y}_2)}} = \frac{1.190 - 1.785}{.1516} = -3.92$$

Therefore $p.value \leq \alpha$ and we reject $H_o$.
**b)** There is sufficient evidence at the $\alpha = .05$ level to conclude flooding tends to lower ATP in birch seedlings.

## Question 7.34

The null and alternative hypotheses are $H_o$: Mean fall in cholesterol is the same on both diets ($\mu_1 = \mu_2$) and $H_a$: Mean fall in cholesterol is not the same on both diets ($\mu_1 \neq \mu_2$).

$$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{\frac{31.1^2}{10} + \frac{29.4^2}{10}} = 13.53$$

Therefore $t = (53.6 - 55.5)/13.53 = -.14$ and $p.value \geq \alpha$ and we fail to reject $H_o$.

## Question 7.35

**a)** True. We would reject $H_o$ because the P-value is less than $\alpha$. [6]
**b)** True. We would reject $H_o$ because the P-value is less than $\alpha$.
**c)** True. This follows directly from the definition of a P-value.

## Question 7.36

**a)** True. We would reject $H_o$ because the P-value is less than $\alpha$.
**b)** False. We do not reject $H_o$ because the P-value is greater than $\alpha$. [7]
**c)** False. the P-value is the probability, under $H_o$ of getting a result as extreme as, or more extreme than, the result that was actually observed.

## Question 7.38

**a)** $H_o$: mean height is the same for control and fertilized plants ($\mu_1 = \mu_2$) $H_a$: mean height is the different for control and fertilized plants ($\mu_1 \neq \mu_2$)

$$SE_{(\bar{y}_1-\bar{y}_2)} = \sqrt{\frac{.65^2}{28} + \frac{.72^2}{28}} = .183$$

Therefore $t = (2.58 - 2.04)/.183 = 2.95$ and $p.value \leq \alpha$ and we reject $H_o$.
**b)** There is evidence at the $\alpha = .05$ level to conclude that the mean height of fertilized radish sprouts is less than that of controls.

## Question 7.42

If we reject $H_o$ (ie. if the drug is approved) then we eliminate the possibility of a Type II error.

[6] If students interpret the p-value as being reported for just one tail, the answer would be False.
[7] If students interpret the p-value as being reported for just one tail, the answer would be True.

### Question 7.44

Yes. Because zero is outside of the confidence interval, we know that the P-value is less than .05 so we reject the hypothesis that $\mu_1 - \mu_2 = 0$.