UCLA STAT 13

Introduction to Statistical Methods for the Life and Health Sciences

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http://www.stat.ucla.edu/~dinov/courses_students.html

Chapter 5

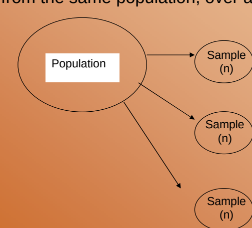
Sampling Distributions

Sampling Distributions

- **Definition:** *Sampling Variability* is the variability among random samples from the same population.
- A probability distribution that characterizes some aspect of sampling variability is called a sampling distribution.
  - tells us how close the resemblance between the sample and the population is likely to be.
- We typically construct a sampling distribution for a statistic.
  - Every statistics has a sampling distribution.

The Meta-Experiment

- All the possible samples that might be drawn from the population (infinity repetitions).
  - In other words if we were to repeatedly take samples of the same size from the same population, over and over.



The Meta-Experiment

- Meta-experiments are important because probability can be interpreted as the long run relative frequency of the occurrence of an event.
- Meta-experiments also let us visualize sampling distributions.
  - and therefore understand the variability among the many random samples of a meta-experiment.

Dichotomous Observations

- Dichotomous - two outcomes
  - (yes or no, good or evil, etc…)
- We use the following notation for a dichotomous outcome

  P population proportion

  $\hat{p}$ sample proportion
- The big question is how close is $\hat{p}$ to P?
- To determine this we need to examine the sampling distribution of $\hat{p}$
- What we want to know is:
  - if we took many samples of size n and observed $\hat{p}$ each time, how would those values of be distributed around p?

Dichotomous Observations

<u>Example</u>: Suppose we would like to estimate the <u>true</u> proportion of male students at UCLA. We could take a random sample of 50 students and calculate the sample proportion of males.

- What is the correct notation for:
  - the true proportion of males?
  - the sample proportion of males?
- Suppose we repeat the experiment over and over. Would we get the same proportion of males for the second sample?

Reece's Pieces Experiment

<u>Example</u>: Suppose we would like to estimate the true proportion of orange reece's pieces in a bag. To investigate we will take a random sample of 10 reece's pieces and count the number of orange. Next we will make an approximation to a sampling distribution with our class results.

What you need to calculate:

- the number of orange
- the sample proportion of orange (number of orange/10)

An Application of a Sampling Distribution

<u>Example</u>: Mendel's pea experiment. Suppose a tall offspring is the event of interest and that the true proportion of tall peas (based on a 3:1 phenotypic ratio) is 3/4 or p = 0.75. If we were to randomly select samples with n = 10 and p = 0.75 we could create a probability distribution as follows:

$\hat{p}$	Number Tall	Number Dwarf	Probability
0.0	0	10	0.000
0.1	1	9	0.000
0.2	2	8	0.000
0.3	3	7	0.003
0.4	4	6	0.016
0.5	5	5	0.058
0.6	6	4	0.146
0.7	7	3	0.250
0.8	8	2	0.282
0.9	9	1	0.188
1.0	10	0	0.056

Lab_Mendel_Pea_Experiment.html (work out in discussion/lab)

Validate using:
http://socr.stat.ucla.edu/Applets.dir/Normal_T_Chi2_F_Tables.htm
E.g., B(n=10, p=0.75, a=6, b=6)=0.146

An Application of a Sampling Distribution

- What is the probability that 5 are tall and 5 are dwarf?

P(5 tall and 5 dwarf) = P($\hat{p}$ = 5/10)
= P($\hat{p}$ = 0.5)
= 0.058

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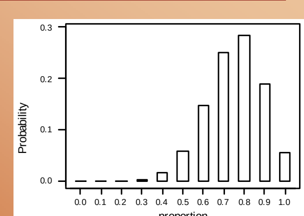
An Application of a Sampling Distribution

- If we think about this in terms of a meta-experiment and we sample 10 offspring over and over, about 5.8% of the $\hat{p}$'s will be 0.5.
  - This is the sampling distribution of sample proportion of tall offspring is the distribution of in repeated samples of size 10.
- If we take a random sample of size 10, what is the probability that six or more offspring are tall?

P($\hat{p}$ $\geq$ 0.6) = 0.146 + 0.250 + 0.282 + 0.188 + 0.056
= 0.922

An Application of a Sampling Distribution

- This table could also be represented as a histogram with probability on the y-axis and proportion on the x-axis.
  - easier to draw these by hand



Relationship to Statistical Inference

- We can also use our sampling distribution of to estimate how much sampling error there is within 5 percentage points of p. Because we knew p from the previous example (p=0.75), we might want to estimate:

  $P(0.7 \le \hat{p} \le 0.8)$
  $= 0.250 + 0.282 = 0.532$

  There is a 53% chance that for a sample of size 10, $\hat{p}$ will be within $\pm$ 0.05 of p.

  - This seems a little crazy, why?

$\hat{p}$	Number Tall	Number Dwarf	Probability
0.0	0	10	0.000
0.1	1	9	0.000
0.2	2	8	0.000
0.3	3	7	0.003
0.4	4	6	0.016
0.5	5	5	0.058
0.6	6	4	0.146
0.7	7	3	0.250
0.8	8	2	0.282
0.9	9	1	0.188
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Relationship to Statistical Inference

- So far we have been using p to determine the sampling distribution of $\hat{p}$.
- Why sample for $\hat{p}$ when we already know p?
  - We don't need to know p to get a good estimate (this will come later).

Sample Size

- As n gets larger, $\hat{p}$ will become a better estimate of p.
- Just to show…

N	$P(0.7 \le \hat{p} \le 0.8)$
10	0.53
20	0.56
50	0.673
100	0.798

*These calculations were done using the SOCR binomial distribution Calculator.
http://socr.stat.ucla.edu/Applets.dir/Normal_T_Chi2_F_Tables.htm

E.g., B(n=20, p=0.75, a=0.7x20=14, b=0.8x20=16)=0.5606

THE POINT: A larger sample improves the chance that $\hat{p}$ is close to p.

- Caution: this doesn't necessarily mean that the estimate will be closer to p, only that there is a better chance that it will be close to p.