

Stats 13.1 Homework 3 Solutions

October 22, 2007

1 Problem 4.3

Setup: $\mu = 1400$ $\sigma = 100 \Rightarrow Y \sim N(1400, 100)$

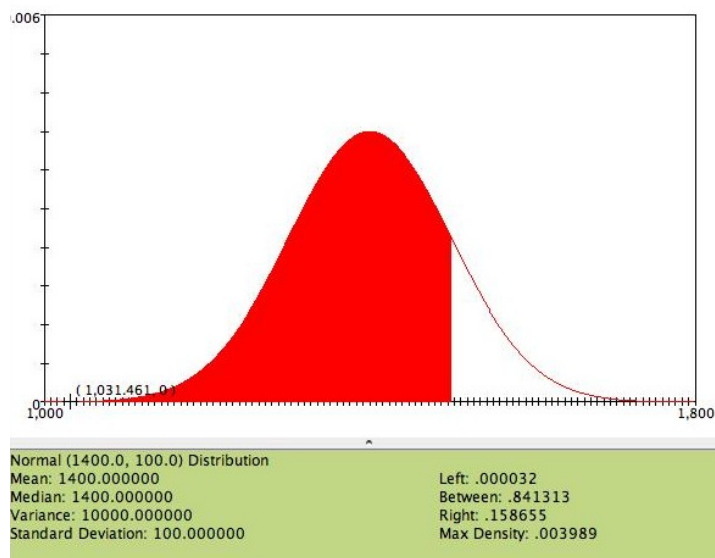
Formulas to use: $z = \frac{y-\mu}{\sigma}$ $P(Y \leq y) = P(z \leq \frac{y-\mu}{\sigma})$

1.1

$P(Y \leq 1500) = P(z \leq \frac{1500-1400}{100}) = P(z \leq 1.00) = .8413$

From Table 3 in the text or SOCR the area is .8413 or 84.13%

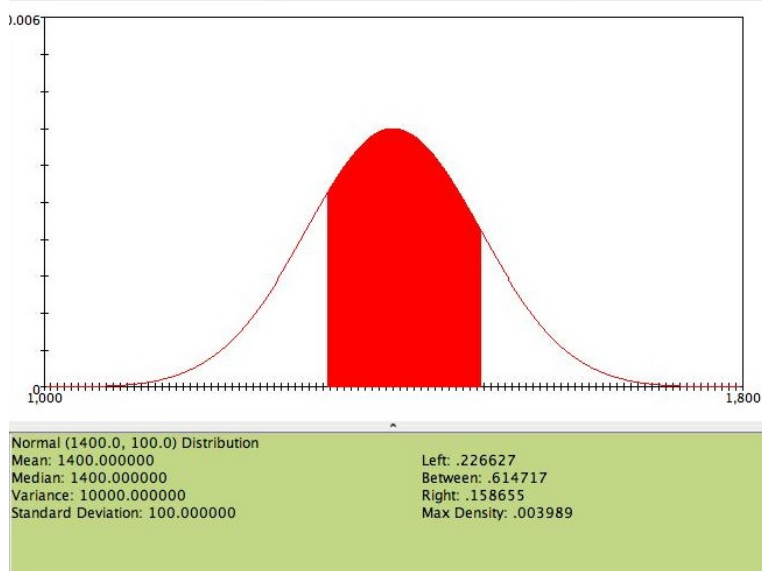
This means that 84.13% of the data (Swedish male brains) is below 1500 grams.



1.2

$$\begin{aligned} P(1325 \leq Y \leq 1500) &= P(Y \leq 1500) - P(Y \leq 1325) \\ &= P\left(z \leq \frac{1500-1400}{100}\right) - P\left(z \leq \frac{1325-1400}{100}\right) \\ &= P(z \leq 1.00) - P(z \leq -.75) = .8413 - .2266 = .6147 \text{ or } 61.47\% \end{aligned}$$

61.47% of the area is between 1325g and 1500g.



1.3

We want to know $P(Y \geq 1325)$

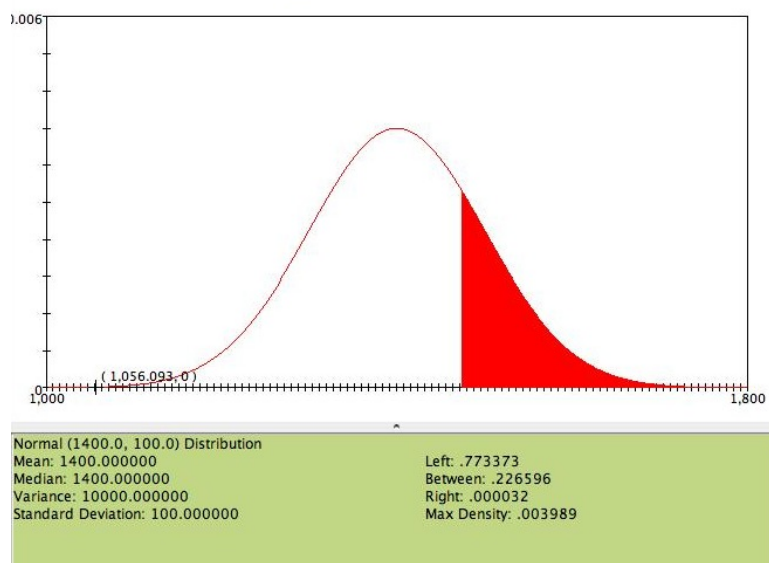
Since we already know $P(Y \leq 1325) = P(z \leq -.75) = .2266$ we can simply

$$P(Y \geq 1325) = 1 - P(Y \leq 1325) = 1 - .2266 = .7734 \text{ 77.34\% of the data is above 1325g.}$$

1.4

$$P(Y \geq 1475) \Rightarrow z = \frac{1475-1400}{100} = .75$$

Note that this is the mirror image (75g over rather than 75g under the mean) of a previous question where we learned that 22.66% of the data is below 1325g. Also note that the z-score is .75 rather than -.75. Therefore our hypothesis is that 22.66% of the data is above 1475g because the normal distribution is symmetrical. If using Table 3 we find that the area below 1475 is .7734. Thus, the percentage greater than 1475 is $1 - .7734 = .2266$.



1.5

$$\begin{aligned}
 P(1475 \leq Y \leq 1600) &= P(Y \leq 1600) - P(Y \leq 1475) \\
 &= P\left(z \leq \frac{1600-1400}{100}\right) - P\left(z \leq \frac{1475-1400}{100}\right) \\
 &= P(z \leq 2.00) - P(z \leq .75) = .9772 - .7734 = .2038
 \end{aligned}$$

The percentage of data between 1600g and 1475g is 20.38%

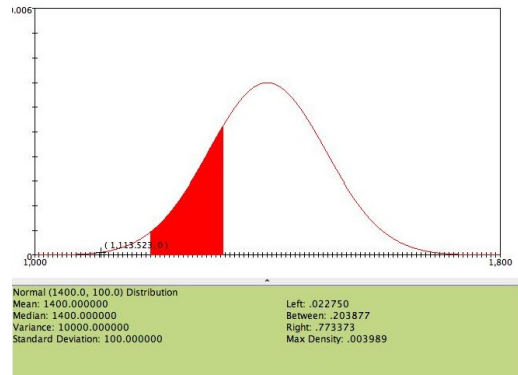
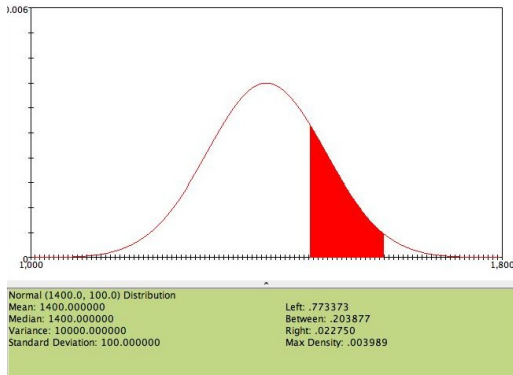
1.6

Since we already know the z-score for 1325g, we can just find the z-score for 1200g and go from there. Note that this is 200g (or 2 standard deviations) below the mean and in the previous question we found the z-score for 1600g, which is also 200g above the mean, so it should have the same z-score just with the opposite sign.

$$z = \frac{1200-1400}{100} = -2.00$$

$$\begin{aligned}
 \text{The area below } -2.00 \text{ is } .0228. \Rightarrow P(1200 \leq Y \leq 1325) &= \\
 P(Y \leq 1325) - P(Y \leq 1200) &= .2266 - .0228 = .2038
 \end{aligned}$$

Another way to look at this problem is that it represents the mirror image from the problem above and so would contain the same percentage of data. SOCR shows this well.



2 Problem 4.9

Setup: $\mu = 176\text{mg/dLi}$ $\sigma = 30\text{mg/dLi}$ $\Rightarrow Y \sim N(176, 30)$

Formulas to use: $z = \frac{y-\mu}{\sigma}$ $P(Y \leq y) = P(z \leq \frac{y-\mu}{\sigma})$

2.1

$$P(Y \geq 186) = 1 - P(Y \leq 186) = 1 - P(z \leq \frac{186-176}{30}) = 1 - P(z \leq .33) = 1 - .6293$$

2.2

$$P(Y \leq 156) = P(z \leq \frac{156-176}{30}) = P(z \leq -.67) = .2514$$

2.3

$$P(Y \leq 216) = P(z \leq \frac{216-176}{30}) = P(z \leq 1.33) = .9082$$

2.4

$$P(Y \geq 121) = 1 - P(Y \leq 121) = 1 - P(z \leq \frac{121-176}{30}) = 1 - P(z \leq -1.83) = 1 - .0336 = .9664$$

2.5

Since we already know the z-scores for both 186 and 216 and the associated percentages of data below both:

$$P(z \leq 1.33) - P(z \leq .33) = .9082 - .6293 = .2789$$

2.6

Since we already know the z-scores for both 121 and 156 and the associated percentages of data below both:

$$.2514 - .0336 = .2178$$

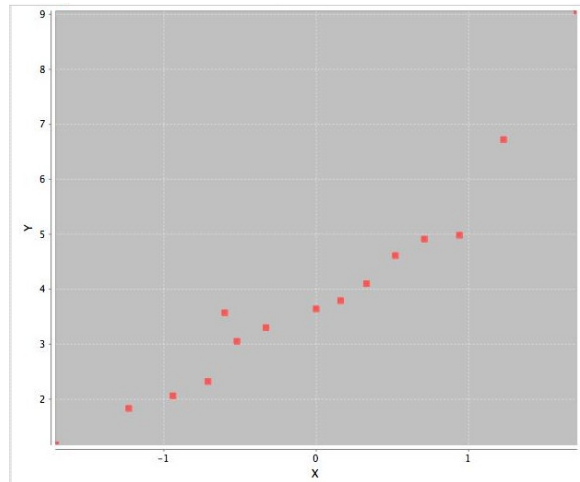
2.7

Since we already know the z-scores for both 156 and 186 and the associated percentages of data below both:

$$.6293 - .2514 = .3779$$

3 Problem 4.19

To determine if a data set is normally distributed a Q-Q plot is used. This is created by plotting the Rainfall data score on the y-axis and the z-score (also called the Normal Score) on the x-axis. If the data is normally distributed the data will be aligned on the $y = x$ line. The Q-Q plot below is slightly curved, indicating the distribution of the sample is skewed to the right. It does not appear that the precipitation distribution is normal.



4 Problem 4.24

Setup: $\mu = 7.8$ $\sigma = 2.3$ $\Rightarrow Y \sim N(7.8, 2.3)$

As the data is discrete (whole numbers) rather than continuous and the Normal distribution

is being used to model it, which is a continuous distribution, a continuity correction needs to be used.

4.1

We wish to find the area under the normal curve to the left of $6+.5 = 6.5$

$$z = \frac{6.5-7.8}{2.3} = -.57$$

From Table 3, 28.43% of mice litters consist of 6 mice or less.

4.2

$P(Y = 6)$ means we want to find the area under the normal curve between $6-.5 = 5.5$ and $6+.5 = 6.5$. We already know the z-score and area to the left of 6.5 (.2843) from the previous question.

$$z = \frac{5.5-7.8}{2.3} = -1.00$$

From Table 3 the area is .1587

Thus, $P(Y = 6) = .2843 - .1587 = .1256$

12.56% of mice litters consist of 6 mice.

4.3

$P(8 \leq 11)$ means we want to find the area under the normal curve between $8-.5 = 7.5$ and $11+.5 = 11.5$.

$$z = \frac{11.5-7.8}{2.3} = 1.61$$

From Table 3, the area is .9463.

$$z = \frac{7.5-7.8}{2.3} = -.13$$

From Table 3, the area is .4483

$\Rightarrow P(8 \leq 11) = .9463 - .4483 = .4980$

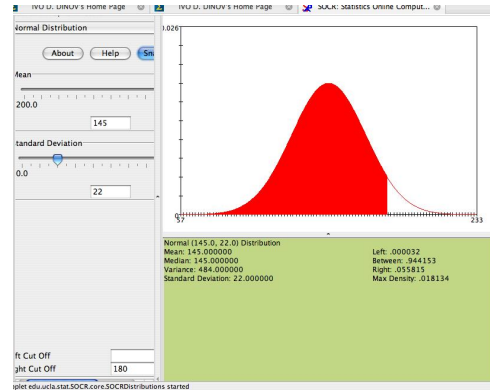
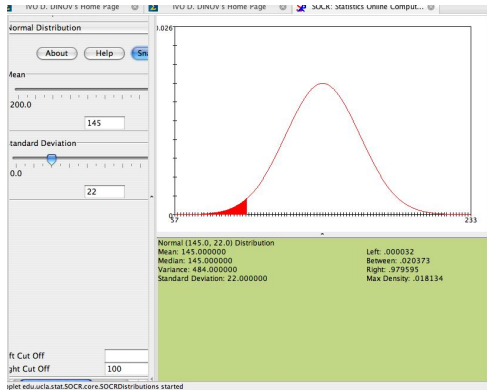
5 Problem 4.29

Setup: $\mu = 145\text{cm}$ $\sigma = 22\text{cm}$ $\Rightarrow N(145, 22)$

5.1

$P(Y \geq 100)$ means we want to find the area under the curve to the right of 100 cm as show in the picture on the right. We can use the z-score and Table 3, which gives us the area under the curve to the left of 100 as show in the picture on the left, to find the answer by subtracting it from one.

$$P(Y \geq 100) = 1 - P(Y \leq 100) = 1 - P(z \leq \frac{100-145}{2.3}) = 1 - P(z \leq -2.05) = 1 - .0202 = .9798$$



5.2

$P(Y \leq 120) = P(z \leq \frac{120-145}{22}) = P(z \leq -1.14) = .1271$
 12.71% of corn plants are shorter than 120 cm.

5.3

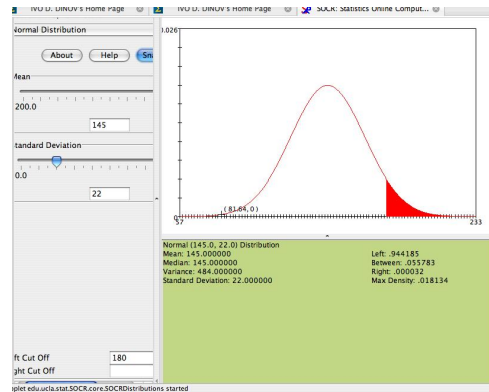
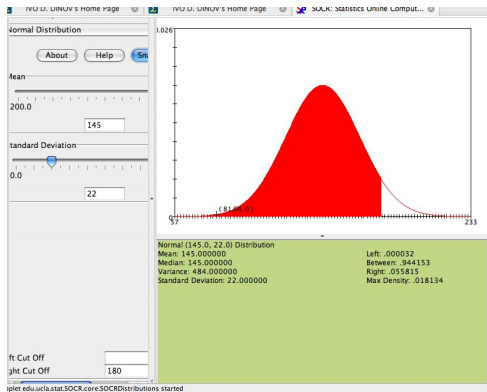
$P(120 \leq Y \leq 150)$
 We already know the z-score and corresponding area for 120 cm.
 $z = \frac{150-145}{22} = .23$
 From Table 3 the area below .23 is .5910.
 $.5910 - .1271 = .4639$
 46.39% of corn plant are between 120 cm and 150 cm in height.

5.4

$P(100 \leq Y \leq 120)$
 We already know the z-score and corresponding area for 100 cm and 120 cm.
 $.1271 - .0202 = .1069$

5.5

$P(Y \geq 180)$
 $z = \frac{180-145}{22} = 1.59$
 From Table 3, the area to the left of 1.59 is .9441, but we are interested in the area to the right (greater than). So we subtract from 1:
 $1 - .9441 = .0559 \Rightarrow 5.59\%$ of corn plants are taller than 180 cm.



5.6

$$P(Y \leq 150)$$

We already know the z-score and corresponding area for 150 cm, .23 and .5910 respectively. Therefore, 59.10% of corn plants are less than 150 cm.

6 Problem 4.41

$$\text{Setup: } \mu = 7.3 \quad \sigma = 11.1 \quad \Rightarrow \quad N(7.3, 11.1)$$

6.1

$$P(Y > 10) = 1 - P(Y < 10) = 1 - P\left(z < \frac{10 - 7.3}{11.1}\right) = 1 - P(z < .24) = 1 - .5937 = .4063$$

40.63% of the change in heart rates is above 10 beats per minute.

6.2

$$P(Y > 20) = 1 - P(Y < 20) = 1 - P\left(z < \frac{20 - 7.3}{11.1}\right) = 1 - P(z < 1.14) = 1 - .8729 = .1271$$

12.71% of the change in heart rates is above 20 beats per minute.

6.3

$$P(5 \leq Y \leq 15) = P(Y \leq 15) - P(Y \leq 5) = P\left(z \leq \frac{15 - 7.3}{11.1}\right) - P\left(z \leq \frac{5 - 7.3}{11.1}\right)$$

$$= P(z \leq .69) - P(z \leq -.21) = .7549 - .4168 = .3381$$

33.81% of the change in heart rates is between 5 and 15 beats per minutes.

