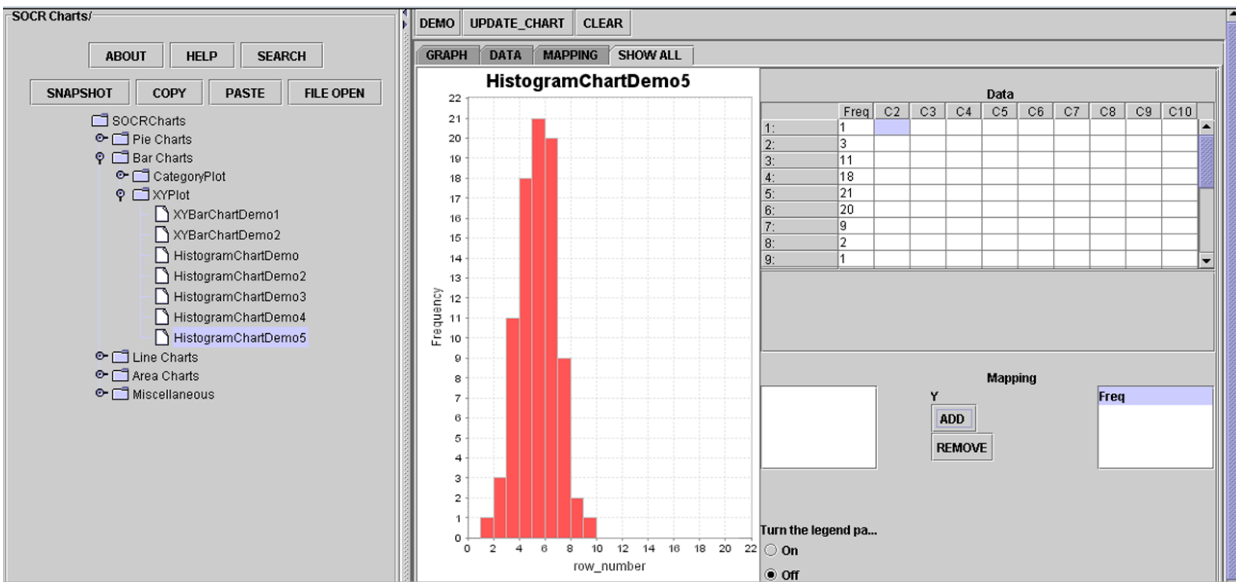
Stats 13.1 Homework 5 Solutions

6.4

Setup: $n = 86 \qquad \bar{y} = 60.43 \qquad s = 3.06$
a) $SE_{\bar{y}} = \frac{s}{\sqrt{n}} = \frac{3.06}{\sqrt{86}} = .33$ mm

b)



6.12

a) Since we are using s instead of σ we will use the T-distribution instead of a z-score.

95% CI $= \bar{y} \pm t(df)_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$

$\bar{y} = 28.7 \qquad s = 4.6 \qquad n = 6 \qquad SE_{\bar{y}} = \frac{4.6}{\sqrt{6}} = 1.9 \qquad t(6-1)_{.05/2} = t(5)_{.025} = 2.571$

$95\%CI = 28.7 \pm (2.571)(1.9) = (23.8, 33.6) \quad \Rightarrow \quad 23.8 < \mu < 33.6 \ \mu g/ml$

We are highly confident (95% confidence) that the true mean of the blood serum concentration of Gentamicin ($\mu g/m$) in three-year-old female Suffolk sheep is between 23.8 and 33.6 $\mu g/m$.

b) The population mean μ is the blood serum concentration of Gentamicin (1.5 hours after injection of 10 mg/kg body weight) in healthy three-year-old female Suffolk sheep.

c) No. The 95% refers to the percentage (in a meta-experiment) of confidence intervals that would contain μ. since the width of a confidence interval depends on n, the percentage of observations contained in the confidence interval also depends on n, and would be very small if n were large.

6.16

We want a 95% CI for the mean difference, so only the numbers in the right column will be used.
95% CI $= \bar{y} \pm t(df)_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$

$a)\bar{y} = 13.0 \quad s = 12.4 \quad n = 10 \quad SE_{\bar{y}} = \frac{12.4}{\sqrt{10}} = 3.92 \quad t(10-1)_{.05/2} = t(9)_{.025} = 2.262$

$95\%CI = 13 \pm (2.262)(3.92) = (4.1, 21.9) \quad \Rightarrow \quad 4.1 < \mu < 21.9 \ pg/ml$

b) We are 95% confident that the average drop in HBE levels from January to May in the population of all participants in physical fitness programs like the one in the study is between 4.1 and 21.9 pg/ml.

6.41

Construct a CI for the proportion $p \quad n = 959 \quad \hat{p} = .157$
For $\hat{p}$
a) $\hat{p} \pm z_{\frac{\alpha}{2}}(SE_{\hat{p}}) \quad z_{\frac{\alpha}{2}} = z_{\frac{.10}{2}} = z_{.05} = 1.645$

$SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{.157(1-.157)}{959}} = .0117$

$.157 \pm (1.645)(.0177) \Rightarrow .1378 < p < .1762$

b) The confidence interval from part (a) is a confidence interval for the probability of interference with the pacemaker for a specific type of cellular phone tested. We are 90% confident that the true population proportion for cell phone interference with pacemakers is between 13.78% and 17.62%.

For $\tilde{p}$

a) $\tilde{p} \pm z_{\frac{\alpha}{2}}(SE_{\tilde{p}})$

$\tilde{p} = \frac{y+0.5\left(z^2_{\frac{\alpha}{2}}\right)}{n+\left(z^2_{\frac{\alpha}{2}}\right)}$

$z_{\frac{\alpha}{2}} = z_{\frac{.10}{2}} = z_{.05} = 1.645$

$y = n * \hat{p} = (959)(.157) = 150.56$ so use y = 151

$\simeq \frac{151+.5(1.645^2)}{959+1.645^2} = .1584$

$SE_{\tilde{p}} = \sqrt{\frac{\tilde{p}(1-\tilde{p})}{n+z^2_{\frac{\alpha}{2}}}} = \sqrt{\frac{.1584(1-.1584)}{959+1.645^2}} = .01178$

$.1584 \pm (1.645^2)(.01178) \Rightarrow .1390 < p < .1778$

b)he confidence interval from part (a) is a confidence interval for the probability of interference with the pacemaker for a specific type of cellular phone tested. We are 90% confident that the true population proportion for cell phone interference with pacemakers is between 13.78% and 17.62%.

6.52 - 95% CI

a) $\bar{y} = 2.275 \quad s = .238 \quad n = 8$
$SE_{\bar{y}} = \frac{s}{\sqrt{n}} = \frac{.238}{\sqrt{2.275}} = .084mm$

b) Since we are using s instead of σ we will use the T-distribution instead of a z-score.

95% CI $= \bar{y} \pm t(df)_{\alpha/2} SE_{\bar{y}} \qquad t(8-1)_{.05/2} = t(7)_{.025} = 2.365$

$2.275 \pm (2.365)(.084) = (2.08, 2.47) \quad \Rightarrow \quad 2.08 < \mu < 2.47$

c) We are 95% confident that the average diameter of the stem of a wheat plant three weeks after flowering is between 2.08 and 2.47 mm.

d) We want a sample size, n, that will give us a margin of error (ME) half the size of the one found in part (b).
$ME = t_{.05}\left(\frac{s}{\sqrt{n}}\right)$ By scanning the .025 column (95% CI) of the t-table you can see that the t-statistics are approximately 2.

$ME_{old} = (2.365)(0.84) = .199 \quad \Rightarrow$

$ME_{new} = \frac{ME_{old}}{2} = \frac{.199}{2} = .0995 \quad \Rightarrow \quad .0995 = (2)\left(\frac{.238}{\sqrt{n}}\right) \quad \Rightarrow \quad \sqrt{n} = \frac{(2)(.238)}{.0995} \quad \Rightarrow \quad n = \left(\frac{(2)(.238)}{.0995}\right)^2 = 22.89 \approx 23$

To have a margin of error half the size of the one found in part (b) or smaller a sample size of 23 or larger is needed.

6.52 - 98% CI

a) $\bar{y} = 2.275 \quad s = .238 \quad n = 8$
$SE_{\bar{y}} = \frac{s}{\sqrt{n}} = \frac{.238}{\sqrt{2.275}} = .084$

b) Since we are using s instead of σ we will use the T-distribution instead of a z-score.

98% CI $= \bar{y} \pm t(df)_{\alpha/2} SE_{\bar{y}} \qquad t(8-1)_{.02/2} = t(7)_{.01} = 2.998$

$2.275 \pm (2.998)(.084) = (2.02, 2.53) \quad \Rightarrow \quad 2.02 < \mu < 2.53$

c) We are 98% confident that the average diameter of the stem of a wheat plant three weeks after flowering is between 2.02 and 2.53 mm.

d) We want a sample size, n, that will give us a margin of error (ME) half the size of the one found in part (b).
$ME = t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$ By scanning the .01 column (98% CI) of the t-table you can see that the t-statistics are approximately 2.5.

$ME_{old} = (2.998)(0.84) = .252 \quad \Rightarrow$

$ME_{new} = \frac{ME_{old}}{2} = \frac{.252}{2} = .126 \quad \Rightarrow \quad .126 = (2.5)\left(\frac{.238}{\sqrt{n}}\right) \quad \Rightarrow \quad \sqrt{n} = \frac{(2.5)(.238)}{.126} \quad \Rightarrow \quad n = \left(\frac{(2.5)(.238)}{.126}\right)^2 = 22.3 \approx 23$

To have a margin of error half the size of the one found in part (b) or smaller a sample size of 36 or larger is needed.

6.64

a) $\bar{y} = \frac{1}{n}\sum_{i=1}^{6} y_i = 62.767 \qquad s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{6}(y_i - \bar{y})^2} = 1.01127$
$SE_{\bar{y}} = \frac{s}{\sqrt{n}} = \frac{1.01127}{\sqrt{6}} = .4128$

b) 90% CI $= \bar{y} \pm t(df)_{\alpha/2} SE_{\bar{y}} \qquad t(6-1)_{.1/2} = t(5)_{.05} = 2.015$

$62.767 \pm (2.015)(.41) = (61.94, 63.60) \quad \Rightarrow \quad 61.94 < \mu < 63.60\%$

6.69

For $\tilde{p}$

$SE_{\tilde{p}} = \sqrt{\frac{\tilde{p}(1-\tilde{p})}{n+z^2_{\frac{\alpha}{2}}}} \Rightarrow \sqrt{\frac{.45(.55)}{n+2^2}} \leqslant .02 \Rightarrow \frac{.45*.55}{.02^2} - 4 \leqslant 614.75$

615 or more students need to be sampled to have a standard of error less than 2 percentage points.

For $\hat{p}$

$SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \Rightarrow \sqrt{\frac{.45(.55)}{n}} \leqslant .02 \Rightarrow \frac{.45*.55}{.02^2} \leqslant 618.75$

619 or more students need to be sampled to have a standard of error less than 2 percentage points.