Stats 13.1 HW 2 Solutions

October 10, 2007

1 3.7

a) The probability that both are affected is 0% because female offspring have no chance of getting the disease.

b) The probability that only one sibling is affected can be written as:

$$
\begin{aligned}
\text{P(one child affected)} &= \text{P(male affected and female not)} + \text{P(female affected and male not)} \\
&= 50\% \cdot 100\% + 0\% \cdot 50\% \\
&= 50\%
\end{aligned}
$$

2 3.8

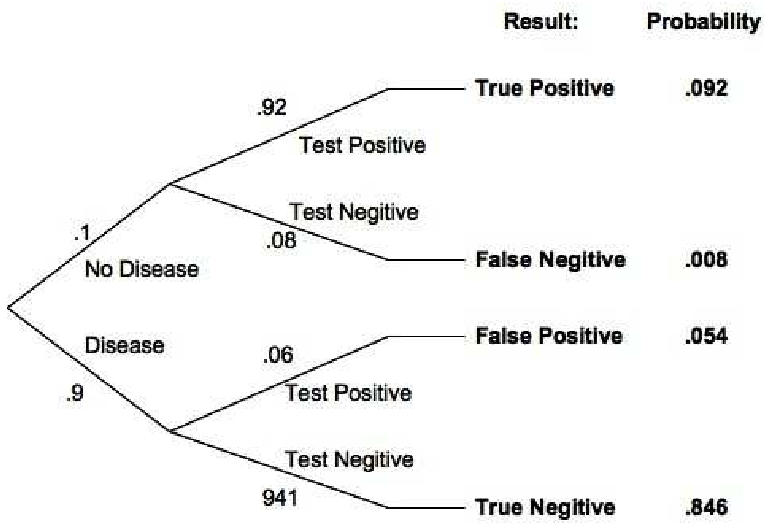
a)

$$
\begin{aligned}
\text{P(get question right)} &= \text{P(studied the question)} + \text{P(didn't study the question \& got it right)} \\
&= 40\% + 60\% \cdot 20\% \\
&= 52\%
\end{aligned}
$$

b)

$$
\begin{aligned}
\text{P(studied it — got it right)} &= \frac{\text{P(studied question \& got it right)}}{\text{P(got it right)}} \\
&= \frac{40\%}{52\%} \\
&= 76.9\%
\end{aligned}
$$

3 3.11

a) 

There are two ways to test positive, a true positive and a false positive. P(test positive) = P(true positive) + P(false positive) = 0.146

b)
$$\begin{aligned}
\text{P(has disease — test positive)} &= \frac{\text{P(has disease \& test positive)}}{\text{P(test positive)}} \\
&= \frac{\text{P(true positive)}}{\text{P(test positive)}} \\
&= \frac{0.092}{0.146} \\
&= 0.63
\end{aligned}$$

4 3.14

The law of independence states:

A and B are independent if and only if P(A+B) = P(A)·P(B).

But,

$$\begin{aligned}
\text{P(husband and wife smoke)} &=? \ \text{P(husband smokes)}\cdot\text{P(wife smokes)} \\
8\% &=? \ 30\% \cdot 20\% \\
8\% &\neq \ 6\%
\end{aligned}$$

So the smoking status of the husband is not independet of the status of the wife.

5 3.18 and 3.20

a)
$$\begin{aligned}
P(Y = 3) &= \frac{\text{\# of broods with } Y = 3}{\text{total \# of broods}} \\
&= \frac{610}{5000} \\
&= 0.122
\end{aligned}$$

b)

$$\begin{aligned} P(Y \leq 7) &= \frac{\text{\# of broods with } Y \leq 3}{\text{total \# of broods}} \\ &= \frac{130+26+3+1}{5000} \\ &= 0.032 \end{aligned}$$

c)

$$\begin{aligned} P(4 \leq Y \leq 6) &= \frac{\text{\# of broods with } 4 \leq Y \leq 6}{\text{total \# of broods}} \\ &= \frac{1400+1760+750}{5000} \\ &= 0.782 \end{aligned}$$

d)

$$\begin{aligned} \text{mean} &= \sum_{Y=1}^{10} Y \cdot (\text{proportion of nests with brood size Y}) \\ &= 1 \cdot \frac{90}{5000} + 2 \cdot \frac{230}{5000} + 3 \cdot \frac{610}{5000} + 4 \cdot \frac{1400}{5000} + 5 \cdot \frac{1760}{5000} + 6 \cdot \frac{750}{5000} \\ &\quad + 7 \cdot \frac{130}{5000} + 8 \cdot \frac{26}{5000} + 9 \cdot \frac{3}{5000} + 10 \cdot \frac{1}{5000} \\ &= 4.487 \end{aligned}$$

6 3.12 and 3.22

a)

$$\begin{aligned} P(Y \geq 2) &= P(Y = 2) + P(Y = 3) \\ &= 0.189 + 0.027 \\ &= 0.973 \end{aligned}$$

b)

$$\begin{aligned} P(\ Y \leq 2) &= P(Y = 0) + P(Y = 1) + P(Y = 2) \\ &= 0.343 + 0.441 + 0.189 \\ &= 0.973 \end{aligned}$$

-or-

$$\begin{aligned} P(\ Y \leq 2) &= 1 - P(Y = 3) \\ &= 1 - 0.027 \\ &= 0.973 \end{aligned}$$

c)

$$\begin{aligned} \text{mean} &= \sum_{y=0}^{3} y \cdot P(\text{\# of black flies} = y) \\ &= 0 \cdot 0.343 + 1 \cdot 0.441 + 2 \cdot 0.189 + 3 \cdot 0.027 \\ &= 0.9 \end{aligned}$$

7 3.28

a) P(all 20 will be cured)
$= P(1^{st}$ will be cured$) \cdot P(2^{nd}$ will be cured$) \cdot \ldots \cdot P(20^{th}$ will be cured$)$
$= 0.9^{20} = 0.1216$

b) P(all but 1 is cured)
$= P($all but the 1^{st} will be cured$) \cdot P($all but the 2^{nd} will be cured$) \cdot \ldots \cdot P($all but the 20^{th} will be cured$)$
$= 0.1 \cdot 0.9^{19} + 0.9 \cdot 0.1 \cdot 0.9^{18} + \ldots + 0.9^{19}\ cdot 0.1$
$= (20) \cdot 0.1 \cdot 0.9^{19} = 0.2702$

c) P(exactly 18 are cured) = (# of ways to chooose the 2 uncured children)$\cdot 0.1^2 \cdot 0.9^{18}$
$= 20 \cdot C_2 \cdot 0.1^2 \cdot 0.9^{18} = 190 \cdot 0.1^2 \cdot 0.9^{18} = 0.2852$

d) P(90% will be cured) = $P(0.9 \cdot 20$ will be cured$)$ = P(18 will be cured) = 0.2852 (part c)

8 3.34

a) Let us consider a success as "high blood lead level." Then we have a binomial problem with n = 16 and p = 1/8. Then,

P(none have high blood lead level)
= P(no successes)
= ${}_{16}C_0 \cdot \left(\frac{7}{8}\right)^{16} = 0.1181$

b) P(one has high blood lead level)
= P(1 success)
= ${}_{16}C_1 \cdot \left(\frac{7}{8}\right)^{15} \cdot \left(\frac{1}{8}\right)^{1} = 0.2699$

c) P(two have high blood lead level)
= P(2 successes)
= ${}_{16}C_2 \cdot \left(\frac{7}{8}\right)^{14} \cdot \left(\frac{1}{8}\right)^{2} = 0.2891$

d) P(three or more have high blood lead level)
= 1 - P(2 or fewer have high blood lead level)
= 1 - (P(0 have high blood lead level) + P(1 have high blood lead level) + P(2 have high blood lead level))
= 1 - (0.1181 + 0.2699 + 0.2891)
= 0.3229