Stat 13.1 F07 HW 4 Solutions

5.3

Let the number of successes in 5 be represented by the random variable X

- $\hat{p}$=0, therefore we have 0 successes out of 5 trials.

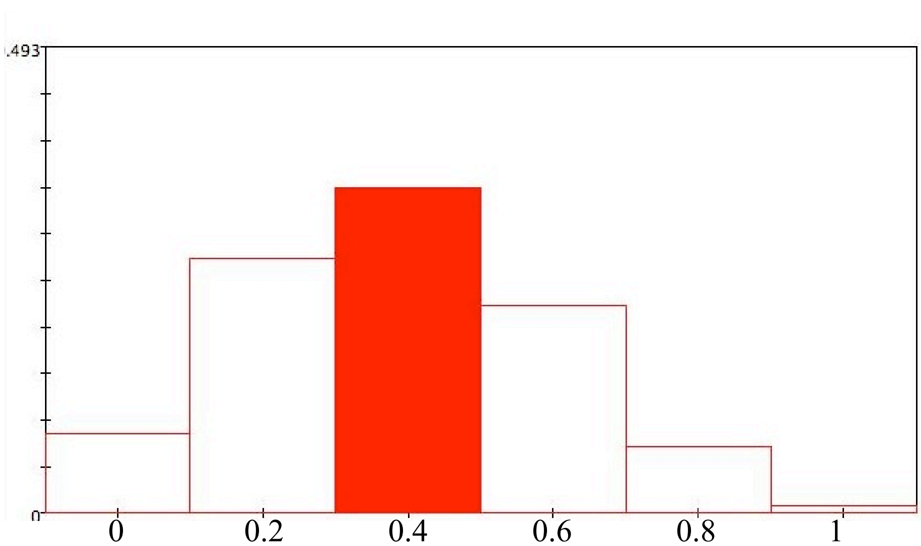
  P($\hat{p}$=0) = P(X=0) = . 077760
- $\hat{p}$=0.2, therefore we have 1 successes out of 5 trials.

  P($\hat{p}$=1) = P(X=1) = .259200
- $\hat{p}$=0.4, therefore we have 2 successes out of 5 trials.

  P($\hat{p}$=2) = P(X=2) = .345600
- $\hat{p}$=0.6, therefore we have 3 successes out of 5 trials.

  P($\hat{p}$=3) = P(X=3) = . 230400
- $\hat{p}$=0.8, therefore we have 4 successes out of 5 trials.

  P($\hat{p}$=4) = P(X=4) = .076800
- $\hat{p}$=1, therefore we have 5 successes out of 5 trials.
- P($\hat{p}$=5) = P(X=5) = . 010240

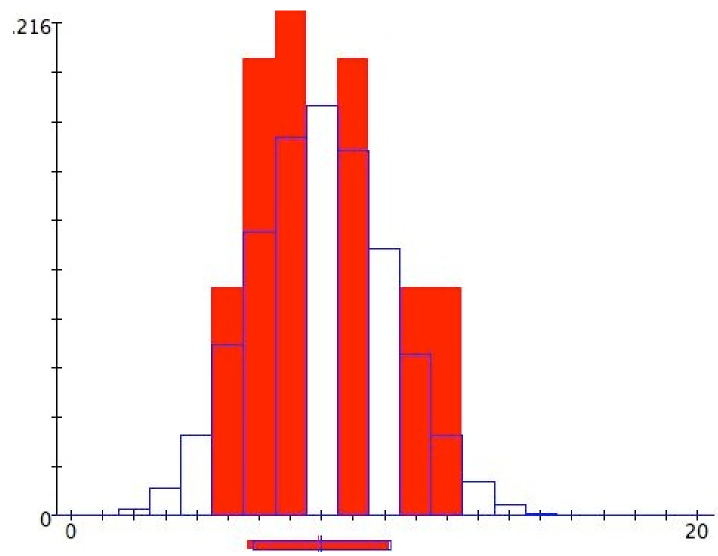


$\hat{p}$

Note: this is just a histogram. Ignore the filled-in box.

My Binomial Coin Experiment Results:

X	Theoretical	Empirical
1	0.00049	0
2	0.00309	0
3	0.01235	0
4	0.03499	0
5	0.07465	0.1
6	0.12441	0.2
7	0.16588	0.3
8	0.17971	0
9	0.15974	0.2
10	0.11714	0
11	0.07099	0.1
12	0.0355	0.1
13	0.01456	0
14	0.00485	0
15	0.00129	0
16	0.00027	0
17	0.00004	0
18	0	0
9	0	0
20	0	0
Mean	8	7.9



- The empirical values in col 3 do not look like the theoretical values in col 2. Many of the empirical values are 0 and the non-zero values are much larger than the theoretical values. They are not close at all.
- With only ten runs, there are only 10 data points to create the empirical distribution, which has 21 possibilities for X (0 through 20 successes). If we increased the number of flips for each run to 100, but still only did 10 runs, the empirical distribution would look even worse because we would now have only 10 runs with 101 possibilities.
- If we instead did 100 runs of 20 flips each, we would now have 100 data points for our empirical distribution histogram, therefore it should much more closely resemble the theoretical distribution.

5.16

$\mu: 3000 \quad \sigma: 400$ Note, E occurs if 2900 < $\overline{Y}$ <3100

a) n=15

$\mu_{\overline{Y}}: 3000 \quad \sigma_{\overline{Y}}: 400/\sqrt{15} = 103.3$

Using SOCR: P(E) = .666983

By hand:

$$z_L = \frac{\overline{y}_L - \mu_{\overline{Y}}}{\sigma_{\overline{Y}}} = \frac{2900 - 3000}{103.3} = -.97$$

$$z_U = \frac{\overline{y}_U - \mu_{\overline{Y}}}{\sigma_{\overline{Y}}} = \frac{3100 - 3000}{103.3} = .97$$

$$P(Z \le z_L) = .1660$$
$$P(Z \le z_U) = .8340$$
$$P(z_L \le Z \le z_U) = .8340 - .1660 = .6680$$
$$\therefore P(\overline{y}_L \le \overline{Y} \le \overline{y}_U) = .6680$$

b) n=60

$\mu_{\overline{Y}}: 3000 \quad \sigma_{\overline{Y}}: 400/\sqrt{60} = 51.64$

Using SOCR: P(E) = .947191

By hand:

$$z_L = \frac{\overline{y}_L - \mu_{\overline{Y}}}{\sigma_{\overline{Y}}} = \frac{2900 - 3000}{51.64} = -1.94$$

$$z_U = \frac{\overline{y}_U - \mu_{\overline{Y}}}{\sigma_{\overline{Y}}} = \frac{3100 - 3000}{51.64} = 1.94$$

$$P(Z \le z_L) = .0262$$
$$P(Z \le z_U) = .9738$$
$$P(z_L \le Z \le z_U) = .9738 - .0262 = .9476$$
$$\therefore P(\overline{y}_L \le \overline{Y} \le \overline{y}_U) = .9476$$

c) As n increases, P(E) increases as well.

5.38

Let E be the event that $\hat{p}$ is closer to ½ than 9/16

a) n=1. E occurs if the number of purple plants is zero, therefore P(E)=7/16=.4375

b) n=64.

$\frac{1}{2}64 = 32$

$\frac{9}{16}64 = 36$

Therefore, if we have a number of plants less than or equal to 33, $\hat{p}$ will be closer to ½ than 9/16.

The normal approximation to the binomial has:

$\mu = np = 64\frac{9}{16} = 36$

$\sigma = \sqrt{np(1-p)} = \sqrt{64\frac{9}{16}\frac{7}{16}} = 3.969$

Then if we let X be the number of purple plants:

$P(X \le 33) \approx P(Y \le 33)$

Where $Y \sim N(36, 3.969)$

and

$$P(Y \le 33) = P\left(\frac{Y-36}{3.969} \le \frac{33-36}{3.969}\right)$$

$$= P(Z \le -.76) = .2236 = P(E)$$

c) n=320.

$\frac{1}{2}320 = 160$

$\frac{9}{16}320 = 180$

Therefore, if we have a number of plants less than or equal to 169, $\hat{p}$ will be closer to ½ than 9/16.

The normal approximation to the binomial has:

$\mu = np = 320\frac{9}{16} = 180$

$\sigma = \sqrt{np(1-p)} = \sqrt{320\frac{9}{16}\frac{7}{16}} = 8.874$

Then if we let X be the number of purple plants:

$P(X \le 169) \approx P(Y \le 169)$
Where $Y \sim N(180, 8.874)$
and
$P(Y \le 169) = P\left(\frac{Y-180}{8.874} \le \frac{169-180}{8.874}\right)$
$= P(Z \le -1.24) = .1075 = P(E)$

5.51

$\mu = 8.3$
$\sigma = 1.7$
$\bar{y}^* = {90g}/{10\text{mice}} = 9g$
So we want :
$P(\bar{Y} \ge \bar{y}^*)$
We also know :
$\sigma_{\bar{Y}} = {1.7}/{\sqrt{10}} = .538$
$\bar{Y} \sim N(8.3, .538)$
$\therefore$
$P(\bar{Y} \ge 9g) = 1 - P(\bar{Y} \le 9g)$
$= 1 - P\left(\frac{\bar{Y}-8.3}{.538} \le \frac{9-8.3}{.538}\right)$
$= 1 - P(Z \le 1.3)$
$= 1 - .9032$
$= .0968 \text{ or } 9.68\%$

Using SOCR: $P(\bar{Y} \ge 9g)$ = .09661