Homework 7

7.51

Let 1 denote experimental (to be hypnotized) and 2 denote control.
a) H_0: Mean ventilation is the same in the to be hypnotized condition and in the control condition. $\mu_1 = \mu_2$
H_A: Mean ventilation is different in the to be hypnotized condition and in the control condition. $\mu_1 \neq \mu_2$

The Standard Error of the difference is:

$$SE_{\bar{y}_1 - \bar{y}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{.621^2}{8} + \frac{.652^2}{8}} = .3183$$

The test statistic is then:

$$t_s = \frac{\bar{y}_1 - \bar{y}_2}{SE_{\bar{y}_1 - \bar{y}_2}} = \frac{6.169 - 5.291}{.3183} = 2.76$$

With df $= n_1 + n_2 - 2 = 14$, Table 4 gives $t(14)_{.01} = 2.624$ and $t(14)_{.005} = 2.977$. Therefore: $.01 < p - value < .02$
(These bracketing numbers are doubled from the table because this is a two tailed test.)
This means that the $p - value < \alpha$ therefore we reject H_0. There is sufficient evidence ($.01 < p - value < .02$) to conclude that the mean ventilation is higher in the "to be hypnotized" condition than in the control group.

b) H_0: Mean ventilation is the same in the to be hypnotized condition and in the control condition. $\mu_1 = \mu_2$
H_A: Mean ventilation is higher in the to be hypnotized condition and in the control condition. $\mu_1 > \mu_2$

We use the same SE and test statistic, neither is changed by changing the alternative hypothesis. The only difference is this is now a one-tailed test instead of a two-tailed test.
Therefore: $.005 < p - value < .01$

This means that the $p - value < \alpha$ therefore we reject H_0. There is sufficient evidence ($.005 < p - value < .01$) to conclude that the mean ventilation is higher in the "to be hypnotized" condition than in the control group.

c) The nondirectional alternative (part (a)) is more appropriate. According to the narrative, the researchers formulated the directional alternative in part (b) *after* they had seen the data. Thus, it wold not be legitimate for them (or us) to use a directional alternative.

7.52

Let 1 denote standard nitrogen and 2 denote extra nitrogen
H_0: Extra nitrogen does not enhance plant growth ($\mu_1 = \mu_2$)
H_A: Extra nitrogen does enhance plant growth ($\mu_1 < \mu_2$)

$$SE_{\bar{y}_1-\bar{y}_2} = \sqrt{\tfrac{.54^2}{5} + \tfrac{.67^2}{5}} = .3848$$

$$t_s = \tfrac{\bar{y}_1-\bar{y}_2}{SE_{\bar{y}_1-\bar{y}_2}} = \tfrac{3.62-4.17}{.3848} = -1.43$$

With df = 5+5-2 = 8, Table 4 gives $t_{.10} = 1.397$ and $t_{.05} = 1.860 \quad \Rightarrow \quad .05 < p{-}value < .10$

H_0 is not rejected because the $p - value > \alpha$. There is insufficient evidence ($.05 < p - value < .10$) to conclude that extra nitrogen enhances plant growth under these conditions.

7.59

Let 1 denote male and 2 denote female.

$$(\bar{y}_1 - \bar{y}_2) \pm t(df)_{\frac{\alpha}{2}} SE_{\bar{y}_1-\bar{y}_2}$$

$$t(498)_{\frac{.05}{2}} \approx t(140)_{.025} = 1.977 \qquad \text{(140 df is the closest lower number to 498 on Table 4)}$$

$$SE_{\bar{y}_1-\bar{y}_2} = \sqrt{SE_1^2 + SE_2^2} = \sqrt{.62^2 + .53^2} = .8157$$
$$(137.21 - 137.18) \pm (1.977)(.1857) \quad \Rightarrow \quad -1.6 < \mu_1 - \mu_2 < 1.6 \text{ beats per minute}$$

We can be 95% confident that the mean difference does not exceed 1.6 beats per minute, which is small and unimportant (in comparison with, for example, ordinary fluctuations in heart rate from one minute to the next).

7.66

The effective size is the difference in population means expressed relative to the common population standard deviation. The difference we are interested in detecting is a change in stem length of 2cm

effect size $= \frac{\mu_1 - \mu_2}{\sigma} = \frac{2}{.8} = 2.5$

(a) We go to Table 5, look up the effect size of 2.5, go across to section of the one tailed $\alpha = .05$ and look for the 90% power column (which is 95 because this is a one tailed test). The table gives n=5.
(b) The required conditions are that the sampled populations are normal with equal standard deviations. The condition of normality can be checked from the pilot data.
(c) Following the same procedure in (a) but going to the $\alpha = .01$ section gives n = 7.

7.79

(a)
H_0: Toluene has no effect on dopamine in rat striatum
H_A: Toluene has some effect on dopamine in rat striatum
Let 1 denote toluene and let 2 denote control.
The ordered arrays of observations are as follows:

Y_1	Y_2
	1397
	1802
	1820
	1843
1811	
	1990
2314	
2464	
	2539
2781	
2803	
3420	

For the K_1 counts we can see from the table that there are 4 Y_2's less than the first Y_1; there are five Y_2's less than the second Y_1; there are five Y_2's less than the third Y_1; etc.

$K_1 = 4 + 5 + 5 + 6 + 6 + 6 = 32$
$K_2 = 0 + 0 + 0 + 1 + 3 = 4$
To check the counts, we verify that:
$K_1 + K_2 = 32 + 4 = 36 = (6)(6) = (n_1)(n_2)$

As this is a non-directional test, the Wilcoxon-Mann-Whitney test statistics is the larger of the two counts, thus $U_s = 32$. Looking in Table 6 under n= 6 and n' = 6, we find that for a nondirectional alternative, the .05 entry is 31 and the .02 entry is 33. Thus, the p-value is bracketed as:
$.02 < p - value < .05$

At the significance level $\alpha = .05$, we reject H_0, since p-value$<$ .05. We note K_1 is larger than K_2, which indicates a tendency for the Y_1's to be larger than the Y_2's. Thus, there is sufficient evidence to conclude that the toluene increases dopamine in rat striatum.

(b) When conducting a non-directionality test, we must check directionality. In this case we note that K_1 is larger than K_2, which indicates a tendency for the Y_1's to be larger than the Y_2's, which is what the directional alternative predicts. We proceed as in part (a), except that we use the "directional" tail probabilities. Thus, $.01 < p - value < .025$. We reject H_0 and conclude that there is sufficient evidence to conclude that toluene increases dopamine in rat striatum.

7.84: n1 = 15, n2 = 11

Let 1 denote joggers and let 2 denote fitness program entrants.
H_0: There is no difference in resting blood concentration of HBE between joggers fitness and program entrants
H_A: There is difference in resting blood concentration of HBE between joggers fitness and program entrants

$K_1 = 71.5 \qquad K_2 = 93.5 \qquad U_s = K_2 = 93.5$
With n= 15 and n' = 11, 108 is under the .20 heading for a nondirectional alternative and is the smallest entry listed. Thus $p - value > .20$ and H_0 is not rejected. There is insufficient evidence to conclude that there is a difference in resting blood concentration of HBE between joggers and fitness program entrants.

7.84: n1 = 14, n2 = 12

Let 1 denote joggers and let 2 denote fitness program entrants.
H_0: There is no difference in resting blood concentration of HBE between joggers fitness

and program entrants

H_A: There is difference in resting blood concentration of HBE between joggers fitness and program entrants

$K_1 = 70.5 \qquad K_2 = 97.5 \qquad U_s = K_2 = 97.5$

With n= 14 and n' = 12, a larger number is under the .20 heading for a nondirectional alternative and is the smallest entry listed. Thus $p - value > .20$ and H_0 is not rejected. There is insufficient evidence to conclude that there is a difference in resting blood concentration of HBE between joggers and fitness program entrants.

7.90

Let 1 denote control and 2 denote stress

H_0: Stress has no effect on growth ($\mu_1 = \mu_2$)

H_A: Stress tends to retard growth ($\mu_1 > \mu_2$)

$$SE_{\bar{y}_1 - \bar{y}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{2.13^2}{13} + \frac{1.73^2}{13}} = .7611$$

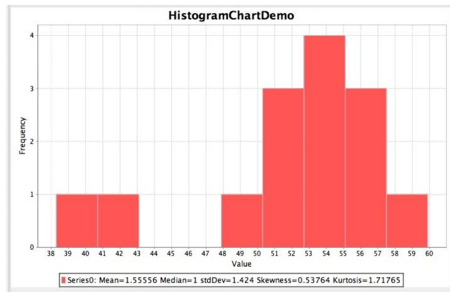
$$t_s = \frac{\bar{y}_1 - \bar{y}_2}{SE_{\bar{y}_1 - \bar{y}_2}} = \frac{30.59 - 27.78}{.7611} = 3.69.$$

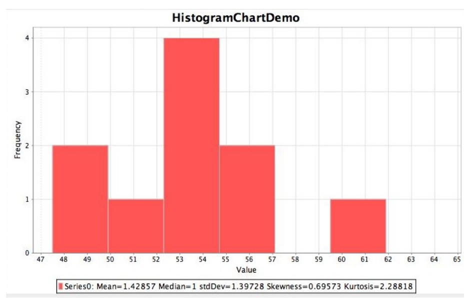
With df = 24, Table 4 give $t_{.005} = 2.797$ and $t_{.0005} = 3.745$. Thus, $.0005 < p - value < .005$, so we reject H_0.

(b) There is sufficient evidence ($.0005 < p - value < .005$) to conclude that stress tends to retard plant growth.

7.99

(a) Designed experiment
(b)





The low chromium diet is skewed left and the normal diet skewed right. Both have potential outliers in the long tails and they appear to be opposite of each other. The variances are similar even though the distributions are skewed in the opposite directions.

(c) Let 1 denote low chromium and 2 denote normal
H_0: Low chromium diet does not affect GITH ($\mu_1 = \mu_2$)
H_A: Low chromium diet does affect GITH ($\mu_1 > \mu_2$)

$\bar{y}_1 = 51.75 \quad s_1 = 5.526 \qquad \bar{y}_2 = 53.17 \quad s_2 = 4.123$

$SE_{\bar{y}_1 - \bar{y}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{5.526^2}{14} + \frac{4.123^2}{10}} = 1.970$

$t_s = \frac{\bar{y}_1 - \bar{y}_2}{SE_{\bar{y}_1 - \bar{y}_2}} = \frac{5.526 - 4.123}{1.97} = -.72.$

With df = 22, $t(22)_{.20} = .858$, so $p - value > .4$. Thus we do not reject H_0. There is insufficient evidence (p-value > .40) to conclude that low chromium diet affects GITH in rats.

(d) $t(22)_{.025} = 2.074$

$(\bar{y}_1 - \bar{y}_2) \pm t(df)_{\frac{\alpha}{2}} SE_{\bar{y}_1 - \bar{y}_2} = (5.526 - 4.123) \pm (2.074)(1.97)$
$\Rightarrow \quad -2.683 < \mu_1 - \mu_2 < 5.489$

If the confidence interval contains 0 we fail to reject H_0, as was seen in part (c).

7.103

(a) Let 1 denote amphetamine and 2 denote control
H_0: Amphetamine is not related to water consumption ($\mu_1 = \mu_2$)
H_A: Amphetamine is associated with decreased water consumption ($\mu_1 < \mu_2$)

$\bar{y}_1 = 123.1 \quad s_1 = 16.23 \qquad \bar{y}_2 = 156.0 \quad s_2 = 25.322$

$SE_{\bar{y}_1 - \bar{y}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{16.23^2}{4} + \frac{25.322^2}{4}} = 15.04$

$t_s = \frac{\bar{y}_1 - \bar{y}_2}{SE_{\bar{y}_1 - \bar{y}_2}} = \frac{129.375 - 156}{15.04} = -2.19.$

With df = 6, Table 4 gives $t_{.05} = 1.943$ and $t_{.025} = 2.447$, so $.025 < p - value < .05$. Thus we reject H_0. There is sufficient evidence ($.025 < p - value < .05$) to conclude that amphetamine is associated with decreased water consumption.
(b) H_0: Amphetamine is not related to water consumption
H_A: Amphetamine is associated with decreased water consumption

$K_1 = 2, K_2 = 14, U_s = 14$; the data deviate from H_0 in the direction specified by H_A. With n = 4, n' = 4, and a directional alternative, the entry 14 is under the .20 heading. Thus, $p - value > .20$ and we do not reject H_0. There is insufficient evidence to conclude that amphetamine is associated with decreased water consumption.