STAT 35A HW3 Solutions

http://www.stat.ucla.edu/~dinov/courses_students.dir/08/Spring/STAT35.dir

Problem 1

a. How many ways are there to select a sample of 5 buses from the 20 for a thorough inspection?
   ${}_{20}C_5 = 15504$
b. In how many ways can a sample of 5 buses contain exactly 4 with visible cracks?
   $12 \times {}_8C_4 = 840$
c. If a sample of 5 buses is chosen at random, what is the probability that exactly 4 of the 5 will have visible cracks?
   840 / 15504 = 0.0542
d. If buses are selected as in part (c), what is the probability that at least 4 of those selected will have visible cracks?
   $(840 + {}_8C_5) / 15504 = 0.0578$

Problem 2

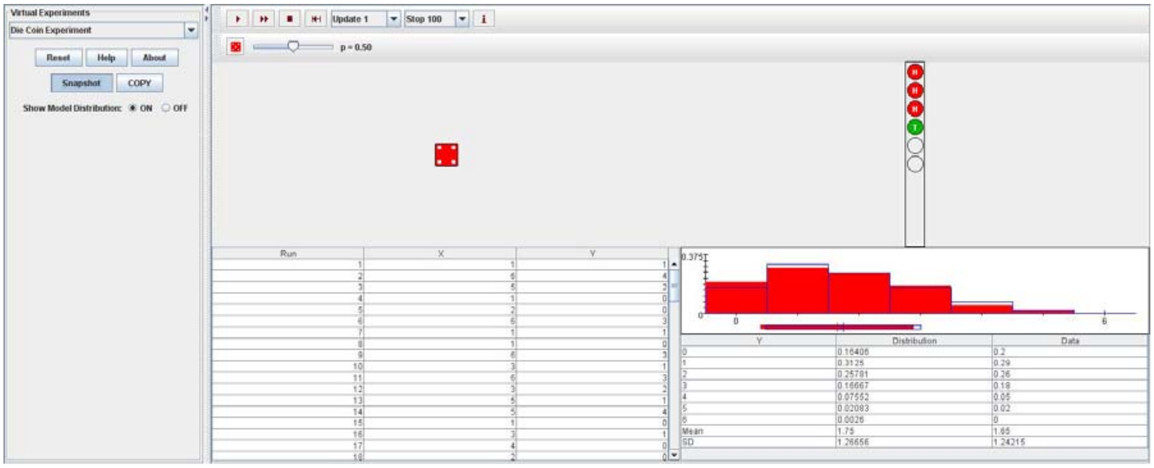
a. How many such chain molecules are there?
   $12! / (3! \times 3! \times 3! \times 3!) = 369600$
b. Suppose a chain molecule of the type desired is randomly selected. What is the probability that all three molecules of each type end up next to one another (such as BBBAAADDDCCC)?
   $4!/ 369600 = 0.00006494$

Problem 3

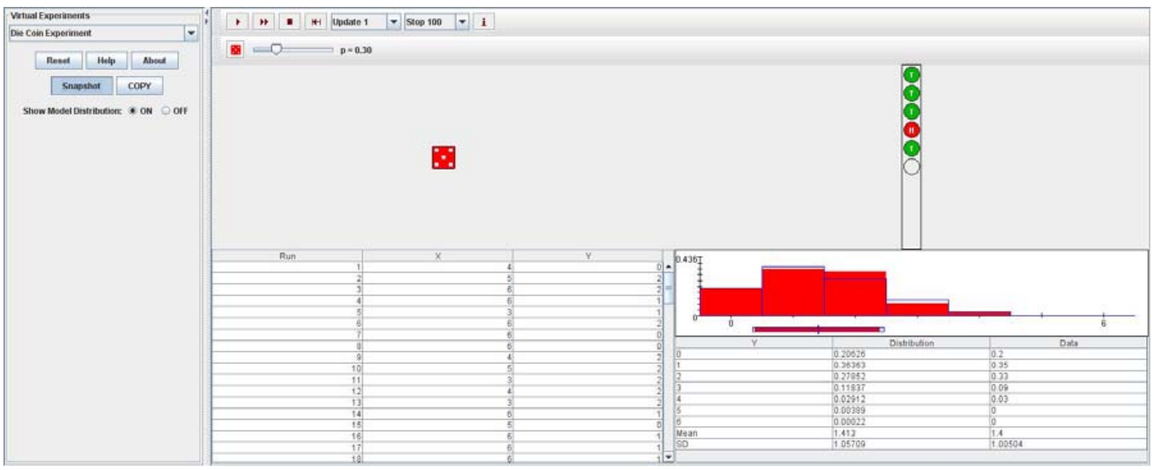
a. Calculate P(A), P(B), and P(A ∩ B).
   P(A) = 0.45, P(B) = 0.25, P(A ∩ B) = 0.10.
b. Calculate both P(A | B) and P(B | A), and explain in context what each of these probabilities represent.
   P(A | B) = 0.4, probability that a black car has automatic transmission.
   P(B | A) = 0.22, probability that a car with automatic transmission is black.
c. Calculate and interpret P(A | C) and P(A | C').
   P(A | C) = 0.5, probability that a white car has *automatic transmission.*
   P(A | C') = 0.43, probability that a car that is not white has automatic transmission.

Problem 4

a. Suppose D3 = event { Die turns up a three (first part of the experiment) } and 2H = event { There are two Heads observed in the tossed coins (second part of the experiment) }. Run the experiment 100 times first with flat-die-probabilities (p=1/6) and a fair-coin (p=0.5). By counting outcomes of interest validate computationally that: P(D3 | 2H) = [P(2H | D3) P(D3)] / P(2H)
   Answer varies.
   In my trial, I obtained P(D3 | 2H) = 0.2308, P(2H | D3) = 0.3158, P(D3) = 0.19, P(2H) = 0.26, which validates the Bayes rule if you plug in the numbers.



b. Now run the experiment 100 times first with a loaded die with probabilities (p1= 0.01, p2=0.05, p3=0.1, p4=0.2, p5=0.34, p6=0.3) and a loaded coin (p=0.3). Again by counting outcomes of interest validate computationally that: P(D3 | 2H) = [P(2H | D3) P(D3)] / P(2H)



Answer varies.

Problem 5

Show empirically the following approximations: Normal to Binomial, Binomial to Hypergeometric, Normal to Poisson and Poisson to Binomial. Use SOCR Distributions and SOCR Experiments. Can you find empirical situations when these approximations fail? Please provide specifics, snapshots, try to explain why, etc. Follow the specific instructions listed here: www.stat.ucla.edu/~dinov/courses_students.dir/08/Spring/STAT35.dir/HWs.dir/HW3.html.

Follow the examples provided on the links.

Normal to Binomial fails when n is small.

Binomial to Hypergeometic fails when population size is small, or number of good objects/population is close to 0 or 1.

Normal to Poisson fails when λ is small.

Poisson to Binomial fails when n is small or p is large or np>10.