

STAT 35A HW4 Solutions

http://www.stat.ucla.edu/~dinov/courses_students.dir/08/Spring/STAT35.dir

Problem 1

- a. $E(X)$, the expected value of the R.V. X .

$$E(X) = 0.08 \times 0 + 0.15 \times 1 + 0.45 \times 2 + 0.27 \times 3 + 0.053 \times 4 = 2.06$$

- b. $V(X)$, the variance of X .

$$V(X) = 0.08 \times (0 - 2.06)^2 + 0.15 \times (1 - 2.06)^2 + 0.45 \times (2 - 2.06)^2 + 0.27 \times (3 - 2.06)^2 + 0.053 \times (4 - 2.06)^2 = 0.9364$$

- c. $SD(X)$, the standard deviation of X .

$$SD(X) = 0.9364^{1/2} = 0.9677$$

- d. $V(X)$, using the shortcut formula, $V(X) = E(X^2) - \mu^2$. Should equal the answer in part (b).

$$V(X) = 0.08 \times 0^2 + 0.15 \times 1^2 + 0.45 \times 2^2 + 0.27 \times 3^2 + 0.053 \times 4^2 - 2.06^2 = 0.9364$$

Problem 2

- a. Compute the first three moments of the random variable X , $E(X)$, $E(X^2)$, $E(X^3)$, and $V(X)$.

$$E(X) = 0.2 \times 13.5 + 0.5 \times 15.9 + 0.3 \times 19.1 = 16.38$$

$$E(X^2) = 0.2 \times 13.5^2 + 0.5 \times 15.9^2 + 0.3 \times 19.1^2 = 272.298$$

$$E(X^3) = 0.2 \times 13.5^3 + 0.5 \times 15.9^3 + 0.3 \times 19.1^3 = 4592.276$$

$$V(X) = 272.298 - 16.38^2 = 3.9936$$

- b. If the price of a freezer having capacity X ft³ is $25X - 8.5$, what is the expected price paid by the next customer to buy a freezer?

$$E(25X - 8.5) = 25 \times 16.38 - 8.5 = 401$$

- c. What is the variance of the price $25X - 8.5$ paid by the next customer? Interpret this value!

$$V(25X - 8.5) = 25^2 \times 3.9936 = 2496$$

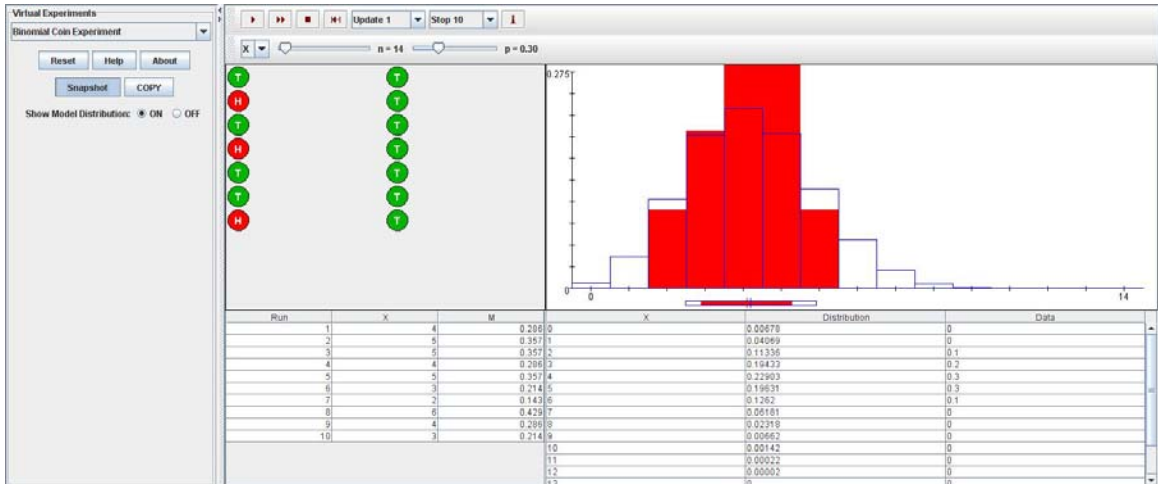
- d. Suppose that although the rated capacity of a freezer is X , the actual capacity is $h(X) = X - 0.01X^2$, slightly under the advertised space. What is the expected actual capacity of the freezer purchased by the next customer?

$$E(X - 0.01X^2) = 16.38 - 0.01 \times 272.298 = 13.657$$

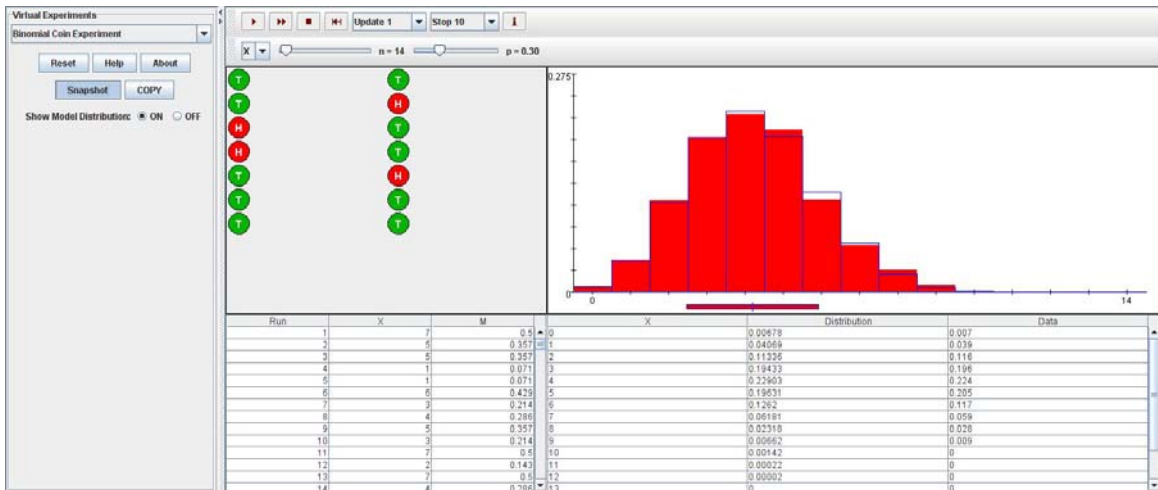
Problem 3

- How different are the population (theoretical) and the sample (observed) mean and standard deviation?

Answer varies.



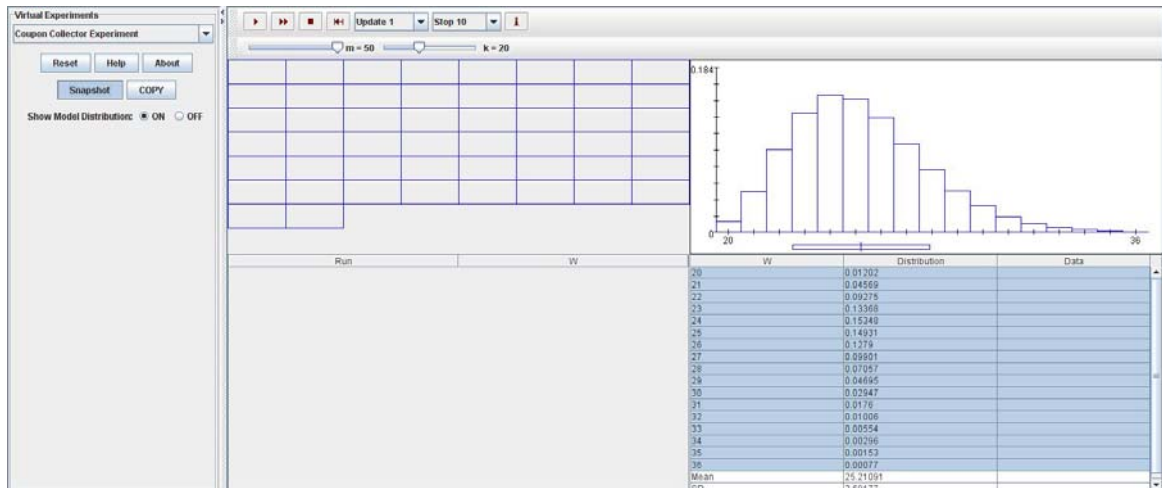
- How many runs of the 14-coin-toss experiment you need to do to get both of these differences within 10-2?
Answer varies, roughly 1,000.



- If you do another 10 runs are these differences guaranteed to stay within this limit (do not refresh the experiment, do an extra 10 runs)? Look at the SOCR Law of Large Numbers Activity and explain in your own words!
We cannot guarantee the difference will stay within this limit; this is because we don't know what kind of results we will get from these 10 runs. It is possible to get some extreme values.

Problem 4

- Select the proper values for the parameters m (population-size) and k (number of distinct objects desired). Look at the corresponding distribution for the random variable W , which represents the total number of coupons clipped by shopping time. Without running any experiments, just look at the blue model distribution graph and the discrete model distribution table for W below, to estimate the odds that you will need to clip at least 35 coupons to get 20 distinct ones, as desired.
0.0023



- b. Now, run 1,000 experiments in auto-mode. In what proportion of these you had to clip at least 35 coupons to get 20 distinct ones, as desired?
Answer varies.
- c. What is the discrepancy between the theoretical probability and the empirically observed odds of having to clip at most 24 coupons, in the above 1,000 experiments?
Answer varies.

Problem 5

- a. Use the Uniform E-Estimate Experiment (see SOCR Law of Large Numbers Activity) to generate an approximation to the natural number (e) with error within 0.01.
Answer varies.
- b. Use the Buffon's Needle Experiment (see the Buffon's Needle Activity) to estimate the number π with error within 0.1.
Answer varies.
- c. Suppose a call service center expects to get 35 calls a minute for questions regarding each of 23 different vendors that rely on this call center for handling their calls. Use the Normal approximation to Poisson distribution to compute the probability that in a 1-minute interval they receive less than 710 calls in total? See this demonstration as example of this approximation.
0.00374

