

**STAT 13, section 1, Winter 2011, UCLA Statistics
HW 3; Problem Solution**

HW 3.1

First, define the notations:

$M = \{\text{A male child born}\}$ (could have been $F = \{\text{A female child born}\}$. No intention of gender discrimination)

$D = \{\text{Have the disease (Affected)}\}$

Then, from the problem:

$$P(M) = 0.51$$

$$P(D|M) = 0.5 ; P(D|M^c) = 0$$

From the given information, we can infer that

$$P(D \cap M) = P(D|M)P(M) = 0.255$$

$$P(D \cap M^c) = P(D|M^c)P(M^c) = 0$$

(a) Since we are told that 2 children are a male and a female,

$$\begin{aligned} P(\text{a male affected} \cap \text{a female affected}) &= P(M \cap D) \cdot P(M^c \cap D) \\ &= 0.255 \cdot 0 = 0 \end{aligned}$$

(b) In this problem, we want to get $P(DD^c)$. Since we are not given the gender of the 2 children, we have to consider 3 cases; male and male, male and female, and female and female.

First, using binomial distribution with $n=2$ and $p=0.51$, we have

$$P(\text{male and male}) = P(MM) = \binom{2}{2} 0.51^2 (1 - 0.51)^0 = 0.2601$$

$$P(MM^c) = \binom{2}{1} 0.51^1 (1 - 0.51)^1 = 0.4998$$

$$P(M^cM^c) = \binom{2}{0} 0.51^0 (1 - 0.51)^2 = 0.2401$$

Also, note that

$$P(DD^c|M^cM^c) = 0$$

$$P(DD^c|MM^c) = \binom{1}{1} 0.5^1 (1 - 0.5)^0 = 0.5$$

$$P(DD^c|MM) = \binom{2}{1} 0.5^1 (1 - 0.5)^1 = 0.5$$

Finally, we combine the above results.

$$\begin{aligned}P(DD^c) &= P(DD^c \cap MM) + P(DD^c \cap MM^c) + P(DD^c \cap M^cM^c) \\ &= P(DD^c|MM)P(MM) + P(DD^c|MM^c)P(MM^c) + P(DD^c|M^cM^c)P(M^cM^c) \\ &= 0.37995\end{aligned}$$

(c) Now we want to get $P(D^cD^c)$. Using similar argument as in (b),

$$\begin{aligned}P(D^cD^c|M^cM^c) &= 1 \\ P(D^cD^c|MM^c) &= \binom{1}{0} 0.5^0 (1-0.5)^1 = 0.5 \\ P(D^cD^c|MM) &= \binom{2}{0} 0.5^0 (1-0.5)^2 = 0.25\end{aligned}$$

$$\begin{aligned}P(D^cD^c) &= P(D^cD^c \cap MM) + P(D^cD^c \cap MM^c) + P(D^cD^c \cap M^cM^c) \\ &= P(D^cD^c|MM)P(MM) + P(D^cD^c|MM^c)P(MM^c) + P(D^cD^c|M^cM^c)P(M^cM^c) \\ &= 0.555025\end{aligned}$$

HW 3.2

First, define the notations,

$D = \{\text{Have the disease}\}$

$P = \{\text{Test Positive}\}$

Then, from the problem, we can infer that

$$P(P|D) = 0.82$$

$$P(P^c|D^c) = 0.91; P(P|D^c) = 0.09$$

$$P(D) = 0.14; P(D^c) = 0.86$$

$$\begin{aligned}\text{(a) } P(P) &= P(P \cap D) + P(P \cap D^c) \\ &= P(P|D)P(D) + P(P|D^c)P(D^c) \\ &= 0.1922\end{aligned}$$

$$\text{(b) } P(D^c | P) = \frac{P(D^c \cap P)}{P(P)} = \frac{P(P|D^c)P(D^c)}{P(P)} = \frac{0.0774}{0.1922} = 0.4027$$

$$P(D | P) = \frac{P(D \cap P)}{P(P)} = \frac{P(P|D)P(D)}{P(P)} = \frac{0.82 * 0.14}{0.1922} = 0.597$$

HW 3.3

Brood Size(Y)	Brood Number	P(Y=y)
1	90	0.018
2	230	0.046
3	610	0.122
4	1300	0.26
5	1810	0.362
6	800	0.16
7	130	0.026
8	26	0.0052
9	3	0.0006
10	1	0.0002
Total	5000	1

(a) $P(Y=4) = 1300/5000 = 0.26$

(b) $P(Y \geq 8) = P(Y=8) + P(Y=9) + P(Y=10) = 0.006$

(c) $P(2 \leq Y < 8) = P(2 \leq Y \leq 7) = 0.976$

(d) $E(Y) = \sum_{y=1}^{10} y \cdot P(Y = y) = 4.517$

HW 3.4

Y(No.Black)	Probability
0	0.343
1	0.441
2	0.189
3	0.027
Total	1

(a) $P(Y \geq 1) = 1 - P(Y=0) = 0.657$

(b) $P(Y < 3) = 1 - P(Y=3) = 0.973$

(c) $E(Y) = \sum_{y=0}^3 y \cdot P(Y = y) = 0.9$

HW 3.5

(a) In SOCR, note that

$P(A) = P(\text{Coin Head}) = P(X=1)$

$P(B) = P(\text{Die}=3) = P(Y=3)$

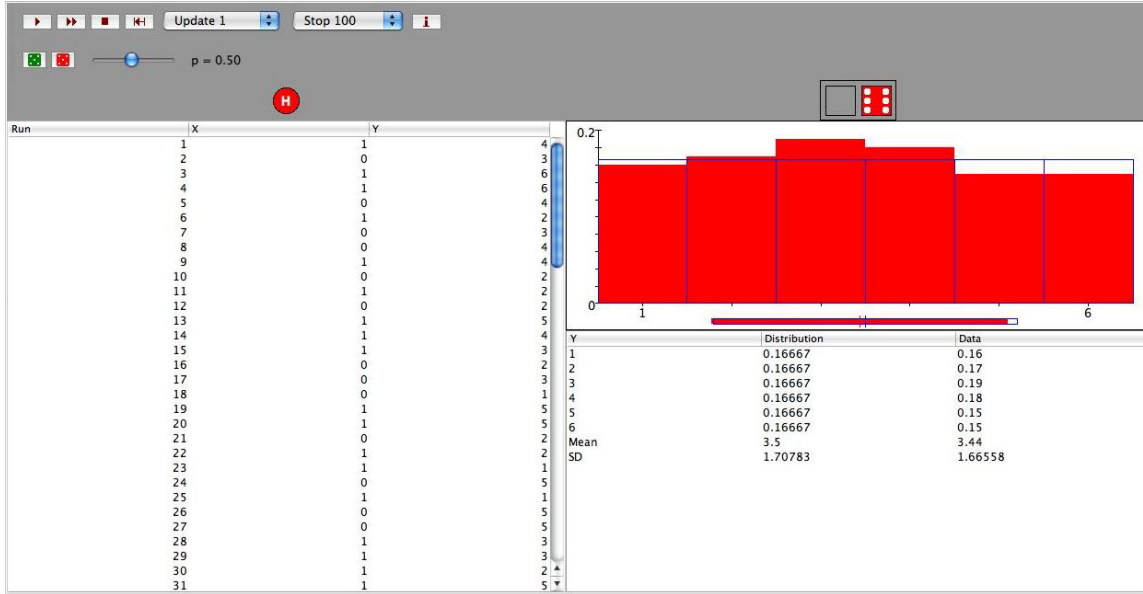
$$P(AB)=P(\text{Coin Head and Die}=3)=P(X=1 \text{ and } Y=3)$$

After 100 simulations, you get empirical probabilities $P(X=1)$ and $P(Y=3)$ by counting certain observations. Then, compare with $P(X=1 \text{ and } Y=3)$ and see if they seem to be independent.

(b) With different setting for green and red die, do the same simulation in (a), and see if they are dependent.

Example)

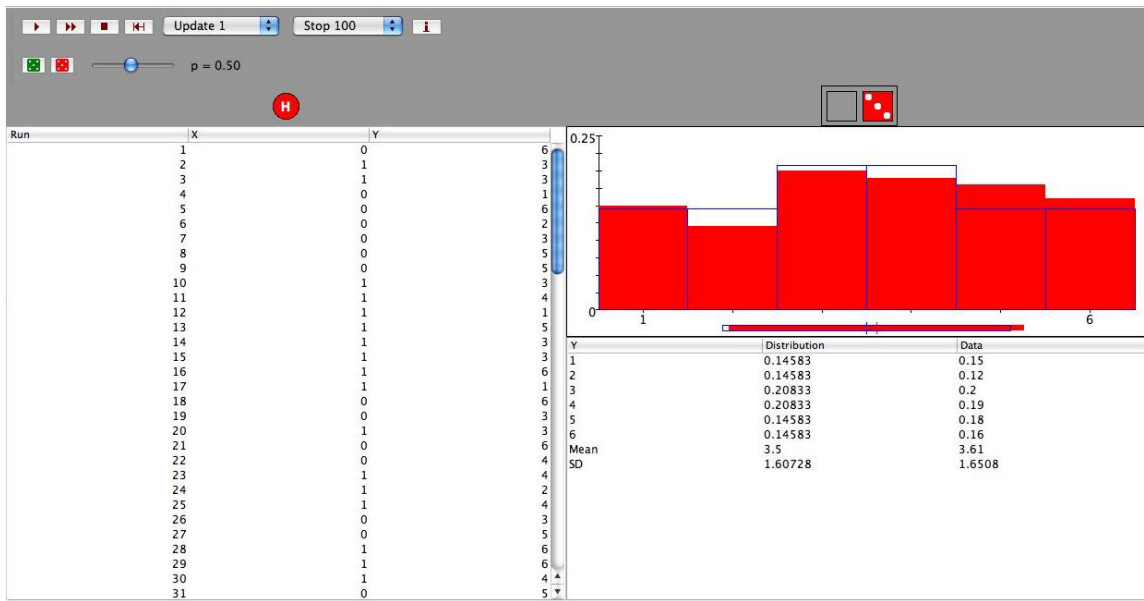
(a) We have both fair dice.



As a result $P(A)=53/100=0.53$, $P(B)=19/100=0.19$, and $P(AB)=9/100=0.09$

Since $P(A)*P(B)=0.10$ which is quite close to $P(AB)=0.09$, we observe some clue that the two events might be independent.

(b) We set fair green die, and 3-4 flat red die.



As a result $P(A)=49/100=0.49$, $P(B)=20/100=0.2$, and $P(AB)=15/100=0.15$

Since $P(A)*P(B)=0.09$ which is quite different from $P(AB)=0.15$, we observe some clue that the two events might be dependent.