

**UCLA Stat 10, Statistical Methods - Final Exam Review**  
**Probabilities and Proportions**

1. Suppose there were 2011 students enrolled in either Stat10 or Stat11 at UCLA. The numbers of female and male students are given in the following table.

|              | Females | Males | Total |
|--------------|---------|-------|-------|
| Stat10       | 604     | 593   | 1197  |
| Stat11       | 387     | 427   | 814   |
| <b>Total</b> | 991     | 1020  | 2011  |

- (a) Convert the above table of counts into a probability table (to 4 dp).

|         | Females | Males | Total |
|---------|---------|-------|-------|
| Stat 10 |         |       |       |
| Stat11  |         |       |       |
| Total   |         |       |       |

- (b) One of the 2011 students is chosen at random. What is the probability that the student chosen is:
- (i) a male taking Stat 10?
  - (ii) a female?
  - (iii) a female taking Stat 11?

- (c) Given that a student is taking Stat 11, what is the probability that they are male?
- (d) What is the probability that a randomly chosen male student is taking Stat 11?

2. Consider drivers stopped at random for breath testing. Below is a partially completed probability table providing information about such drivers, with regards to their age (40 or under, over 40) and whether they were (or were not) wearing seat belts.

|                       | 40 or under | Over 40 | Total |
|-----------------------|-------------|---------|-------|
| Wearing a seat belt   | 0.484       |         | 0.853 |
| Not wearing seat belt |             | 0.081   |       |
| <b>Total</b>          |             |         | 1     |

- (a) Complete the table.
- (b) What is the probability that a driver stopped at random is not wearing a seat belt?
- (c) If a driver stopped at random is not wearing a seat belt, then what is the probability the driver is over 40?
- (d) What is the probability that a driver stopped at random is 40 or under?

3. The medical records of a group of diabetic patients presenting at a clinic showed that 50 presented as serious cases, while 36 presented as mild cases. Of the 31 patients aged under 40, 16 presented as mild cases.

(a) Present this information in the table below.

|  |  |  |
|--|--|--|
|  |  |  |
|  |  |  |
|  |  |  |

(b) A patient is chosen at random. Find the probabilities that:

(i) the patient is under 40 and has a mild case.

(ii) the patient is at least 40 years old or has a serious case.

(iii) the patient has a serious case and is at least 40 years old.

(c) Of those presenting with serious cases, what proportion are aged under 40?

(d) Of those aged at least 40, what proportion present with mild cases?

4. A bank classifies borrowers as high-risk or low-risk. Of all its loans, 5% are in default. Forty percent (40%) of those loans in default are to high-risk borrowers, while 77% of loans not in default are to low-risk borrowers.

(a) Complete the table.

|  |  |  |
|--|--|--|
|  |  |  |
|  |  |  |
|  |  |  |

(b) What percentage of loans is made to borrowers in the high-risk category?

(c) What is the probability that a high-risk borrower will default on his or her loan?

5. According to recent figures from the National Centre of Educational Statistics (US), 17.5% of all bachelor's degrees are in business. 27% of bachelor's degrees in business are obtained by women and 48.75% of other degrees are obtained by men.

(a) Complete the table.

|  |  |  |
|--|--|--|
|  |  |  |
|  |  |  |
|  |  |  |

(b) What is the probability that a randomly selected recent bachelor's degree graduate will be a man?

(c) What is the probability that a randomly selected recent bachelor's degree graduate will be a man with a degree in business?

(d) What is the probability that a randomly selected female recent bachelor's degree graduate will have a degree in business?

6. .

A drinking pattern found by a survey is that 19% of male drinkers and 10% of female drinkers drink alcohol daily. Also, 51% of all drinkers are male (a 'drinker' was defined as someone who had consumed alcohol in the previous 12 months).

The probability that a randomly selected drinker from this survey who drinks alcohol daily is female is:

- (1) 0.3448
- (2) 0.3358
- (3) 0.0490
- (4) 0.1459
- (5) 0.2041

# UCLA Stat 10 Review Final Exam

## Confidence Intervals

### Section A: Confidence intervals for a mean, proportion and difference between means

1. An exam had a possible total of 64 points.

A random sample of 30 scores was selected from all of the exams. The data collected and its summary is as follows:

46 32 24 20 51 33 35 43 26 29 59 41 30 35 49  
53 32 50 52 23 25 53 51 34 26 29 40 38 45 42

| sample size | sample mean | sample standard deviation |
|-------------|-------------|---------------------------|
| 30          | 38.20       | 10.85                     |

You will use this sample to construct a 95% confidence interval for the mean.

- (a) State the parameter  $\theta$  (using a symbol and in words).
  
  
  
  
  
  
  
  
  
  
- (b) State the estimate  $\hat{\theta}$  (using a symbol, in words and as a number).
  
  
  
  
  
  
  
  
  
  
- (c) Calculate  $se(\hat{\theta})$ .
  
  
  
  
  
  
  
  
  
  
- (d) State the value of  $df$ .
  
  
  
  
  
  
  
  
  
  
- (e) Use the table for the Student's  $t$ -distribution to write down the value of the  $t$ -multiplier.

- (f) Calculate the 95% confidence interval for the mean.
  
  
  
  
  
  
  
  
  
  
- (g) Interpret the confidence interval.
  
  
  
  
  
  
  
  
  
  
- (h) Does the confidence interval contain the true mean? Discuss briefly.

2. Tuberculosis (TB) is known to be a highly contagious disease. In 1995 a study was carried out on a random sample of 1074 Spanish prisoners. The study investigated factors that might be associated with the tuberculosis infection. Some of the results follow.

|        | Prisoners with tuberculosis | Total number of prisoners |
|--------|-----------------------------|---------------------------|
| Male   | 556                         | 984                       |
| Female | 36                          | 90                        |

You will use this sample to construct a 95% confidence interval for the proportion of female prisoners who had tuberculosis.

- (a) State the parameter  $\theta$  (using a symbol and in words).
- (b) State the estimate  $\hat{\theta}$  (using a symbol, in words and as a number).
- (c) Calculate  $se(\hat{\theta})$ .
- (d) Use the table for the Student's  $t$ -distribution to write down the value of the  $z$ -multiplier.

- (e) Calculate the 95% confidence interval for the proportion of female prisoners who had tuberculosis.

- (f) Interpret the confidence interval.

- (g) Does the confidence interval contain the true proportion? Discuss briefly.

3. .

Banford et al. [1982] noted that thiol concentrations within human blood cells are seldom determined in clinical studies, in spite of the fact that they are believed to play a key role in many vital processes. They reported a new reliable method for measuring thiol concentration and demonstrated that, in one disease at least (rheumatoid arthritis), the change in thiol status in the lysate from packed blood cells is substantial. There were two groups of volunteers, the first group being “normal” and the second suffering from rheumatoid arthritis. We shall treat the two groups as random samples from the normal and rheumatoid populations respectively (for the area in which the study was undertaken) and will estimate  $\mu_R - \mu_N$ , the difference in true mean thiol levels between the rheumatoid and normal populations.

### Computer Output

Two sample T for Rheumatoid vs Normal

|          | N | Mean   | StDev  | SE Mean |
|----------|---|--------|--------|---------|
| Rheumato | 6 | 3.465  | 0.440  | 0.18    |
| Normal   | 7 | 1.9214 | 0.0756 | 0.029   |

95% CI for mu Rheumato - mu Normal: ( 1.08, 2.012)

T-Test mu Rheumato = mu Normal (vs not =): T = 8.48 P = 0.0004 DF = 5

(a) Interpret the confidence interval.

(b) Does the confidence interval contain the difference in true mean thiol levels between the rheumatoid and normal populations? Discuss briefly.

4. .

Consider constructing a confidence interval for the mean of a population. Which of the following would have an effect on the width of the confidence interval?

- 
- I: The size of the sample used to construct the interval.
  - II: The confidence level for the interval.
  - III: The amount of variability in the population.
- 

- (1) I only.
- (2) I and II only.
- (3) I and III only.
- (4) I, II, and III.
- (5) II and III only.

5. .

A 95% confidence interval for the difference between the mean haemoglobin levels of people with Type III and people with Type II sickle cell disease,  $\mu_{\text{Type III}} - \mu_{\text{Type II}}$ , is [-0.80, 2.39]. A **correct** interpretation of this interval would be:

- (1) Since zero is in the interval, there is a difference between the average haemoglobin levels for people with Type II sickle cell disease and people with Type III sickle cell disease.
- (2) We estimate, with 95% confidence, the average haemoglobin level for people with Type III sickle cell disease to be somewhere between 0.80g/dL lower and 2.39g/dL higher than the average haemoglobin level for people with Type II sickle cell disease.
- (3) We estimate, with 95% confidence, the average haemoglobin level for people with Type II sickle cell disease to be somewhere between 0.80g/dL lower and 2.39g/dL higher than the average haemoglobin level for people with Type III sickle cell disease.
- (4) On average, people with Type II sickle cell disease have a lower haemoglobin level than people with Type III sickle cell disease.
- (5) Since zero is in the interval, there is no difference between the average haemoglobin levels for people with Type II sickle cell disease and people with Type III sickle cell disease.

Questions 6 and 7 refer to the following information.

In 1990 *CNN/Time* sought information on how young American adults viewed their parents' marriage. In a telephone poll, one of the questions they asked of six hundred and two (602) 18-29 year old Americans was "Would you like to have a marriage like the one your parents have?" Forty-four percent (44%) responded "Yes".

6. *CNN/Time* were interested in determining what proportion of the 18-29 year old American population would answer "Yes" to this question. Which **one** of the following statements is **false**?

- (1) The value of the parameter of interest is an unknown quantity.
- (2) In this context, 0.44 is an estimate for the parameter of interest.
- (3) The parameter of interest depends on the sample and hence is a random quantity.
- (4) A confidence interval for the parameter of interest will give a range of possible values for this parameter.
- (5) The parameter of interest is the proportion of 18-29 year old Americans who would have answered "Yes" in 1990.

7. An approximate 95% confidence interval for the proportion of the 18-29 year old American population who would have answered "Yes" to this question in 1990 is [0.400, 0.480]. If two thousand four hundred (2400) 18-29 year old Americans had been sampled instead of six hundred and two (602) 18-29 year old Americans, then the new 95% confidence interval would be approximately:

- (1) twice as wide.
- (2) one-quarter as wide.
- (3) half as wide.
- (4) four times as wide.
- (5) equally as wide.

8. .

The *Listener/Heylen* poll from August 6, 1994 reported results on what New Zealanders think about the "Ten Commandments" from a sample of 1000 randomly chosen New Zealanders. A 99% confidence interval for the proportion of New Zealanders who believed that the commandment "I am the Lord your God; worship no god but me" fully applied to them,  $p_G$ , is given by (0.282, 0.358). Which **one** of the following statements is **true**?

- (1) The interval (0.282, 0.358) will cover the true, but unknown parameter  $p_G$  for 99% of samples taken.
- (2) Between 28.2 and 35.8 per cent of New Zealanders believe that this commandment fully applies to them 99% of the time.
- (3) A 95% confidence interval for  $p_G$  would be wider than this interval.
- (4) The probability that the interval (0.282, 0.358) covers the sample proportion is 0.99.
- (5) The probability that another interval calculated in the same way from a new sample of 1000 New Zealanders covers  $p_G$  is 0.99.

9. .

The results of a survey of 1146 New Zealanders were published in the 23 March 1992 issue of *Time* magazine. In response to the question "Is it a good time to buy a major household item?" 585 respondents replied "yes", 332 replied "no" and 229 replied "don't know".

Let  $p$  represent the true proportion of New Zealanders who think it is a good time to buy a major household item. Using the results of this survey a 99% confidence interval for  $p$  and a 95% confidence interval for  $p$  were constructed. A two standard error interval for  $p$  was also constructed.

Which **one** of the following statements is **true**?

The 99% confidence interval would:

- (1) be completely contained by the corresponding 95% confidence interval for  $p$ .
- (2) be narrower than the corresponding two standard error interval for  $p$ .
- (3) be wider if a much larger sample had been taken.
- (4) be wider than the corresponding 95% confidence interval for  $p$ .
- (5) have confidence limits which are twice as far apart as the confidence limits for the corresponding 95% confidence interval for  $p$ .

**Section B: Confidence interval for a difference in proportions**

1. In 1991 a random sample of New Zealand adults were surveyed about their working hours and the number of jobs they had. A similar survey was carried out in 1994.

Identify the sampling situation as:

Situation (a): *Two independent samples,*

Situation (b): *Single sample, several response categories,*

Situation (c): *Single sample, two or more Yes/No items,*

in the following cases.

- (a) We want to compare the proportion of females working 1-39 hours in 1994 with the proportion of females working 40 hours or more in 1994.
- (b) We want to compare the proportion of males working 40 hours or more in 1991 with the proportion of females working 40 hours or more in 1991.
- (c) In the same survey people were also asked if they had 2 or more jobs. We want to compare the proportion of people who had 2 or more jobs in 1994 with the proportion of people who worked 40 hours or more per week in 1994.
- (d) We want to compare the proportion of females working 40 hours or more in 1994 with the proportion of females working 40 hours or more in 1991.

**Questions 2 to 6 refer to the following information.**

Tuberculosis (TB) is known to be a highly contagious disease. In 1995 a study was carried out on a random sample of 1074 Spanish prisoners. The study investigated factors that might be associated with the tuberculosis infection. The results follow.

| Variable               |        | Prisoners with tuberculosis | Total number of prisoners |
|------------------------|--------|-----------------------------|---------------------------|
| Gender                 | Male   | 556                         | 984                       |
|                        | Female | 36                          | 90                        |
| Race                   | White  | 496                         | 886                       |
|                        | Gypsy  | 74                          | 152                       |
|                        | Other  | 22                          | 36                        |
| Intravenous Drug Users | Yes    | 361                         | 629                       |
|                        | No     | 231                         | 445                       |
| HIV Positive           | Yes    | 186                         | 294                       |
|                        | No     | 406                         | 780                       |
| Re-imprisonment        | Yes    | 272                         | 456                       |
|                        | No     | 320                         | 618                       |

2. Identify the sampling situation as:

Situation (a): *Two independent samples,*

Situation (b): *Single sample, several response categories,*

Situation (c): *Single sample, two or more Yes/No items,*

in the following cases:

- (a) Of those prisoners who had TB, we want to compare the proportion of white prisoners with the proportion of Gypsy prisoners.
- (b) We want to compare the proportion of male prisoners who had TB with the proportion of female prisoners who had TB.
- (c) We want to compare the proportion of prisoners who were intravenous drug users with the proportion of prisoners who had been re-imprisoned.
- (d) We want to compare the proportion of white prisoners who had TB with the proportion of Gypsy prisoners who had TB.
- (e) Of those prisoners who had TB, we want to compare the proportion who were intravenous drug users with the proportion who were HIV-positive.
- (f) We want to compare the proportion of Gypsy prisoners with the proportion of prisoners whose race was categorised as "other".

3. The standard error of the difference between the proportion of prisoners who have TB that are intravenous drug users and the proportion of prisoners who have TB that are HIV positive is:

(1) 
$$\sqrt{\frac{0.6098(1-0.6098) + 0.3142(1-0.3142)}{592}}$$

(2) 
$$\sqrt{\frac{0.6098 + 0.3142 - (0.6098 - 0.3142)^2}{592}}$$

(3) 
$$\sqrt{\frac{0.6098 + 0.3142 + (0.6098 - 0.3142)^2}{592}}$$

(4) 
$$\sqrt{\frac{0.6098(1-0.6098)}{629} + \frac{0.3142(1-0.3142)}{294}}$$

(5) 
$$\sqrt{\frac{0.6098^2 + 0.3142^2}{592}}$$



4. Construct a 95% confidence interval for the difference between the proportion of White prisoners who were infected with TB and the proportion of Gypsy prisoners who were infected with TB. State what your interval tells you in plain English.

(a) State the parameter  $\theta$  (using symbols and in words).

(b) State the estimate  $\hat{\theta}$  (using symbols, in words and as a number).

(c) Calculate  $se(\hat{\theta})$ .

(d) Use the table for the Student's  $t$ -distribution to write down the value of the  $z$ -multiplier.

(e) Calculate the confidence interval.

(f) Interpret the confidence interval.

5. Construct a 95% confidence interval for the difference in the proportion of prisoners infected with TB who were white and the proportion of prisoners infected with TB who were Gypsy.

(a) State the parameter  $\theta$  (using symbols and in words).

(b) State the estimate  $\hat{\theta}$  (using symbols, in words and as a number).

(c) Calculate  $se(\hat{\theta})$ .

(d) Use the table for the Student's  $t$ -distribution to write down the value of the  $z$ -multiplier.

(e) Calculate the confidence interval.

(f) Interpret the confidence interval.

6. Let  $p_Y$  be the proportion of intravenous drug user prisoners who were infected with TB, and  $p_N$  be the proportion of non-intravenous drug user prisoners who were infected with TB. The *Excel* worksheet below shows the calculations for a 95% confidence interval based on the data shown on the first page.

Two population proportions

| Input data |             |
|------------|-------------|
| X1_sample  | 361         |
| X2_sample  | 231         |
| n1_total   | 629         |
| n2_total   | 445         |
| p1_ratio   | 0.573926868 |
| p2_ratio   | 0.519101124 |
| pdiff      | 0.054825744 |

Alpha 0.05

| Calculated value |             |
|------------------|-------------|
| se               | 0.03081794  |
| t-multiplier     | 1.959961082 |

| Confidence Interval |             |
|---------------------|-------------|
| Lower limit         | -0.00557622 |
| Upper limit         | 0.115227708 |

- (a) Which sampling situation applies here? Briefly explain why.

- (b) Interpret the confidence interval.

- (c) Is it plausible that  $p_Y$  is equal to  $p_N$ ? Justify your answer.

7. .

A *Time/CNN* poll was based on a telephone survey of 800 adult Hong Kong residents conducted two weeks before the hand over of Hong Kong to China.  $p_c$  is the proportion of people in Hong Kong who think "Corruption" is the issue which worries them most, and  $p_f$  is the proportion of people in Hong Kong who think "Reduced personal freedoms" is the issue which worries them most.

A 95% confidence interval for  $p_c - p_f$  is (0.012, 0.088). Which **one** of the following statements is **false**?

- (1) In repeated sampling, we would expect that 95% of the 95% confidence intervals produced contain the true value of  $p_c - p_f$ .
- (2) In light of the data, the interval (0.012, 0.088) contains the most plausible values for  $p_c - p_f$ .
- (3) The true value of  $p_c - p_f$  must be in the interval (0.012, 0.088).
- (4) At this level of confidence, statements such as " $p_c$  is bigger than  $p_f$  by somewhere between 0.012 and 0.088" are true, on average, 19 out of 20 times.
- (5) With 95% confidence, the true value of  $p_c - p_f$  is 0.05 with a margin of error of  $\pm 0.038$ .

8. .

In a Time Morgan poll (July 1994) 662 voters were interviewed by telephone and asked whether *developing the economy* or *protecting the environment* would be more important in the short term. There were 238 National and 162 Labour supporters in the poll.

Let  $p_N$  be the true proportion of National supporters and let  $p_L$  be the true proportion of Labour supporters who think that *protecting the environment* is more important in the short term. A 95% confidence interval for the difference between the proportions  $p_N - p_L$  is given by [-0.1526, 0.0326]. Which **one** of the following interpretations is **true**?

- (1) With a probability of 0.95, the true difference of proportions  $p_N - p_L$  lies between -0.1526 and 0.0326.
- (2) In repeated sampling the 95% confidence interval [-0.1526, 0.0326] will contain the true difference in proportions in 95% of the samples taken.
- (3) In repeated sampling the true proportion  $p_N$  will be somewhere between 0.1526 larger and 0.0326 smaller than  $p_L$ .
- (4) With 95% confidence the true proportion  $p_N$  is somewhere between 0.1526 smaller and 0.0326 larger than  $p_L$ .
- (5) With 95% confidence the true proportion  $p_N$  is 0.1852 larger than  $p_L$ .

## UCLA STAT 10 Final Exam Review

### Sampling Distributions of Estimates, CLT

1. A random sample of size  $n$  is drawn from a population with mean,  $\mu$ , and standard deviation,  $\sigma$ . Let  $\bar{X}$  be the sample mean.
  - (a) What is the:
    - (i) mean of  $\bar{X}$ ?
    - (ii) standard deviation of  $\bar{X}$ ?
  - (b) If we are sampling from a Normal distribution then  $\bar{X}$  is **exactly** / **approximately** (circle **one**) Normally distributed.
  - (c) (i) If we are sampling from a non-Normal distribution then for large samples (ie,  $n$  is large)  $\bar{X}$  is **exactly** / **approximately** (circle **one**) Normally distributed.  
 (ii) The result in (i) is called the
2. A random sample of size  $n$  is drawn from a population in which a proportion  $p$  has a characteristic of interest. Let  $\hat{P}$  be the sample proportion.
  - (a) What is the:
    - (i) mean of  $\hat{P}$ ?
    - (ii) standard deviation of  $\hat{P}$ ?
  - (b) For large samples  $\hat{P}$  is **exactly** / **approximately** (circle **one**) Normally distributed.
3. A \_\_\_\_\_ is a numerical characteristic of a population.
4. An \_\_\_\_\_ is a \_\_\_\_\_ quantity calculated from data in order to estimate an unknown \_\_\_\_\_.
5. Suppose that  $X_1, X_2, \dots, X_{16}$  is a random sample from a Normal distribution with mean of 50 and a standard deviation of 10. Then the distribution of the sample mean  $\bar{X} = \frac{X_1 + X_2 + \dots + X_{16}}{16}$  has mean,  $\mu_{\bar{X}}$ , and standard deviation,  $\sigma_{\bar{X}}$ , given by:
  - (1)  $\mu_{\bar{X}} = 50, \quad \sigma_{\bar{X}} = 6.25$
  - (2)  $\mu_{\bar{X}} = 50, \quad \sigma_{\bar{X}} = 0.625$
  - (3)  $\mu_{\bar{X}} = 800, \quad \sigma_{\bar{X}} = 10$
  - (4)  $\mu_{\bar{X}} = 50, \quad \sigma_{\bar{X}} = 2.5$
  - (5) cannot be determined because  $n = 16$  is too small for the central limit effect to take effect.

6. The distribution of all long-distance telephone calls is approximately Normally distributed with a mean of 280 seconds and a standard deviation of 60 seconds. A random sample of sixteen calls is chosen from telephone company records. Let  $\bar{X}$  be the sample mean of sixteen such calls.

- (a) Describe the distribution of  $\bar{X}$ .
- (b) Calculate the probability that the sample mean exceeds 240 seconds. Use the output below to help you.

#### Cumulative Distribution Function

Normal with mean = 280.000 and standard deviation = 60.0000

| x        | P( X <= x) |
|----------|------------|
| 240.0000 | 0.2525     |

#### Cumulative Distribution Function

Normal with mean = 280.000 and standard deviation = 15.0000

| x        | P( X <= x) |
|----------|------------|
| 240.0000 | 0.0038     |

#### Cumulative Distribution Function

Normal with mean = 280.000 and standard deviation = 3.75000

| x        | P( X <= x) |
|----------|------------|
| 240.0000 | 0.0000     |

7. The fuel consumption, in litres per 100 kilometres, of all cars of a particular model has mean of 7.15 and a standard deviation of 1.2. A random sample of these cars is taken.

- (a) Calculate the mean and standard deviation of the sample if:
  - (i) one observation is taken.
  - (ii) four observations are taken.
  - (iii) sixteen observations are taken.

(b) In what way do your answers in (a) differ? Why?

8. About 65% of all university students belong to the student loan scheme. Consider a random sample of 50 students. Let  $\hat{P}$  be the proportion of these 50 students who belong to the student loan scheme.

(a) In words, describe  $p$ .

(b) State the distribution of  $\hat{P}$ .

(c) What is the probability that the sample proportion is more than 70%? Use the output below to help you.

(d) What is the probability that the sample proportion is between 0.45 and 0.55? Use the output below to help you.

**Cumulative Distribution Function**

Normal with mean = 0.650000 and standard deviation = 0.0674537

| x      | P( X <= x) |
|--------|------------|
| 0.4000 | 0.0001     |
| 0.4500 | 0.0015     |
| 0.5000 | 0.0131     |
| 0.5500 | 0.0691     |
| 0.6000 | 0.2293     |
| 0.6500 | 0.5000     |
| 0.7000 | 0.7707     |
| 0.7500 | 0.9309     |

9. The owner of a large fleet of courier vans is trying to estimate her costs for next year's operations. Fuel purchases are a major cost. A random sample of 8 vans yields the following fuel consumption data (in km/L):

10.3 9.7 10.8 12.0 13.4 7.5 8.2 9.1

Assume that the distribution of fuel consumption of the vans is approximately Normal.

(a) Calculate the sample mean and the sample standard deviation.

(b) Construct a two-standard-error interval for the mean fuel consumption of all of her vans.

(c) Without doing any calculations specify what happens to the width of the two-standard error interval in the following cases:

(i) the sample standard deviation increases.

(ii) the sample mean decreases.

(iii) the sample size increases.

10. A large department store wants to estimate the proportion of their customers who have a charge card for the store. They take a random sample of 120 shoppers. They find that 36 of these shoppers have a charge card for the store. Construct a two-standard-error interval for the proportion of all of their customers who have a charge card for the store.

11. Which **one** of the following statements is **true**?

(1) In a poll, all estimates of population proportions, including estimates for subgroups of the population, will have the same standard error.

(2) If  $X$  is Normal, then the Student's  $t$ -distribution is used instead of the standard Normal distribution for the distribution of  $\frac{\bar{X} - \mu}{\sigma_x / \sqrt{n}}$  when the population standard deviation is replaced by the sample standard deviation.

(3) The Central Limit Effect can only be detected for sample sizes that are greater than 30.

(4) When sampling, taking a large sample guarantees an accurate estimate of the parameter of interest.

(5) For small samples, the shape of the distribution of the sample mean,  $\bar{X}$ , is always Normal regardless of the shape of the distribution of the random variable  $X$ .

# UCLA Stat 10 Final Exam Review

## Hypothesis Testing

**Section A:**

1. What is the difference between the null hypothesis and the alternative hypothesis?
  
2. List the three alternative hypotheses that are possible when testing the null hypothesis  $H_0 : \mu = \theta_0$ .
  - (a)
  - (b)
  - (c)
  
3. On what basis do we decide whether to do a one-tailed or a two-tailed test?

$\theta$

4. Use the Formulae Sheet to fill in the formula for the  $t$ -test statistic:

(a) In words:

(b) In symbols:

5. Fill in the gaps in this description of the  $P$ -value from a  $t$ -test:

The  $P$ -value is the \_\_\_\_\_ that, if the \_\_\_\_\_  
 \_\_\_\_\_ was true, \_\_\_\_\_  
 would produce an estimate that is \_\_\_\_\_  
 \_\_\_\_\_ from the \_\_\_\_\_ than our  
 \_\_\_\_\_.

6. What does the  $P$ -value measure?

7. In the table below, fill in the descriptions used for the given  $P$ -values:

| $P$ -value     | Evidence against $H_0$ |
|----------------|------------------------|
| $> 0.12$       |                        |
| $\approx 0.10$ |                        |
| $\approx 0.05$ |                        |
| $\approx 0.01$ |                        |
| $< 0.01$       |                        |

8. What does the  $P$ -value tell us about the size of the effect?

9. What tool do we use to tell us about the size of an effect?

10. What is a significance test?

11. What does a significant test reveal?

**Section B:**

1. Tuberculosis (TB) is known to be a highly contagious disease. In 1995 a study was carried out on a random sample of 1074 Spanish prisoners. The study investigated factors that might be associated with the tuberculosis infection. The results follow.

| Variable               |        | Prisoners with tuberculosis | Total number of prisoners |
|------------------------|--------|-----------------------------|---------------------------|
| Gender                 | Male   | 556                         | 984                       |
|                        | Female | 36                          | 90                        |
| Race                   | White  | 496                         | 886                       |
|                        | Gypsy  | 74                          | 152                       |
|                        | Other  | 22                          | 36                        |
| Intravenous Drug Users | Yes    | 361                         | 629                       |
|                        | No     | 231                         | 445                       |
| HIV Positive           | Yes    | 186                         | 294                       |
|                        | No     | 406                         | 780                       |
| Re-imprisonment        | Yes    | 272                         | 456                       |
|                        | No     | 320                         | 618                       |

Is there any evidence to suggest that the race of the prisoner (White or Gypsy) makes any difference to whether they contracted tuberculosis? Carry out a significance test to answer this question and then calculate an appropriate 95% confidence interval.

Let  $p_W$  be the proportion of White prisoners infected with TB and  $p_G$  be the proportion of Gypsy prisoners infected with TB.

- (a) Identify the parameter  $\theta$ .
  
  
  
  
  
  
  
  
  
  
- (b) State the hypotheses.
  
  
  
  
  
  
  
  
  
  
- (c) Write down the estimate and its value.

- (d) Calculate the value of the  $t$ -test statistic.

- (e) Find the  $P$ -value.

- (f) Interpret the  $P$ -value.

- (g) Answer the original question.

- (h) Calculate a 95% confidence interval for the parameter.

- (i) Interpret the 95% confidence interval.

2. In a poll conducted for TIME and CNN (TIME 17 September 1990, page 51), 1009 residents of New York City were asked "If you could choose where you live, would you live in New York City or move somewhere else?" 595 of the residents said that they would move somewhere else. Could you conclude that this is the opinion of a majority of residents of New York City?

The information below is a Computer output for a significance test and a 95% confidence interval. Use this output to answer the questions below.

### Test and Confidence Interval for One Proportion

Test of  $p = 0.5$  vs  $p \text{ not } = 0.5$

| Sample | X   | N    | Sample p | 95.0 % CI            | Z-Value | P-Value |
|--------|-----|------|----------|----------------------|---------|---------|
| 1      | 595 | 1009 | 0.589693 | (0.559342, 0.620044) | 5.70    | 0.000   |

- State the parameter used in this analysis.
- State the hypotheses used in this  $t$ -test.
- Write down the estimate and its value.
- Write down the value of the test statistic and the  $P$ -value.
- Answer the original question. (I.e., could you conclude that this is the opinion of a majority of residents of New York City?)
- Interpret the confidence interval.

3. A businessperson is interested in buying a coin-operated laundry and has a choice of two different businesses. The businessperson is interested in comparing the average daily revenue of the two laundries, so she collects some data. A simple random sample for 50 days from the records for the past five years of the first laundry and a simple random sample for 30 days from the records for the past three years of the second laundry reveal the following summary statistics:

|           | Sample size | Sample mean | Sample standard deviation |
|-----------|-------------|-------------|---------------------------|
| Laundry 1 | 50          | \$635.40    | \$71.90                   |
| Laundry 2 | 30          | \$601.60    | \$77.70                   |

### Computer output

#### Two Sample T-Test and Confidence Interval

Two sample T for Laundry1 vs Laundry2

|          | N  | Mean  | StDev | SE Mean |
|----------|----|-------|-------|---------|
| Laundry1 | 50 | 635.4 | 71.9  | 10      |
| Laundry2 | 30 | 601.6 | 77.7  | 14      |

95% CI for  $\mu$  Laundry1 -  $\mu$  Laundry2: ( -1, 69)

T-Test  $\mu$  Laundry1 =  $\mu$  Laundry2 (vs not =): T = 1.94 P = 0.057 DF = 57

Stem-and-leaf of Laundry1 N = 50  
Leaf Unit = 10

```

1  4 7
1  4
2  5 0
5  5 233
6  5 5
10 5 6777
16 5 889999
18 6 01
(10) 6 2222233333
22 6 444444555
13 6 67
11 6 889
8  7 1
7  7 23
5  7 5
4  7 77
2  7 99

```

Stem-and-leaf of Laundry2 N = 30  
Leaf Unit = 10

```

1  4 3
2  4 6
9  5 012344
15 5 558889
15 6 113344444
6  6 77
4  7 023
|  1  7 5

```

- (a) State the parameter used in this analysis.
- (b) State the hypotheses used in this  $t$ -test.
- (c) Write down the estimate and its value.
- (d) Write down the value of the test statistic.
- (e) Interpret the test.
- (f) Interpret the confidence interval.
- (g) Do the stem-and-leaf plots give you any reasons for doubting the validity of the results of this analysis? Briefly explain.
- (h) If this analysis were done by hand the value of  $df$  would have been 29. Why does the output show that  $df = 57$ ?

### Section C:

1. Which **one** of the following statements regarding significance testing is **false**?
  - (1) A highly significant test result means that the size of the difference between the estimated value of the parameter and the hypothesised value of the parameter is significant in a practical sense.
  - (2) A  $P$ -value of less than 0.01 is often referred to as a highly significant test result.
  - (3) A nonsignificant test result does not necessarily mean  $H_0$  is true.
  - (4) A two-tail test of  $H_0: \theta = \theta_0$  is significant at the 5% significance level if and only if  $\theta_0$  lies outside a 95% confidence interval for  $\theta$ .
  - (5) Testing at the 5% level of significance means that the null hypothesis is rejected whenever a  $P$ -value smaller than 5% is obtained.
2. Which **one** of the following statements is **false**?
  - (1) In hypothesis testing, a nonsignificant result implies that  $H_0$  is true.
  - (2) In hypothesis testing, a two-tail test should be used when the idea for doing the test has been triggered by the data.
  - (3) In surveys, the nonsampling error is often greater than the sampling error.
  - (4) Larger sample sizes lead to smaller standard errors.
  - (5) In hypothesis testing, statistical significance does not necessarily imply practical significance.
3. Which **one** of the following statements regarding significance testing is **false**?
  - (1) Formal tests can help determine whether effects we see in our data may just be due to sampling variation.
  - (2) The  $P$ -value associated with a two-sided alternative hypothesis is obtained by doubling the  $P$ -value associated with a one-sided alternative hypothesis.
  - (3) The  $P$ -value says nothing about the size of an effect.
  - (4) The data should be carefully examined in order to determine whether the alternative hypothesis needs to be one-sided or two-sided.
  - (5) The  $P$ -value describes the strength of evidence against the null hypothesis.