**Stat 100a, introduction to Probability.**

<u>Outline for the day:</u>

1. teams, emails, bruin.
2. Midterm 1 and hw2.
3. Markov and Chebyshev inequalities.
4. Luck and skill in poker.
5. Lederer and Minieri.
6. Booth and Ivey.
7. Binomial random variables.
8. Geometric random variables.
9. Moment generating functions.
10. Practice problems.

Homework 2 is on the course website. It is due Thu Nov 2.
Midterm 1 is Tue Nov 7 in class.

♠ ♣ ♥ ♦

1. teams, emails, bruin.

The teams will be posted tonight in teams.txt on the course website.

I will post your email addresses there too until Thursday night. If you do not want yours listed for privacy reasons, please let me know by tonight and I will not list yours.

1. R project. Teams, emails, and bruin.

The project is problem 8.2, page 249.

You need to write code to go all in or fold. In R, try:

```
install.packages(holdem)
library(holdem)
library(help="holdem")
```

gravity, timemachine, tommy, ursula, vera, william, and xena are examples.

crds1[1,1] is your higher card (2-14).

crds1[2,1] is your lower card (2-14).

crds1[1,2] and crds1[2,2] are suits of your higher card & lower card.

```
help(tommy)
tommy
```

```
function (numattable1, crds1, board1, round1, currentbet, mychips1,
  pot1, roundbets, blinds1, chips1, ind1, dealer1, tablesleft)
{ a1 = 0
  if (crds1[1, 1] == crds1[2, 1])
     a1 = mychips1
  a1
}
```

```
help(vera)
All in with a pair, any suited cards, or if the smaller card is at least 9.
 function (numattable1, crds1, board1, round1, currentbet, mychips1,
   pot1, roundbets, blinds1, chips1, ind1, dealer1, tablesleft)
{a1 = 0
 if ((crds1[1, 1] == crds1[2, 1]) || (crds1[1, 2] == crds1[2,2]) ||
     (crds1[2, 1] > 8.5)) a1 = mychips1
   a1
}
```

You need to email me your function, to frederic@stat.ucla.edu, by Sat Dec 2, 8pm. It should be written (or cut and pasted) simply into the body of the email. If you write it in Word, save as text first, and then paste it into the email. Just submit one email per team.

For instance, if your function is named "bruin", you might do:

```
bruin = function (numattable1, crds1, board1, round1, currentbet,
      mychips1, pot1, roundbets, blinds1, chips1, ind1, dealer1,
      tablesleft) {
## all in with any pair higher than 7s, or if lower card is J or higher
a1 = 0
if ((crds1[1, 1] == crds1[2, 1]) && (crds1[1, 1] > 6.5)) a1 = mychips1
if (crds1[2,1] > 10.5) a1 = mychips1
a1
} ## end of bruin
```

2. Midterm 1 and homework 2.

Midterm 1 is one hour and 15 min, on Tue Nov 7.

Around 12 multiple choice questions all worth the same amount.

You can use any books and notes you want, but no computers, tablet, ipads, phones, or anything that can surf the net or do email.

**Bring a calculator** and a pen or pencil.

None of the above is an option but it is hardly ever the answer.

Answers are rounded to 2 decimal places.

Homework 2 is problems 4.6, 4.8, 4.26 and 5.2. 4.26 is only in the $2^{nd}$ edition, but it is also in hw2.pdf. 5.2 is tricky. On problem 5.2, b) let Z = the time until you have been dealt a pocket pair and you have also been dealt two black cards.

Consider $P(Z > k)$, and $P(Z > k-1)$. These are actually easier to derive in this case than $P(Z = k)$. Can you get $P(Z = k)$ in terms of these?

**3. Markov and Chebyshev inequalities. Ch 4.6.**

The Markov inequality states

If X takes only non-negative values, and c is any number > 0, then

$P(X \geq c) \leq E(X)/c$.

**Proof.** The discrete case is given on page 123.

If X is discrete and nonnegative, then

$$
\begin{aligned}
E(X) &= \sum_b b\, P(X = b) \\
&= \sum_{b<c} b\, P(X = b) \quad + \quad \sum_{b \geq c} b\, P(X = b) \\
&\geq \sum_{b \geq c} b\, P(X = b) \\
&\geq \sum_{b \geq c} c\, P(X = b) \\
&= c \sum_{b \geq c} P(X = b) \\
&= c\, P(X \geq c).
\end{aligned}
$$

Here is a proof for the case where X is continuous with pdf f(y).

$E(X) = \int y\, f(y)\, dy$

$= \int_0^c yf(y)dy + \int_c^\infty yf(y)dy$

$\geq \int_c^\infty yf(y)dy$

$\geq \int_c^\infty cf(y)dy$

$= c \int_c^\infty f(y)dy$

$= c\, P(X \geq c)$.

Thus, $P(X \geq c) \leq E(X) / c$.

## 3. Markov and Chebyshev inequalities. Ch 4.6.

The Markov inequality states

If X takes only non-negative values, and c is any number $> 0$, then

$P(X \geq c) \leq E(X)/c$.

The Chebyshev inequality states

For any random variable Y with expected value $\mu$ and variance $\sigma^2$, and any real number $a > 0$,

$P(\,|\,Y - \mu\,|\geq a) \leq \sigma^2 / a^2$.

**Proof.** The Chebyshev inequality follows directly from the Markov equality by letting $c = a^2$ and $X = (Y-\mu)^2$.

Examples of the use of the Markov and Chebyshev inequalities are on pages 123-125.

## 4. Luck and skill in poker. ♠ ♣ ♥ ♦

Let equity = your expected portion of the pot after the hand, assuming no future betting.
= your expected number of chips after the hand - chips you had before the hand
= pot * p, where p = your probability of winning if nobody folds.
I define luck as the expected profit gained during the dealing of the cards,
= equity gained during the dealing of the cards.
Skill = expected profit gained during the betting rounds.

Example.
You have Q♣ Q♦. I have 10♠ 9♠. Board is 10♦ 8♣ 7♣ 4♣. Pot is $5.
The river is 2♦, you bet $3, and I call.
On the river, how much expected profit did you gain by luck and how much by skill?

Expected profit by luck on river = your equity after 2♦ is exposed – your equity just pre-2♦
= 100% ($5) - 35/44 ($5) = $1.02.
Why 35/44? I can win with a 10, 9, 6, or J that is not a club. There are 1 + 2 + 3 + 3 = 9 of these cards, so the remaining 35 cards give you the win.

Expected profit by skill on river
= increase in pot on river * P(you win) - your cost
= $6 * 100% - $3 = $3.

**Luck and skill in poker, continued.** ♠ ♣ ♥ ♦

Example.

You have Q♣ Q♦. I have 10♠ 9♠. Board is 10♦ 8♣ 7♣ 4♣. Pot is \$5.

The river is 2♦, you bet \$3, and I call.

On the river, how much expected profit did you gain by luck and how much by skill?

Alternatively, let x = the number of chips you have after your \$3 bet on the river.

Before this bet, you had x + \$3 chips.

Expected profit gained by skill on river = your equity after all the betting is over - your equity when the 2♦ is dealt

= your expected number of chips after all the betting is over – your expected number of chips when the 2♦ is dealt

= (100%)(x + \$11) – (100%)(x + \$3 + \$5)

= \$3.

5. Lederer and Minieri.

I define luck as the expected profit gained during the dealing of the cards.
Skill = expected profit gained during the betting rounds.

Are there any problems with these definitions?

6. Bluffing. Ivey and Booth.

Mike Cloud raised to 15,000 with A♣ A♠, Hellmuth called with A♥ K♠, Daniel Negreanu called from the big blind with 6♦ 4♥, and the flop came K♣ 8♥ K♥. Before the flop, the pot was 57,000 chips.

After the flop, all three players checked, the turn was the J♥, Negreanu checked, Cloud bet 15,000, Hellmuth called, and Negreanu folded.

The river was the 7♠, Cloud checked, Hellmuth bet 37,000, and Cloud called. How much expected profit did Hellmuth gain due to luck and how much due to skill on the river?

Answer—When the turn was dealt, Hellmuth's probability of winning in a showdown was $41/42 \sim 97.62\%$. After the betting on the turn was over, the pot was 87,000 chips. When the 7♠ was revealed on the river, Hellmuth's equity increased from $97.62\% \times 87{,}000 = 84{,}929.4$ to $100\% \times 87{,}000$, for an increase of 2070.6 chips due to luck.

Hellmuth's expected profit gained due to skill on the river is simply 37,000 chips: the pot size increased by 74,000 while Hellmuth had a 100% chance of winning, but the cost to Hellmuth was 37,000, so his profit was 37,000.

## 7. Facts about expected value.

For any random variable X and any constants a and b,

$E(aX + b) = aE(X) + b.$

Also, $E(X+Y) = E(X) + E(Y)$,

unless $E(X) = \infty$ and $E(Y) = -\infty$, in which case $E(X)+E(Y)$ is undefined.

Thus $\sigma^2 = E[(X-\mu)^2]$

$$= E[(X^2 - 2\mu X + \mu^2)]$$

$$= E(X^2) - 2\mu E(X) + \mu^2$$

$$= E(X^2) - 2\mu^2 + \mu^2$$

$$= E(X^2) - \mu^2.$$

## 8. Bernoulli Random Variables, ch. 5.1.

If X = 1 with probability p, and X = 0 otherwise, then X = *Bernoulli (p)*.

Probability mass function (pmf):

$P(X = 1) = p$

$P(X = 0) = q$, where $p+q = 100\%$.

**If X is Bernoulli (p), then $\mu = E(X) = p$, and $\sigma = \sqrt{(pq)}$.**

For example, suppose X = 1 if you have a pocket pair next hand; X = 0 if not.

$p = 5.88\%$. So, $q = 94.12\%$.

[Two ways to figure out p:

(a) Out of choose(52,2) combinations for your two cards, 13 * choose(4,2) are pairs.

13 * choose(4,2) / choose(52,2) = 5.88%.

(b) Imagine *ordering* your 2 cards. No matter what your 1st card is, there are 51 equally likely choices for your 2nd card, and 3 of them give you a pocket pair. 3/51 = 5.88%.]

$\mu = E(X) = .0588$. $SD = \sigma = \sqrt{(.0588 * 0.9412)} = 0.235$.

**9. Binomial Random Variables, ch. 5.2.**

Suppose now X = # of times something with prob. p occurs, out of n independent trials

Then X = *Binomial (n.p)*.

e.g. the number of pocket pairs, out of 10 hands.

Now X could = 0, 1, 2, 3, …, or n.

pmf: $P(X = k) = \text{choose}(n, k) * p^k q^{n-k}$.

e.g. say n=10, k=3: $P(X = 3) = \text{choose}(10,3) * p^3 q^7$.

Why? Could have 1 1 1 0 0 0 0 0 0 0, or 1 0 1 1 0 0 0 0 0 0, etc.

choose(10, 3) choices of places to put the 1's, and for each the prob. is $p^3 q^7$.

Key idea: $X = Y_1 + Y_2 + \ldots + Y_n$, where the $Y_i$ are independent and *Bernoulli* (p).

If X is Bernoulli (p), then $\mu = p$, and $\sigma = \sqrt{(pq)}$.

**If X is Binomial (n,p), then $\mu = np$, and $\sigma = \sqrt{(npq)}$.**

**Binomial Random Variables**, continued.

Suppose X = the number of pocket pairs you get in the next 100 hands.

What's P(X = 4)? What's E(X)? σ? X = Binomial (100, 5.88%).

$P(X = k) = \text{choose}(n, k) * p^k q^{n-k}$.

So, $P(X = 4) = \text{choose}(100, 4) * 0.0588^4 * 0.9412^{96} = 13.9\%$, or 1 in **7.2.**

$E(X) = np = 100 * 0.0588 = \mathbf{5.88}$. $\sigma = \sqrt{(100 * 0.0588 * 0.9412)} = \mathbf{2.35}$.

So, out of 100 hands, you'd *typically* get about 5.88 pocket pairs, +/- around 2.35.