

Stat 100a, Introduction to Probability. Rick Paik Schoenberg

Outline for the day:

1. Discuss addition and hw1 terminology.
2. Basic principle of counting.
3. Permutations and combinations.
4. R .
5. Conditional probability, independence, and multiplication rule.
6. Independence and dependence examples.
7. Negreanu and Elezra.
8. Odds ratios.
9. $P(\text{have AA and flop a full house})?$
10. $P(A^\spadesuit \text{ after } 1^{\text{st}} \text{ ace})?$



Hw1 terms.

Assume you never fold. I say this so one can't object "But I would never play 7♦ 5♦."

flop a straight flush. For example, you have 7♦ 5♦ and the flop is 4♦ 8♦ 6♦ .

flopping 2 pairs. For example, you have 7♦ 7♥ and the flop is 3♥ 3♠ J♥.

Or, you have 7♦ 3♥ and the flop is 7♥ 3♠ J♥.

pocket pair. When your two cards form a pair by themselves, like 7♦ 7♥.

face cards. K, Q, or J.

the nuts. Given the board, the best possible hand you could currently have in terms of the ranking order of poker hands, not in terms of probability of winning or improving in the future. For example, if the board is 7♥ 3♠ J♥ 8♦, then if you have 10♦ 9♦, then you have the nuts. If you have 10♥ 9♥, it would be slightly better in terms of probability of winning, but either way you have the nuts.

the unbreakable nuts. When you are guaranteed to win no matter what your opponent might have and no matter what board cards might come. In the above example where you have 10♥ 9♥ and the board is 7♥ 3♠ J♥ 8♦, you do not have the unbreakable nuts because you could lose for instance if the river is 9♠ and your opponent has Q♠ 10♦. However, if the board is 8♥ 7♥ 6♥ and you have 10♥ 9♥, then you have the unbreakable nuts.

in terms of. 3.2b is not easy. Assuming A and B are independent, you have to express the odds against (AB) using only O_A and O_B . You can't use any other variables. In part a you expressed it in terms of $P(A)$ and $P(B)$, so just figure out how to convert $P(A)$ into an expression of O_A .

1. Addiction handout.

2. **Basic Principle of Counting.**

If there are a_1 distinct possible outcomes on trial #1, and for each of them, there are a_2 distinct possible outcomes on trial #2, then there are $a_1 \times a_2$ distinct possible *ordered* outcomes on both.

e.g. you get 1 card, opp. gets 1 card. # of distinct possibilities?
 52×51 . [ordered: $(A\clubsuit, K\heartsuit) \neq (K\heartsuit, A\clubsuit)$.]

In general, with j trials, each with a_i possibilities,
the # of distinct outcomes *where order matters* is $a_1 \times a_2 \times \dots \times a_j$.

3. Permutations and Combinations.

e.g. you get 1 card, opp. gets 1 card.

of distinct possibilities?

52×51 . [ordered: (A♣, K♥) \neq (K♥, A♣) .]

Each such outcome, where order matters, is called a *permutation*.

Number of permutations of the deck? $52 \times 51 \times \dots \times 1 = 52!$

$$\sim 8.1 \times 10^{67}$$

A combination is a collection of outcomes, where order *doesn't* matter.

e.g. in hold'em, how many *distinct* 2-card hands are possible?

52 x 51 if order matters, but then you'd be double-counting each

[since now $(A\clubsuit, K\heartsuit) = (K\heartsuit, A\clubsuit)$.]

So, the number of *distinct* hands where *order doesn't matter* is

$52 \times 51 / 2$.

In general, with n distinct objects, the # of ways to choose k *different* ones, *where order doesn't matter*, is

“n choose k” = $\text{choose}(n,k) = \frac{n!}{k! (n-k)!}$.

4. *R*. To download and install *R*, go directly to cran.stat.ucla.edu, or as it says in the book at the bottom of p240 (157 of 1st edition), you can start at www.r-project.org, in which case you click on “download *R*”, scroll down to UCLA, and click on cran.stat.ucla.edu. From there, click on “download *R* for ...”, and then get the latest version.

The screenshot shows the homepage of 'The R Project for Statistical Computing'. The browser window has a title bar 'The R Project for Statistical Computing' and a search bar with 'Google'. The main content area features the R logo on the left, a navigation menu with links like 'About R', 'What is R?', 'Contributors', 'Screenshots', 'What's new?', 'Download, Packages', 'CRAN', 'R Project Foundation', 'Members & Donors', 'Mailing Lists', 'Bug Tracking', 'Developer Page', 'Conferences', and 'Search'. The main content area displays several statistical plots: a PCA plot titled 'PCA 5 vars' showing variables like Fertility, Catholic, Examination, Education, and Agriculture; a clustering dendrogram titled 'Clustering 4 groups'; a scatter plot titled 'Factor 1 [41%]' and 'Factor 3 [19%]'; and two histograms showing the distribution of data. The bottom section is titled 'Getting Started:' and contains two bullet points: 'R is a free software environment for statistical computing and graphics. It compiles and runs on a wide variety of UNIX platforms, Windows and MacOS. To **download R**, please choose your preferred [CRAN mirror](#).' and 'If you have questions about R like how to download and install the software, or what the license terms are, please read our [answers to frequently asked questions](#) before you send an email.'

The R Project for Statistical Computing

PCA 5 vars
princomp(x = data, cor = cor)

Fertility
Catholic
Examination
Education
Agriculture
(1-3) 60%

Clustering 4 groups

Factor 1 [41%]
Factor 3 [19%]

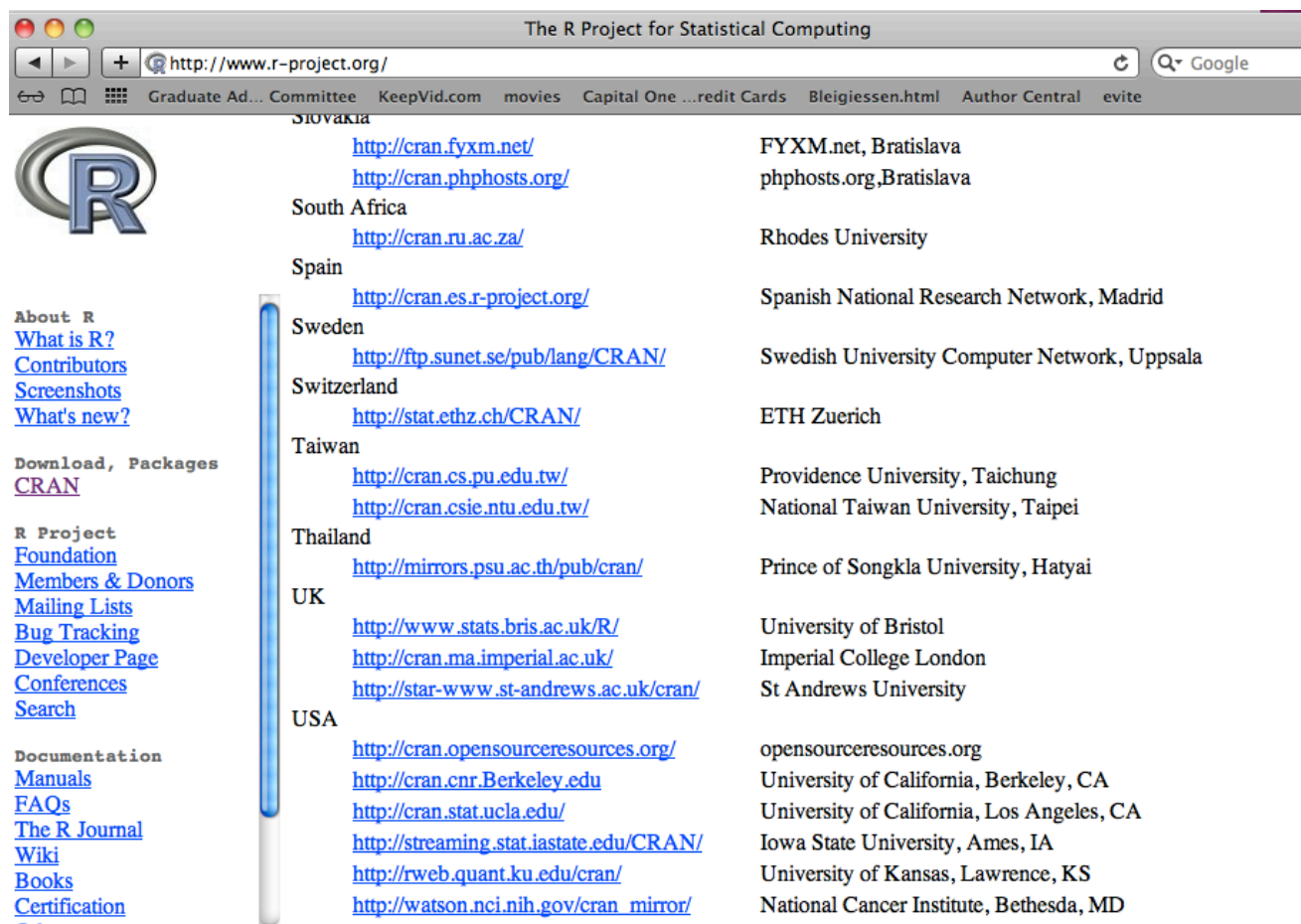
Groups
28
16
1
2

Getting Started:

- R is a free software environment for statistical computing and graphics. It compiles and runs on a wide variety of UNIX platforms, Windows and MacOS. To **download R**, please choose your preferred [CRAN mirror](#).
- If you have questions about R like how to download and install the software, or what the license terms are, please read our [answers to frequently asked questions](#) before you send an email.

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The screenshot shows a web browser window titled "The R Project for Statistical Computing". The address bar displays <http://www.r-project.org/>. The browser's bookmark bar includes links like "Graduate Ad...", "Committee", "KeepVid.com", "movies", "Capital One ...redit Cards", "Bleigiessen.html", "Author Central", and "evite".

The main content area features the R logo on the left and a list of mirrors on the right. The mirrors are organized by country, with links to local CRAN mirrors. The countries listed are Slovakia, South Africa, Spain, Sweden, Switzerland, Taiwan, Thailand, UK, and USA.

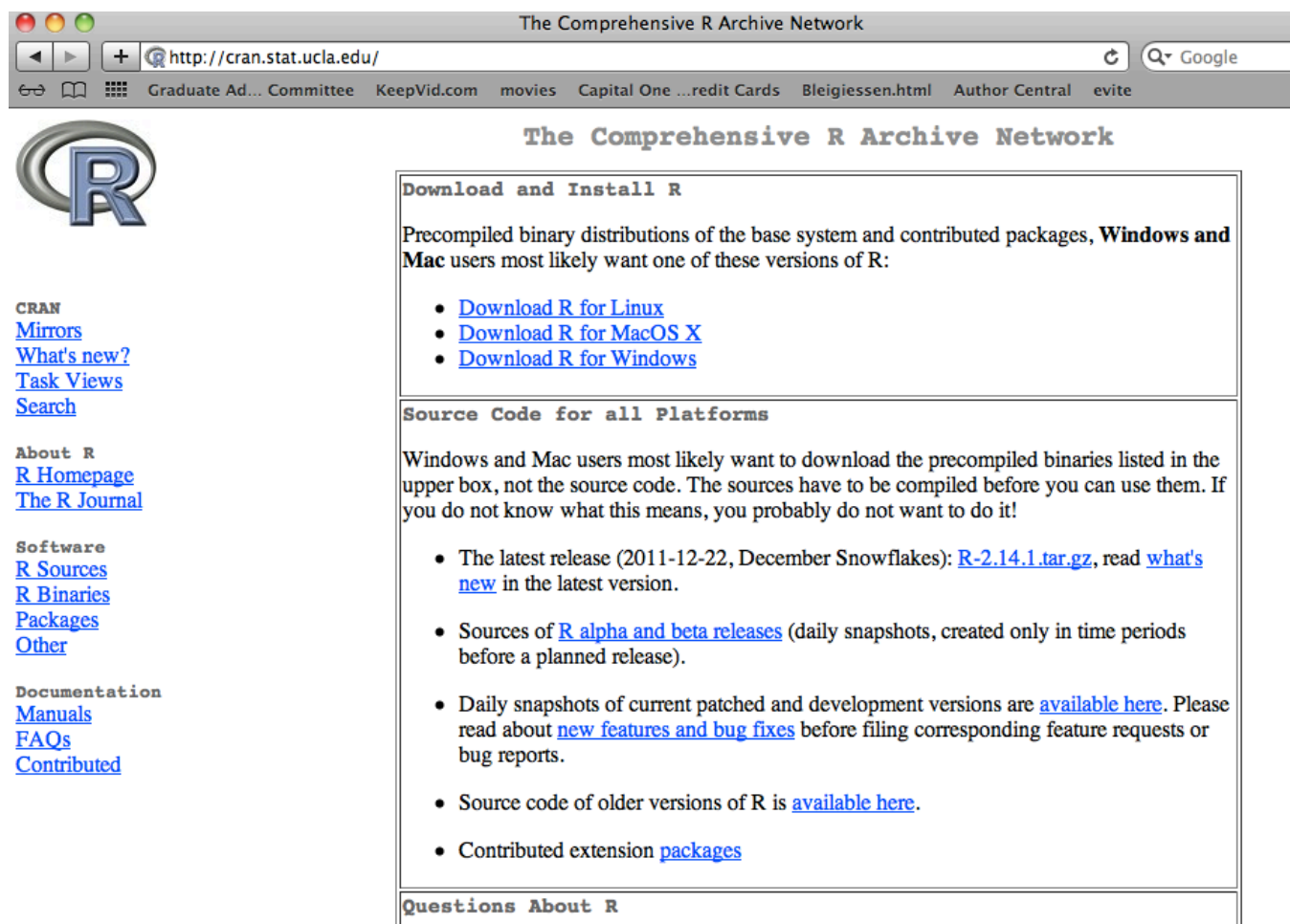
On the left side of the page, there are several navigation links:

- About R
 - [What is R?](#)
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The mirror list on the right includes the following entries:

- Slovakia**
 - <http://cran.fyxm.net/> FYXM.net, Bratislava
 - <http://cran.phphosts.org/> phphosts.org, Bratislava
- South Africa**
 - <http://cran.ru.ac.za/> Rhodes University
- Spain**
 - <http://cran.es.r-project.org/> Spanish National Research Network, Madrid
- Sweden**
 - <http://ftp.sunet.se/pub/lang/CRAN/> Swedish University Computer Network, Uppsala
- Switzerland**
 - <http://stat.ethz.ch/CRAN/> ETH Zuerich
- Taiwan**
 - <http://cran.cs.pu.edu.tw/> Providence University, Taichung
 - <http://cran.csie.ntu.edu.tw/> National Taiwan University, Taipei
- Thailand**
 - <http://mirrors.psu.ac.th/pub/cran/> Prince of Songkla University, Hatyai
- UK**
 - <http://www.stats.bris.ac.uk/R/> University of Bristol
 - <http://cran.ma.imperial.ac.uk/> Imperial College London
 - <http://star-www.st-andrews.ac.uk/cran/> St Andrews University
- USA**
 - <http://cran.opensourceresources.org/> opensourceresources.org
 - <http://cran.cnr.Berkeley.edu> University of California, Berkeley, CA
 - <http://cran.stat.ucla.edu/> University of California, Los Angeles, CA
 - <http://streaming.stat.iastate.edu/CRAN/> Iowa State University, Ames, IA
 - <http://rweb.quant.ku.edu/cran/> University of Kansas, Lawrence, KS
 - http://watson.nci.nih.gov/cran_mirror/ National Cancer Institute, Bethesda, MD

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The screenshot shows a web browser window titled "The Comprehensive R Archive Network" with the address bar displaying <http://cran.stat.ucla.edu/>. The browser's address bar also shows a search bar with the text "Google". The website's header features the R logo and the title "The Comprehensive R Archive Network". The main content area is divided into two sections: "Download and Install R" and "Source Code for all Platforms". The "Download and Install R" section provides precompiled binary distributions for Windows and Mac users, with links to download R for Linux, macOS X, and Windows. The "Source Code for all Platforms" section explains that Windows and Mac users should download precompiled binaries instead of source code, and provides links to the latest release (R-2.14.1.tar.gz), sources of R alpha and beta releases, daily snapshots of current patched and development versions, source code of older versions of R, and contributed extension packages. A sidebar on the left contains links to CRAN Mirrors, What's new?, Task Views, Search, About R, R Homepage, The R Journal, Software, R Sources, R Binaries, Packages, Other, Documentation, Manuals, FAQs, and Contributed.

The Comprehensive R Archive Network

Download and Install R

Precompiled binary distributions of the base system and contributed packages, **Windows and Mac** users most likely want one of these versions of R:

- [Download R for Linux](#)
- [Download R for MacOS X](#)
- [Download R for Windows](#)

Source Code for all Platforms

Windows and Mac users most likely want to download the precompiled binaries listed in the upper box, not the source code. The sources have to be compiled before you can use them. If you do not know what this means, you probably do not want to do it!

- The latest release (2011-12-22, December Snowflakes): [R-2.14.1.tar.gz](#), read [what's new](#) in the latest version.
- Sources of [R alpha and beta releases](#) (daily snapshots, created only in time periods before a planned release).
- Daily snapshots of current patched and development versions are [available here](#). Please read about [new features and bug fixes](#) before filing corresponding feature requests or bug reports.
- Source code of older versions of R is [available here](#).
- Contributed extension [packages](#)

Questions About R

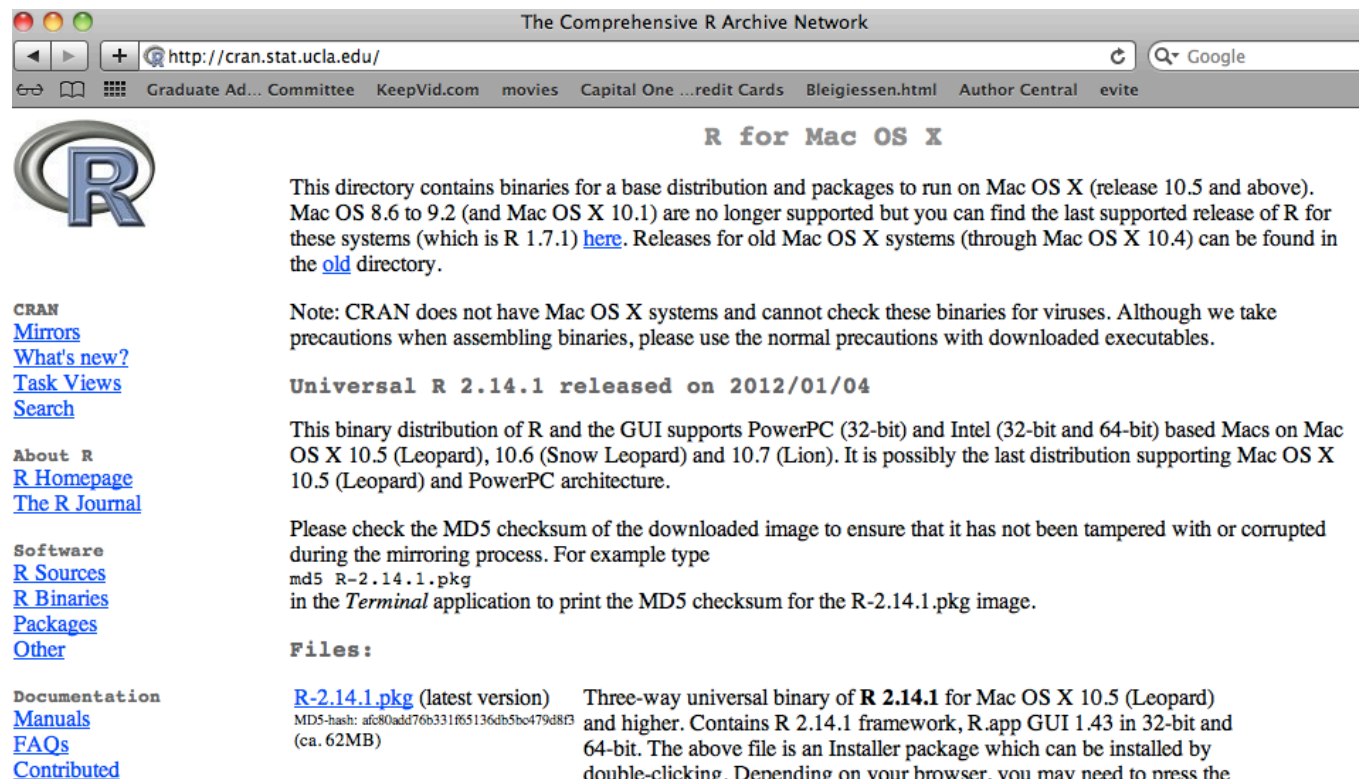
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The Comprehensive R Archive Network

<http://cran.stat.ucla.edu/>

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R for Mac OS X

This directory contains binaries for a base distribution and packages to run on Mac OS X (release 10.5 and above). Mac OS 8.6 to 9.2 (and Mac OS X 10.1) are no longer supported but you can find the last supported release of R for these systems (which is R 1.7.1) [here](#). Releases for old Mac OS X systems (through Mac OS X 10.4) can be found in the [old](#) directory.

Note: CRAN does not have Mac OS X systems and cannot check these binaries for viruses. Although we take precautions when assembling binaries, please use the normal precautions with downloaded executables.

Universal R 2.14.1 released on 2012/01/04

This binary distribution of R and the GUI supports PowerPC (32-bit) and Intel (32-bit and 64-bit) based Macs on Mac OS X 10.5 (Leopard), 10.6 (Snow Leopard) and 10.7 (Lion). It is possibly the last distribution supporting Mac OS X 10.5 (Leopard) and PowerPC architecture.

Please check the MD5 checksum of the downloaded image to ensure that it has not been tampered with or corrupted during the mirroring process. For example type

```
md5 R-2.14.1.pkg
```

in the *Terminal* application to print the MD5 checksum for the R-2.14.1.pkg image.

Files:

R-2.14.1.pkg (latest version) MD5-hash: afc80add76b33185136db5bc479d8f7 (ca. 62MB)	Three-way universal binary of R 2.14.1 for Mac OS X 10.5 (Leopard) and higher. Contains R 2.14.1 framework, R.app GUI 1.43 in 32-bit and 64-bit. The above file is an Installer package which can be installed by double-clicking. Depending on your browser, you may need to press the control key and click on this link to download the file.
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This package **only** contains the R framework, 32-bit GUI (R.app) and 64-bit GUI (R64.app). For Tcl/Tk libraries (needed if you want to use tcltk) and GNU Fortran (needed if you want to compile packages from sources that contain FORTRAN code) please see [the tools](#)

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5. Conditional probability, independence, & multiplication rule.

$P(A \& B)$ is often written “ $P(AB)$ ”.

“ $P(A \cup B)$ ” means $P(A \text{ or } B \text{ [or both]})$.

Conditional Probability:

$P(A \text{ given } B)$ [written “ $P(A|B)$ ”] = $P(AB) / P(B)$.

Independent: A and B are “independent” if $P(A|B) = P(A)$.

Fact (*multiplication rule for independent events*):

If A and B are independent, then $P(AB) = P(A) \times P(B)$

Fact (*general multiplication rule*):

$$P(AB) = P(A) P(B|A)$$

$$P(ABC\dots) = P(A) \times P(B|A) \times P(C|A\&B) \dots$$

6. Independence and dependence examples.

Independence: $P(A \mid B) = P(A)$ [and $P(B \mid A) = P(B)$].

So, when independent, $P(A \& B) = P(A)P(B \mid A) = P(A)P(B)$.

Reasonable to assume the following are independent:

- a) Outcomes on different rolls of a die.
- b) Outcomes on different flips of a coin.
- c) Outcomes on different spins of a spinner.
- d) Outcomes on different poker hands.
- e) Outcomes when sampling from a large population.

Ex: $P(\text{you get AA on 1st hand and I get AA on 2nd hand})$

$$= P(\text{you get AA on 1st}) \times P(\text{I get AA on 2nd})$$

$$= 1/221 \times 1/221 = 1/48841.$$

$P(\text{you get AA on 1st hand and I get AA on 1st hand})$

$$= P(\text{you get AA}) \times P(\text{I get AA} \mid \text{you have AA})$$

$$= 1/221 \times 1/(50 \text{ choose } 2) = 1/221 \times 1/1225 = 1/270725.$$

7. Negreanu and Elezra example: High Stakes Poker, 1/8/07.

Greenstein folds, Todd Brunson folds, Harman folds. Elezra calls \$600, Farha (K♠ J♥) raises to \$2600, Sheikhan folds. Negreanu calls, Elezra calls. Pot is \$8,800.

Flop: 6♠ 10♠ 8♥ .

Negreanu bets \$5000. Elezra raises to \$15000. Farha folds.

Negreanu thinks for 2 minutes..... then goes all-in for another \$88,000.

Elezra: 8♣ 6♣. (Elezra calls. Pot is \$214,800.)

Negreanu: A♦ 10♥ .

At this point, the odds on tv show 73% for Elezra and 25% for Negreanu.

They “run it twice”. First: 2♠ 4♥. Second time? A♥ 8♦!

P(Negreanu hits an A or 10 on turn & still loses)?

Given both their hands, and the flop, and the first “run”, what is $P(\text{Negreanu hits an A or 10 on the turn \& loses})$?

Since he can't lose if he hits a 10 on the turn, it's:

$P(\text{A on turn \& Negreanu loses})$

$$= P(\text{A on turn}) \times P(\text{Negreanu loses} \mid \text{A on the turn})$$

$$= \frac{3}{43} \times \frac{4}{42}$$

$$= 0.66\% \text{ (1 in 150.5)}$$

Note: this is very different from:

$P(\text{A or 10 on turn}) \times P(\text{Negreanu loses}),$

which would be about $\frac{5}{43} \times 73\% = 8.49\% \text{ (1 in 12)}$

8. Odds ratios.

Odds ratio of A = $P(A)/P(A^c)$

Odds *against* A = Odds ratio of $A^c = P(A^c)/P(A)$.

Ex: (from Phil Gordon's *Little Blue Book*, p189)

Day 3 of the 2001 WSOP, \$10,000 No-limit holdem championship.

613 players entered. Now 13 players left, at 2 tables.

Phil Gordon's table has 5 other players. Blinds are 3,000/6,000 + 1,000 antes.

Matusow has 400,000; Helmuth has 600,000; Gordon 620,000.

(the 3 other players have 100,000; 305,000; 193,000).

Matusow raises to 20,000. Next player folds.

Gordon's next, in the *cutoff seat* with $K\clubsuit K\spadesuit$ and re-raises to 100,000.

Next player folds. Helmuth goes all-in. Big blind folds. Matusow folds.

Gordon's decision.... Fold!

Odds against Gordon winning, if he called and Helmuth had AA?

What were the odds against Gordon winning, if he called and Helmuth had AA?

$P(\text{exactly one K, and no aces}) = 2 \times C(44,4) / C(48,5) \sim 15.9\%$.

$P(\text{two Kings on the board}) = C(46,3) / C(48,5) \sim 0.9\%$.

[also some chance of a straight, or a flush...]

Using www.cardplayer.com's poker odds calculator,

$P(\text{Gordon wins})$ is about 18%, so the odds against this are:

$$P(A^c)/P(A) = 82\% / 18\% = 4.6 \text{ (or “4.6 to 1” or “4.6:1”).}$$

9. $P(\text{you get dealt AA and flop a full house})?$

This = $P(\text{you get dealt AA}) \times P(\text{you flop a full house} \mid \text{AA})$

$$= C(4,2) / C(52,2) * P(\text{triplet or Axx} \mid \text{AA})$$

$$= 6/1326 * (12 * C(4,3) + 2*12*C(4,2))/C(50,3)$$

$$= 0.00443\%.$$

10. Deal til first ace appears. Let X = the *next* card after the ace.

$P(X = A\spadesuit)?$ $P(X = 2\clubsuit)?$

11. Which is more likely, given no info about your cards:

* flopping 3 of a kind,

or

* eventually making 4 of a kind?