Stat 100a, Introduction to Probability. Rick Paik Schoenberg

Outline for the day:

1. Discuss addiction and hw1 terminology.
2. Basic principle of counting.
3. Permutations and combinations.
4. *R*.
5. Conditional probability, independence, and multiplication rule.
6. Independence and dependence examples.
7. Negreanu and Elezra.
8. Odds ratios.
9. P(have AA and flop a full house)?
10. P(A♠ after 1st ace)?

♠ ♣ ♥ ♦

Hw1 terms.

Assume you never fold. I say this so one can't object "But I would never play 7♦ 5♦."

flop a straight flush. For example, you have 7♦ 5♦ and the flop is 4♦ 8♦ 6♦ .

flopping 2 pairs. For example, you have 7♦ 7♥ and the flop is 3♥ 3♠ J♥.

Or, you have 7♦ 3♥ and the flop is 7♥ 3♠ J♥.

pocket pair. When your two cards form a pair by themselves, like 7♦ 7♥.

face cards. K, Q, or J.

the nuts. Given the board, the best possible hand you could currently have in terms of the ranking order of poker hands, not in terms of probability of winning or improving in the future. For example, if the board is 7♥ 3♠ J♥ 8♦, then if you have 10♦ 9♦, then you have the nuts. If you have 10♥ 9♥, it would be slightly better in terms of probability of winning, but either way you have the nuts.

the unbreakable nuts. When you are guaranteed to win no matter what your opponent might have and no matter what board cards might come. In the above example where you have 10♥ 9♥ and the board is 7♥ 3♠ J♥ 8♦, you do not have the unbreakable nuts because you could lose for instance if the river is 9♠ and your opponent has Q♠ 10♦. However, if the board is 8♥ 7♥ 6♥ and you have 10♥ 9♥, then you have the unbreakable nuts.

in terms of. 3.2b is not easy. Assuming A and B are independent, you have to express the odds against (AB) using only $O_{A'}$ and $O_{B'}$. You can't use any other variables. In part a you expressed it in terms of P(A) and P(B), so just figure out how to convert P(A) into an expression of $O_{A'}$.

1. Addiction handout.

2. **Basic Principle of Counting.**

If there are a_1 distinct possible outcomes on trial #1, and for each of them, there are a_2 distinct possible outcomes on trial #2, then there are $a_1 \times a_2$ distinct possible *ordered* outcomes on both.

e.g. you get 1 card, opp. gets 1 card. # of distinct possibilities?
52×51. [ordered: (A♣, K♥) ≠ (K♥ , A♣) .]

In general, with j trials, each with a_i possibilities,
the # of distinct outcomes *where order matters* is $a_1 \times a_2 \times \ldots \times a_j$.

3. Permutations and Combinations.

e.g. you get 1 card, opp. gets 1 card.

# of distinct possibilities?

52 x 51. [ordered: (A♣, K♥) ≠ (K♥ , A♣) .]

Each such outcome, where order matters, is called a *permutation*.

Number of permutations of the deck? $52 \times 51 \times \ldots \times 1 = 52!$

$\sim 8.1 \times 10^{67}$

A *combination* is a collection of outcomes, where order *doesn't* matter.

e.g. in hold'em, how many *distinct* 2-card hands are possible?

52 x 51 if order matters, but then you'd be double-counting each

[since now (A♣, K♥) = (K♥ , A♣) .]

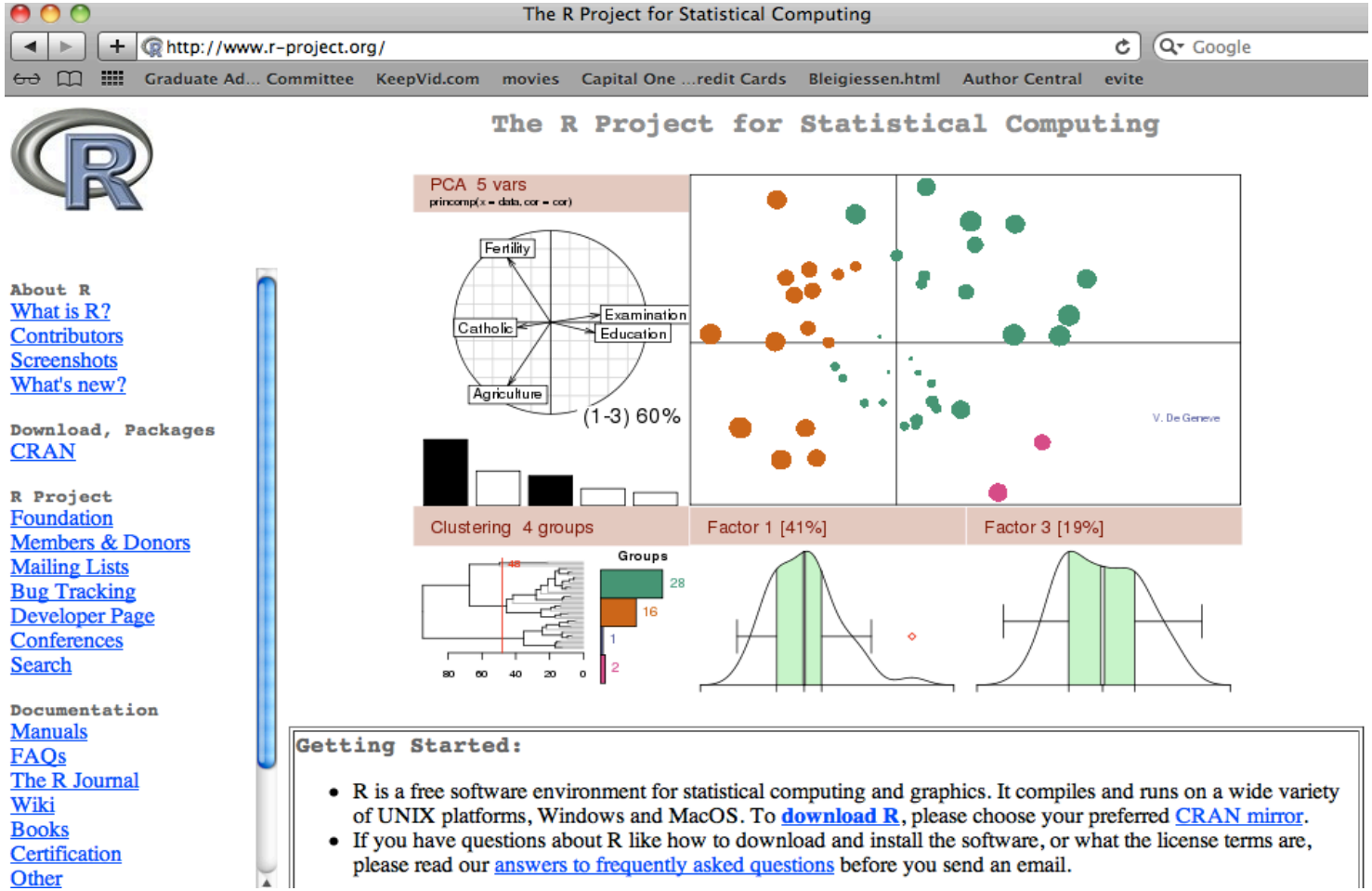
So, the number of *distinct* hands where *order doesn't matter* is

52 x 51 / 2.

In general, with n distinct objects, the # of ways to choose k *different* ones, *where order doesn't matter*, is

"n choose k" = choose(n,k) $= \frac{n!}{k!\,(n-k)!}$.

4. *R*. To download and install *R*, go directly to cran.stat.ucla.edu, or as it says in the book at the bottom of p240 (157 of 1st edition), you can start at www.r-project.org, in which case you click on "download *R*", scroll down to UCLA, and click on cran.stat.ucla.edu. From there, click on "download R for …", and then get the latest version.



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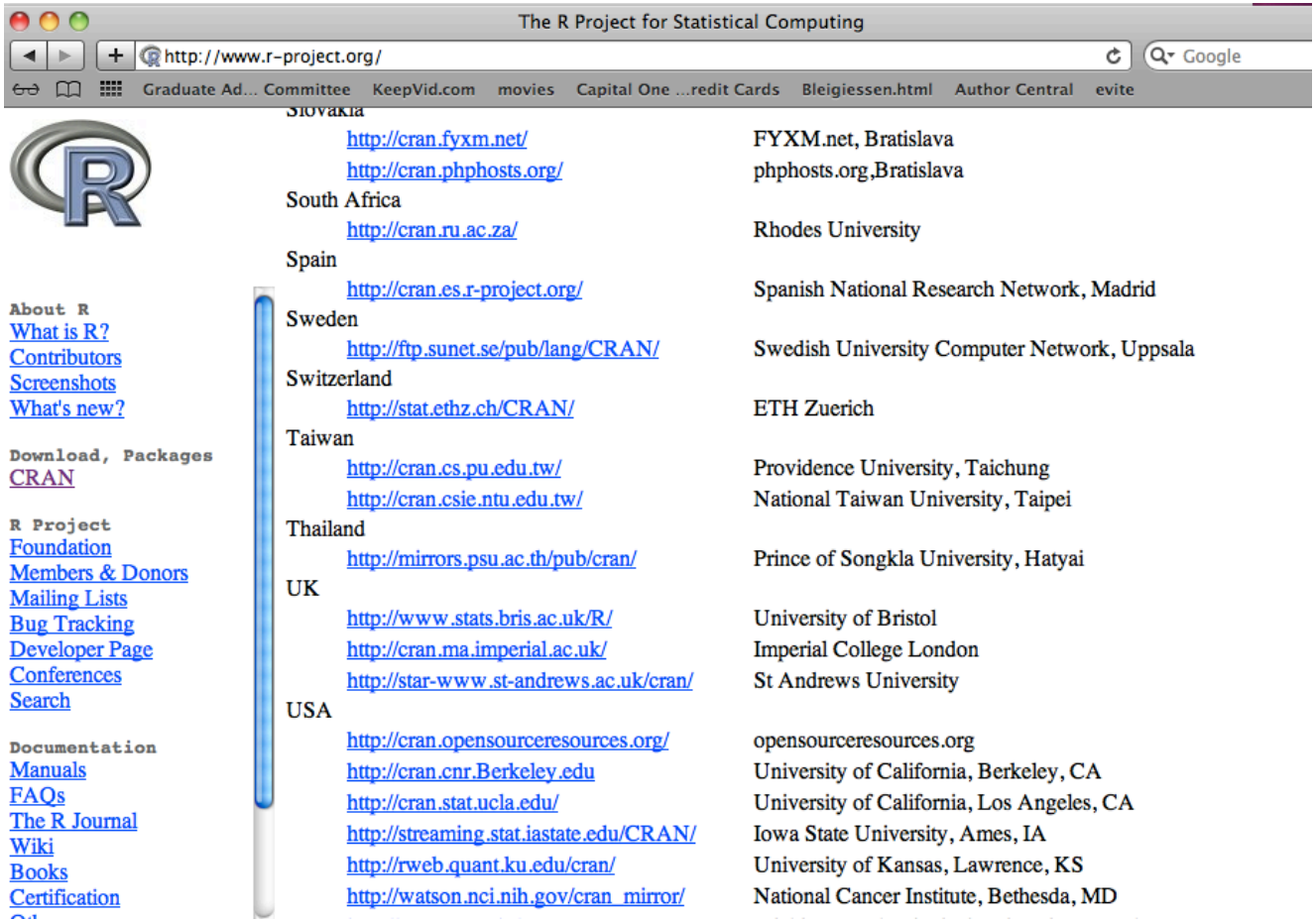

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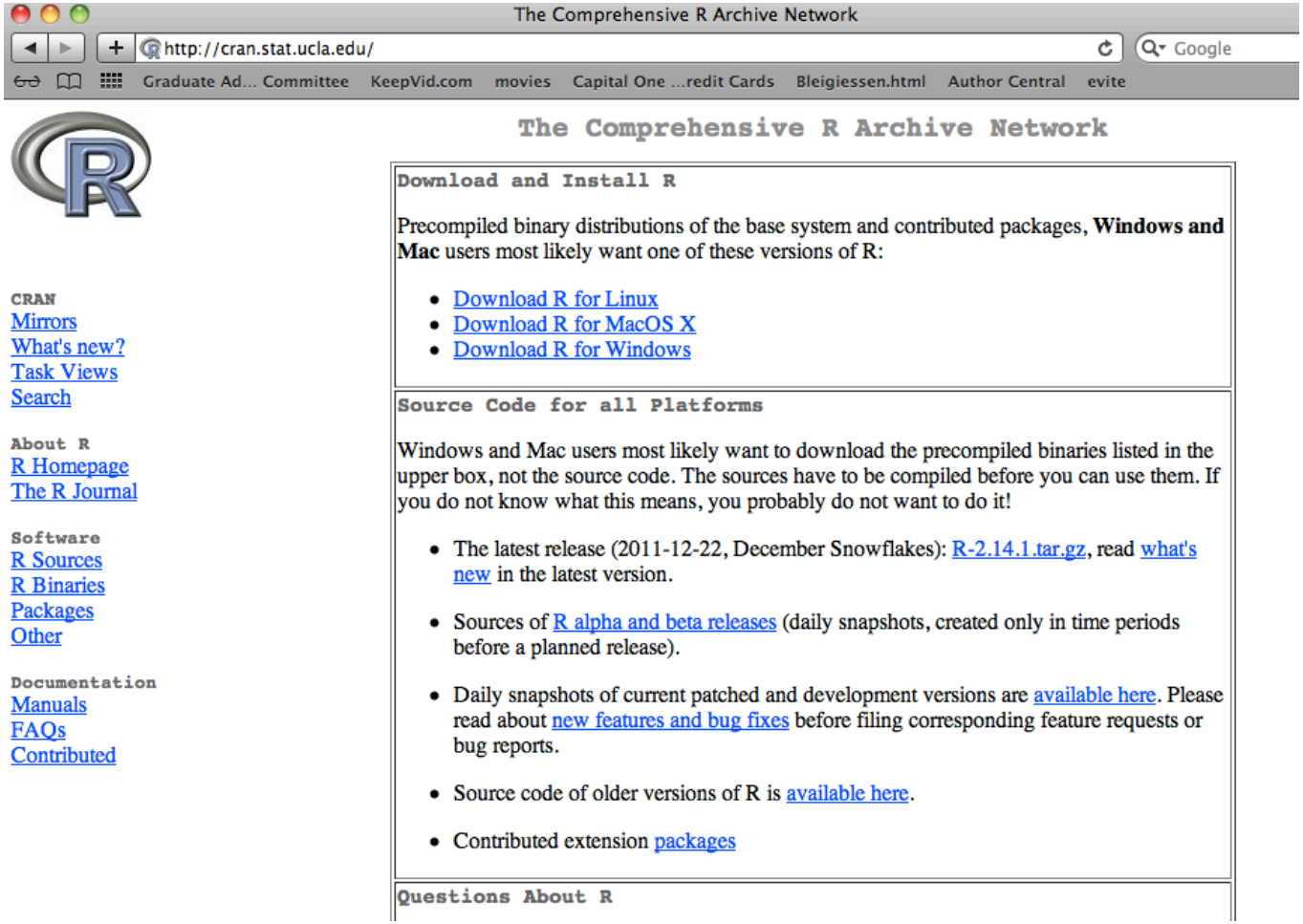

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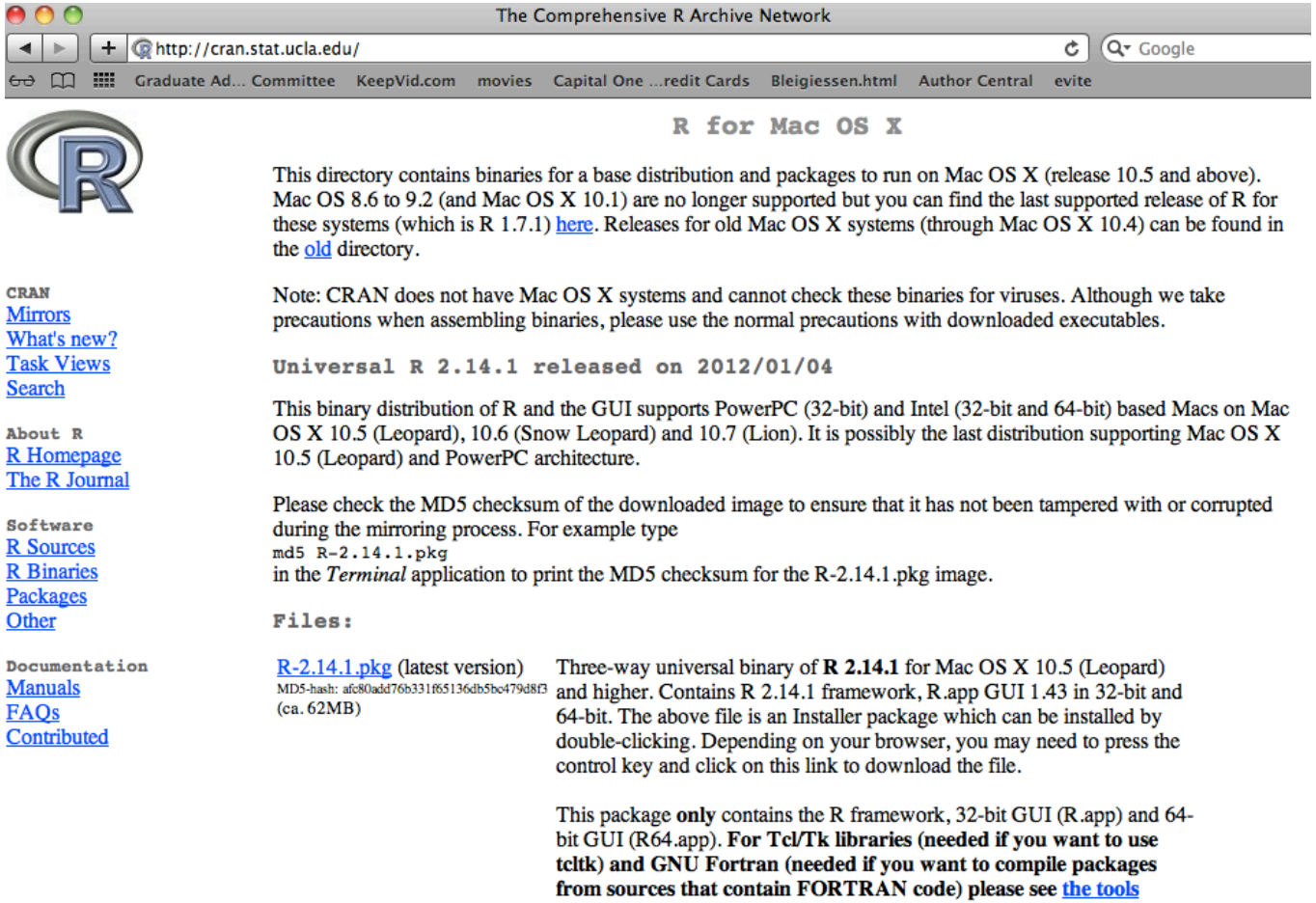

5. Conditional probability, independence, & multiplication rule.

P(A & B) is often written "P(AB)".

"P(A U B)" means P(A or B [or both]).

Conditional Probability:

P(A given B) [written"P(A|B)"] = P(AB) / P(B).

Independent: A and B are "independent" if P(A|B) = P(A).

Fact (*multiplication rule for independent events*):

If A and B are independent, then P(AB) = P(A) x P(B)

Fact (general *multiplication rule*):

P(AB) = P(A) P(B|A)

P(ABC…) = P(A) x P(B|A) x P(C|A&B) …

6. Independence and dependence examples.

Independence: $P(A \mid B) = P(A)$ [and $P(B|A) = P(B)$].

So, when independent, $P(A\&B) = P(A)P(B|A) = P(A)P(B)$.

Reasonable to assume the following are independent:

a) Outcomes on different rolls of a die.

b) Outcomes on different flips of a coin.

c) Outcomes on different spins of a spinner.

d) Outcomes on different poker hands.

e) Outcomes when sampling from a large population.

Ex: P(you get AA on 1st hand and I get AA on 2nd hand)

= P(you get AA on 1st) x P(I get AA on 2nd)

= 1/221 x 1/221 = 1/48841.

P(you get AA on 1st hand and I get AA on 1st hand)

= P(you get AA) x P(I get AA | you have AA)

= 1/221 x 1/(50 choose 2) = 1/221 x 1/1225 = 1/270725.

7. Negreanu and Elezra example: High Stakes Poker, 1/8/07.

Greenstein folds, Todd Brunson folds, Harman folds. Elezra calls $600, Farha (K♠ J♥) raises to $2600, Sheikhan folds. Negreanu calls, Elezra calls. Pot is $8,800.

Flop: 6♠ 10♠ 8♥ .

Negreanu bets $5000. Elezra raises to $15000. Farha folds.
Negreanu thinks for 2 minutes….. then goes all-in for another $88,000.
Elezra: 8♣ 6♣. (Elezra calls. Pot is $214,800.)
Negreanu: A♦ 10♥ .

At this point, the odds on tv show 73% for Elezra and 25% for Negreanu.
They "run it twice". First: 2♠ 4♥. Second time? A♥ 8♦!

P(Negreanu hits an A or 10 on turn & still loses)?

Given both their hands, and the flop, and the first "run", what is P(Negreanu hits an A or 10 on the turn & loses)?

Since he can't lose if he hits a 10 on the turn, it's:

P(A on turn & Negreanu loses)

= P(A on turn) x P(Negreanu loses | A on the turn)

= 3/43 x 4/42

= 0.66% (1 in 150.5)

Note: this is very different from:

P(A or 10 on turn) x P(Negreanu loses),

which would be about 5/43 x 73% = 8.49% (1 in 12)

8. Odds ratios.

Odds ratio of A = $P(A)/P(A^c)$

Odds *against* A = Odds ratio of A^c = $P(A^c)/P(A)$.

Ex: (from Phil Gordon's *Little Blue Book*, p189)

Day 3 of the 2001 WSOP, $10,000 No-limit holdem championship.

613 players entered. Now 13 players left, at 2 tables.

Phil Gordon's table has 5 other players. Blinds are 3,000/6,000 + 1,000 antes.

Matusow has 400,000; Helmuth has 600,000; Gordon 620,000.

(the 3 other players have 100,000; 305,000; 193,000).

Matusow raises to 20,000. Next player folds.

Gordon's next, in the *cutoff seat* with K♣ K♠ and re-raises to 100,000.

Next player folds. Helmuth goes all-in. Big blind folds. Matusow folds.

Gordon's decision…. Fold!

Odds against Gordon winning, if he called and Helmuth had AA?

What were the odds against Gordon winning, if he called and Helmuth had AA?

P(exactly one K, and no aces) = 2 x C(44,4) / C(48,5) ~ 15.9%.

P(two Kings on the board) = C(46,3) / C(48,5) ~ 0.9%.

[also some chance of a straight, or a flush…]

Using www.cardplayer.com's poker odds calculator,

P(Gordon wins) is about 18%, so the odds against this are:

$P(A^c)/P(A) = 82\% / 18\% = 4.6$ (or "*4.6 to 1" or "4.6:1"*).

9. P(you get dealt AA and flop a full house)?

This = P(you get dealt AA) x P(you flop a full house | AA)

= C(4,2) / C(52,2) * P(triplet or Axx | AA)

= 6/1326 * (12 * C(4,3) + 2*12*C(4,2))/C(50,3)

= 0.00443%.

10. Deal til first ace appears. Let X = the *next* card after the ace.

P(X = A♠)? P(X = 2♣)?

11. Which is more likely, given no info about your cards:

* flopping 3 of a kind,

or

* eventually making 4 of a kind?