Stat 100a, Introduction to Probability.

Outline for the day:

1. P(AA and full house) and P(A♦ K♦ and royal flush).
2. Negreanu and Elezra example.
3. Odds ratios.
4. Daniel vs. Gus.
5. P(flop 3 of a kind).
6. P(eventually make 4 of a kind).
7. P(A♠ after first ace).

Finish chapters 1-3 and start on ch4.

For problem 2.4, consider a royal flush an example of a straight flush. That is, calculate P(straight flush or royal flush). ♠ ♣ ♥ ♦

1. P(you get dealt AA and flop a full house)

= P(you get dealt AA) * P(you flop a full house | AA)

= C(4,2) / C(52,2) * P(triplet or Axx | AA)

= 6/1326 * (12 * C(4,3) + 2*12*C(4,2))/C(50,3)

= .00433%.

P(you are dealt A♦ K♦ and flop a royal flush)? This relates to the unbreakable nuts hw question in a way.

= P(you get dealt A♦ K♦) * P(you flop a royal flush | you have A♦ K♦)

= P(you get dealt A♦ K♦) * P(flop contains Q♦ J♦ 10♦ | you have A♦ K♦)

= 1 / C(52,2) * 1/C(50,3)

= 1 / 25,989,600.

Deal til first ace appears. Let X = the *next* card after the ace.

P(X = A♠)? P(X = 2♣)?

2. Negreanu and Elezra example: High Stakes Poker, 1/8/07.

Greenstein folds, Todd Brunson folds, Harman folds. Elezra calls $600, Farha (K♠ J♥) raises to $2600, Sheikhan folds. Negreanu calls, Elezra calls. Pot is $8,800.

Flop: 6♠ 10♠ 8♥ .

Negreanu bets $5000. Elezra raises to $15000. Farha folds.
Negreanu thinks for 2 minutes….. then goes all-in for another $88,000.
Elezra: 8♣ 6♣. (Elezra calls. Pot is $214,800.)
Negreanu: A♦ 10♥ .

At this point, the odds on tv show 73% for Elezra and 25% for Negreanu.
They "run it twice". First: 2♠ 4♥. Second time? A♥ 8♦!

P(Negreanu hits an A or 10 on turn & still loses)?

Given both their hands, and the flop, and the first "run", what is P(Negreanu hits an A or 10 on the turn & loses)?

Since he can't lose if he hits a 10 on the turn, it's:

P(A on turn & Negreanu loses)

= P(A on turn) x P(Negreanu loses | A on the turn)

= 3/43 x 4/42

= 0.66% (1 in 150.5)

Note: this is very different from:

P(A or 10 on turn) x P(Negreanu loses),

which would be about 5/43 x 73% = 8.49% (1 in 12)

3. Odds ratios.

Odds ratio of A = $P(A)/P(A^c)$

Odds *against* A = Odds ratio of A^c = $P(A^c)/P(A)$.

Ex: (from Phil Gordon's *Little Blue Book*, p189)

Day 3 of the 2001 WSOP, $10,000 No-limit holdem championship.

613 players entered. Now 13 players left, at 2 tables.

Phil Gordon's table has 5 other players. Blinds are 3,000/6,000 + 1,000 antes.

Matusow has 400,000; Helmuth has 600,000; Gordon 620,000.

(the 3 other players have 100,000; 305,000; 193,000).

Matusow raises to 20,000. Next player folds.

Gordon's next, in the *cutoff seat* with K♣ K♠ and re-raises to 100,000.

Next player folds. Helmuth goes all-in. Big blind folds. Matusow folds.

Gordon's decision…. Fold!

Odds against Gordon winning, if he called and Helmuth had AA?

What were the odds against Gordon winning, if he called and Helmuth had AA?

P(exactly one K, and no aces) = 2 x C(44,4) / C(48,5) ~ 15.9%.

P(two Kings on the board) = C(46,3) / C(48,5) ~ 0.9%.

[also some chance of a straight, or a flush…]

Using www.cardplayer.com's poker odds calculator,

P(Gordon wins) is about 18%, so the odds against this are:

$P(A^c)/P(A) = 82\% / 18\% = 4.6$ (or "*4.6 to 1" or "4.6:1"*).

4. High Stakes Poker, Daniel vs. Gus.

Which is more likely, given no info about your cards:

* flopping 3 of a kind,

or

* eventually making 4 of a kind?

5. P(flop 3-of-a-kind)?

[including case where all 3 are on board, and *not including full houses*]

Key idea: forget order! Consider all combinations of your 2 cards and the flop.

Sets of 5 cards. Any such combo is equally likely! choose(52,5) different ones.

P(flop 3 of a kind) = # of different 3 of a kinds / choose(52,5)

How many different 3 of a kind combinations are possible?

13 * choose(4,3) different choices for the triple.

For each such choice, there are choose(12,2) choices left for the numbers on the other 2 cards, and for each of these numbers, there are 4 possibilities for its suit.

So, P(flop 3 of a kind) = 13 * choose(4,3) * choose(12,2) * 4 * 4 / choose(52,5)

~ 2.11%, or 1 in 47.3.

P(flop 3 of a kind or a full house) = 13 * choose(4,3) * choose(48,2) / choose(52,5)

~ 2.26%, or 1 in 44.3.

6. P(eventually make 4-of-a-kind)? [including case where all 4 are on board]

Again, just forget card order, and consider all collections of 7 cards.

Out of choose(52,7) different combinations, each equally likely, how many of them involve 4-of-a-kind?

13 choices for the 4-of-a-kind.

For each such choice, there are choose(48,3) possibilities for the other 3 cards.

So, P(4-of-a-kind) = 13 * choose(48,3) / choose(52,7) ~ 0.168%, or 1 in 595.

P(you get dealt AA and flop a full house)?

This = P(you get dealt AA) x P(you flop a full house | AA)

= C(4,2) / C(52,2) * P(triplet or Axx | AA)

= 6/1326 * (12 * C(4,3) + 2*12*C(4,2))/C(50,3)

= 0.00443%.

7. Deal til first ace appears. Let X = the *next* card after the ace.

P(X = A♠)? P(X = 2♣)?

Deal til first ace appears. Let X = the *next* card after the ace.

P(X = A♠)? P(X = 2♣)?

(a) How many permutations of the 52 cards are there?

52!

(b) How many of these perms. have A♠ right after the 1st ace?

(i) How many perms of the *other* 51 cards are there?

51!

(ii) For *each* of these, imagine putting the A♠ right after the 1st ace.

1:1 correspondence between permutations of the other 51 cards & permutations of 52 cards such that A♠ is right after 1st ace.

So, the answer to question (b) is 51!.

Answer to the overall question is 51! / 52! = 1/52.

Obviously, same goes for 2♣.