Stat 100a, Introduction to Probability.

Outline for the day:

1. Hansen and Negreanu.
2. Flop 3 of a kind or eventually get 4 of a kind.
3. Bayes's rule.
4. Random variables.
5. cdf, pmf, and density.

HW1 is due Tue Oct15 in the first 5 min of class.
Exam1 is Tue Oct29.
No lecture or OH Thu Oct24!
Read through chapter 5. ♠ ♣ ♥ ♦

1. High Stakes Poker, Daniel vs. Gus.

Which is more likely, given no info about your cards:

* flopping 3 of a kind,

or

* eventually making 4 of a kind?

2. P(flop 3-of-a-kind)?

[including case where all 3 are on board, and *not including full houses*]

<u>Key idea</u>: forget order! Consider all combinations of your 2 cards and the flop.

Sets of 5 cards. Any such combo is equally likely! choose(52,5) different ones.

P(flop 3 of a kind) = # of different 3 of a kinds / choose(52,5)

How many different 3 of a kind combinations are possible?

13 * choose(4,3) different choices for the triple.

For each such choice, there are choose(12,2) choices left for the numbers on the other 2 cards, and for each of these numbers, there are 4 possibilities for its suit.

So, P(flop 3 of a kind) = 13 * choose(4,3) * choose(12,2) * 4 * 4 / choose(52,5)

~ 2.11%, or 1 in 47.3.

P(flop 3 of a kind or a full house) = 13 * choose(4,3) * choose(48,2) / choose(52,5)

~ 2.26%, or 1 in 44.3.

P(eventually make 4-of-a-kind)? [including case where all 4 are on board]

Again, just forget card order, and consider all collections of 7 cards.

Out of choose(52,7) different combinations, each equally likely, how many of them involve 4-of-a-kind?

13 choices for the 4-of-a-kind.

For each such choice, there are choose(48,3) possibilities for the other 3 cards.

So, P(4-of-a-kind) = 13 * choose(48,3) / choose(52,7) ~ 0.168%, or 1 in 595.

3. Bayes's rule.

Suppose that B_1, B_2 , B_n are disjoint events and that exactly one of them must occur.

Suppose you want $P(B_1 | A)$, but you only know $P(A | B_1)$, $P(A | B_2)$, etc.,

and you also know $P(B_1), P(B_2), \ldots, P(B_n)$.

Bayes' Rule: If $B_{1, \ldots,} B_n$ are disjoint events with $P(B_1 \text{ or } \ldots \text{ or } B_n) = 1$, then

$P(B_i | A) = P(A | B_i) * P(B_i) \div [\sum P(A | B_j)P(B_j)]$.

Why? Recall: $P(X | Y) = P(X \& Y) \div P(Y)$. So $P(X \& Y) = P(X | Y) * P(Y)$.

$$P(B_1 | A) = P(A \& B_1) \div P(A)$$

$$= P(A \& B_1) \div [\, P(A \& B_1) + P(A \& B_2) + \ldots + P(A \& B_n)\,]$$

$$= P(A | B_1) * P(B_1) \div [\, P(A | B_1)P(B_1) + P(A | B_2)P(B_2) + \ldots + P(A | B_n)P(B_n)\,].$$

Bayes's rule, continued.

Bayes's rule: If $B_1, \ldots, B_n$ are disjoint events with $P(B_1 \text{ or } \ldots \text{ or } B_n) = 1$, then

$P(B_i \mid A) = P(A \mid B_i) * P(B_i) \div [\sum P(A \mid B_j)P(B_j)]$.

See example 3.4.1, p50. If a test is 95% accurate and 1% of the pop. has a condition, then given a random person from the population,

P(she has the condition | she tests positive)

= P(cond | +)

= P(+ | cond) P(cond) ÷ [P(+ | cond) P(cond) + P(+ | no cond) P(no cond)]

= 95% x 1% ÷ [95% x 1% + 5% x 99%]

~ 16.1%.

Tests for rare conditions must be extremely accurate.

Bayes' rule example.

Suppose P(your opponent has the nuts) = 1%, and P(opponent has a weak hand) = 10%.

Your opponent makes a huge bet. Suppose she'd only do that with the nuts or a weak hand, and that P(huge bet | nuts) = 100%, and P(huge bet | weak hand) = 30%.

What is P(nuts | huge bet)?

P(nuts | huge bet) =

$$\frac{P(\text{huge bet} \mid \text{nuts}) * P(\text{nuts})}{P(\text{huge bet} \mid \text{nuts})\, P(\text{nuts}) + P(\text{huge bet} \mid \text{horrible hand})\, P(\text{horrible hand})}$$

$$= \frac{100\% * 1\%}{100\% * 1\% + 30\% * 10\%}$$

= **25%.**

4. Random variables.

A *variable* is something that can take different numeric values.

A *random variable* (X) can take different numeric values with different probabilities.

X is *discrete* if all its possible values can be listed. If X can take any value in an interval like say [0,1], then X is *continuous*.

Ex. Two cards are dealt to you. Let X be 1 if you get a pair, and X is 0 otherwise.

P(X is 1) = 3/51 ~ 5.9%. P(X is 0) ~ 94.1%.

Ex. A coin is flipped, and X=20 if heads, X=10 if tails.

The *distribution* of X means all the information about all the possible values X can take, along with their probabilities.

5. cdf, pmf, and density (pdf).

Any random variable has a *cumulative distribution function* (cdf):

$$F(b) = P(X \leq b).$$

If X is discrete, then it has a *probability mass function* (pmf):

$$f(b) = P(X = b).$$

Continuous random variables are often characterized by their *probability density functions* (pdf, or *density*):

a function f(x) such that $P(X \text{ is in } B) = \int_B f(x)\,dx$.