Stat 100a, Introduction to Probability. Rick Paik Schoenberg

Outline for the day:

1. Discuss addiction.
2. Permutations and combinations.
3. Conditional probability, independence, and multiplication rule.
4. Independence and dependence examples.
5. Negreanu and Elezra.
6. Odds ratios.
7. P(have AA and flop a full house)?
8. P(A♠ after 1st ace)?
9. 3 of a kind on the flop vs. 4 of a kind eventually?

http://www.stat.ucla.edu/~frederic/221/F21

♠ ♣ ♥ ♦

1. Addiction handout.

2. Permutations and Combinations.

e.g. you get 1 card, opp. gets 1 card.

# of distinct possibilities?

52 x 51. [ordered: (A♣, K♥) ≠ (K♥ , A♣) .]

Each such outcome, where order matters, is called a *permutation*.

Number of permutations of the deck? 52 x 51 x … x 1 = 52!

$\sim 8.1 \times 10^{67}$

A *combination* is a collection of outcomes, where order *doesn't* matter.

e.g. in hold'em, how many *distinct* 2-card hands are possible?

52 x 51 if order matters, but then you'd be double-counting each

[since now (A♣, K♥) = (K♥ , A♣) .]

So, the number of *distinct* hands where *order doesn't matter* is

52 x 51 / 2.

In general, with n distinct objects, the # of ways to choose k *different* ones, *where order doesn't matter*, is

"n choose k" = C(n,k) = choose(n,k) $= \dfrac{n!}{k!\,(n-k)!}$.

3. Conditional probability, independence, & multiplication rule.

P(A & B) is often written “P(AB)”.

“P(A U B)” means P(A or B [or both]).

Conditional Probability:

P(A given B) [written“P(A|B)”] = P(AB) / P(B).

Independent: A and B are “independent” if P(A|B) = P(A).

Fact (*multiplication rule for independent events*):

If A and B are independent, then P(AB) = P(A) x P(B)

Fact (general *multiplication rule*):

P(AB) = P(A) P(B|A)

P(ABC…) = P(A) x P(B|A) x P(C|A&B) …

4. Independence and dependence examples.

Independence: P(A | B) = P(A) [and P(B|A) = P(B)].

So, when independent, P(A&B) = P(A)P(B|A) = P(A)P(B).

Reasonable to assume the following are independent:

a) Outcomes on different rolls of a die.

b) Outcomes on different flips of a coin.

c) Outcomes on different spins of a spinner.

d) Outcomes on different poker hands.

e) Outcomes when sampling from a large population.

Ex: P(you get AA on 1st hand and I get AA on 2nd hand)

= P(you get AA on 1st) x P(I get AA on 2nd)

= 1/221 x 1/221 = 1/48841.

P(you get AA on 1st hand and I get AA on 1st hand)

= P(you get AA) x P(I get AA | you have AA)

= 1/221 x 1/(50 choose 2) = 1/221 x 1/1225 = 1/270725.

5. Negreanu and Elezra example: High Stakes Poker, 1/8/07.

Greenstein folds, Todd Brunson folds, Harman folds. Elezra calls $600, Farha (K♠ J♥) raises to $2600, Sheikhan folds. Negreanu calls, Elezra calls. Pot is $8,800.

Flop: 6♠ 10♠ 8♥ .

Negreanu bets $5000. Elezra raises to $15000. Farha folds.

Negreanu thinks for 2 minutes….. then goes all-in for another $88,000.

Elezra: 8♣ 6♣. (Elezra calls. Pot is $214,800.)

Negreanu: A♦ 10♥ .

At this point, the odds on tv show 73% for Elezra and 25% for Negreanu.

They "run it twice". First: 2♠ 4♥. Second time? A♥ 8♦!

P(Negreanu hits an A or 10 on turn & still loses)?

Given both their hands, and the flop, and the first “run”, what is P(Negreanu hits an A or 10 on the turn & loses)?

Since he can’t lose if he hits a 10 on the turn, it’s:

P(A on turn & Negreanu loses)

= P(A on turn) x P(Negreanu loses | A on the turn)

= 3/43 x 4/42

= 0.66% (1 in 150.5)

Note: this is very different from:

P(A or 10 on turn) x P(Negreanu loses),

which would be about 5/43 x 73% = 8.49% (1 in 12)

6. Odds ratios.

Odds ratio of A = P(A)/P(A^c)

Odds *against* A = Odds ratio of A^c = P(A^c)/P(A).

Ex: (from Phil Gordon's *Little Blue Book*, p189)

Day 3 of the 2001 WSOP, $10,000 No-limit holdem championship.

613 players entered. Now 13 players left, at 2 tables.

Phil Gordon's table has 5 other players. Blinds are 3,000/6,000 + 1,000 antes.

Matusow has 400,000; Helmuth has 600,000; Gordon 620,000.

(the 3 other players have 100,000; 305,000; 193,000).

Matusow raises to 20,000. Next player folds.

Gordon's next, in the *cutoff seat* with K♣ K♠ and re-raises to 100,000.

Next player folds. Helmuth goes all-in. Big blind folds. Matusow folds.

Gordon's decision…. Fold!

Odds against Gordon winning, if he called and Helmuth had AA?

What were the odds against Gordon winning, if he called and Helmuth had AA?

P(exactly one K, and no aces) = 2 x C(44,4) / C(48,5) ~ 15.9%.

P(two Kings on the board) = C(46,3) / C(48,5) ~ 0.9%.

[also some chance of a straight, or a flush…]

Using www.cardplayer.com's poker odds calculator,

P(Gordon wins) is about 18%, so the odds against this are:

$P(A^c)/P(A)$ = 82% / 18% = 4.6 (or *"4.6 to 1" or "4.6:1")*.

7. P(you get dealt AA and flop a full house)?

This = P(you get dealt AA) x P(you flop a full house | AA)

= C(4,2) / C(52,2) * P(triplet or Axx | AA)

= 6/1326 * (12 * C(4,3) + 2*12*C(4,2))/C(50,3)

= 0.00443%.

8. Deal til first ace appears. Let X = the *next* card after the ace.

P(X = A♠)? P(X = 2♣)?

9. Which is more likely, given no info about your cards:

* flopping 3 of a kind,

or

* eventually making 4 of a kind?