Stat 100a, Introduction to Probability.

Outline for the day:

1. P(AA and full house) and P(A♦ K♦ and royal flush).
2. Daniel vs. Gus.
3. P(flop 3 of a kind).
4. P(eventually make 4 of a kind).
5. P(A♠ after first ace).

Finish chapters 1-3 and start on ch4.

For problem 2.4, consider a royal flush an example of a straight flush. That is, calculate P(straight flush or royal flush). ♠ ♣ ♥ ♦

1. P(you get dealt AA and flop a full house)

= P(you get dealt AA) * P(you flop a full house | AA)

= C(4,2) / C(52,2) * P(triplet or Axx | AA)

= 6/1326 * (12 * C(4,3) + 2*12*C(4,2))/C(50,3)

= .00433%.

P(you are dealt A♦ K♦ and flop a royal flush)? This relates to the unbreakable nuts hw question in a way.

= P(you get dealt A♦ K♦) * P(you flop a royal flush | you have A♦ K♦)

= P(you get dealt A♦ K♦) * P(flop contains Q♦ J♦ 10♦ | you have A♦ K♦)

= 1 / C(52,2) * 1/C(50,3)

= 1 / 25,989,600.

2. High Stakes Poker, Daniel vs. Gus.

Which is more likely, given no info about your cards:

* flopping 3 of a kind,

or

* eventually making 4 of a kind?

3. P(flop 3 of a kind)?

[including case where all 3 are on board, and *not including full houses*]

Key idea: forget order! Consider all combinations of your 2 cards and the flop.

Sets of 5 cards. Any such combo is equally likely! choose(52,5) different ones.

P(flop 3 of a kind) = # of different 3 of a kinds / choose(52,5)

How many different 3 of a kind combinations are possible?

13 * choose(4,3) different choices for the triple.

For each such choice, there are choose(12,2) choices left for the numbers on the other 2 cards, and for each of these numbers, there are 4 possibilities for its suit.

So, P(flop 3 of a kind) = 13 * choose(4,3) * choose(12,2) * 4 * 4 / choose(52,5)

~ 2.11%, or 1 in 47.3.

P(flop 3 of a kind or a full house) = 13 * choose(4,3) * choose(48,2) / choose(52,5)

~ 2.26%, or 1 in 44.3.

4. P(eventually make 4 of a kind)?   [including case where all 4 are on board]

Again, just forget card order, and consider all collections of 7 cards.

Out of choose(52,7) different combinations, each equally likely, how many of them involve 4-of-a-kind?

13 choices for the 4-of-a-kind.

For each such choice, there are choose(48,3) possibilities for the other 3 cards.

So, P(4-of-a-kind) = 13 * choose(48,3) / choose(52,7) ~ 0.168%, or 1 in 595.

5. Deal til first ace appears. Let X = the *next* card after the ace.

P(X = A♠)? P(X = 2♣)?

(a) How many permutations of the 52 cards are there?

52!

(b) How many of these perms. have A♠ right after the 1st ace?

(i) How many perms of the *other* 51 cards are there?

51!

(ii) For *each* of these, imagine putting the A♠ right after the 1st ace.

1:1 correspondence between permutations of the other 51 cards & permutations of 52 cards such that A♠ is right after 1st ace.

So, the answer to question (b) is 51!.

Answer to the overall question is 51! / 52! = 1/52.

Obviously, same goes for 2♣.