Stat 100a: Introduction to Probability.

Outline for the day:

1. Expected value and pot odds. 2006 WSOP, Elezra and Violette.
2. P(flop 4 of a kind).
3. Variance and SD.

I will assign you to teams on Tuesday.

HW1 is due Oct14 in the first 5 min of class.

Exam1 is Oct26.

Read through chapter 5. ♠ ♣ ♥ ♦

Expected value and pot odds.

Suppose someone bets (or raises) you, going all-in. What should your chances of winning be in order for you to correctly call?

Let B = the amount bet to you, i.e. the additional amount you'd need to put in if you want to call. So, if you bet 100 & your opponent with 800 left went all-in, B = 700.

Let POT = the amount in the pot right now (including your opponent's bet).

Let p = your probability of winning the hand if you call. So prob. of losing = 1-p.

Let CHIPS = the number of chips you have right now.

If you call, then E[your chips at end] = (CHIPS - B)(1-p) + (CHIPS + POT)(p)

= CHIPS(1-p+p) - B(1-p) + POT(p) = CHIPS - B + Bp + POTp

If you fold, then E[your chips at end] = CHIPS.

You want your expected number of chips to be maximized, so it's worth calling if

-B + Bp + POTp > 0, i.e. if **p > B / (B+POT).**

From previous slide, to call an all-in, need P(win) > B ÷ (B+pot).
Expressed as an *odds ratio*, this is sometimes referred to as *pot odds* or *express odds*.

If the bet is not all-in & another betting round is still to come, need

P(win) > wager ÷ (wager + winnings),

where winnings = pot + amount you'll win on later betting rounds,
wager = total amount you will wager including the current round & later rounds, assuming no folding.

The terms *Implied-odds* / *Reverse-implied-odds* describe the cases where *winnings > pot* or where *wager > B*, respectively. See p66.

You will not be tested on implied or reverse implied odds in this course.

Example: 2006 World Series of Poker (WSOP). ♠ ♣ ♥ ♦

Blinds: 200,000/400,000, + 50,000 ante.

Jamie Gold (4♠ 3♣): 60 million chips. Calls.

Paul Wasicka (8♠ 7♠): 18 million chips. Calls.

Michael Binger (A♦ 10♦): 11 million chips. Raises to \$1,500,000.

Gold & Wasicka call. (pot = 4,650,000) Flop: 6♠ 10♣ 5♠.

- Wasicka checks, Binger bets \$3,500,000. (pot = 8,150,000)
- Gold moves all-in for 16,450,000. (pot = 24,600,000)
- Wasicka folds. Q: Based on expected value, should he have called?

If Binger will fold, then Wasicka's chances to beat Gold must be at least

16,450,000 / (24,600,000 + 16,450,000) = 40.1%.

If Binger calls, it's a bit complicated, but basically Wasicka's chances must be at least 16,450,000 / (24,600,000 + 16,450,000 + 5,950,000) = 35.0%.

Pot odds example, *Poker Superstars Invitational Tournament*, FSN, October 2005.

Ted Forrest: 1 million chips
Freddy Deeb: 825,000 Blinds: 15,000 / 30,000
Cindy Violette: 650,000
Eli Elezra: 575,000

* Elezra raises to 100,000
* Forrest folds.
* Deeb, the small blind, folds.
* Violette, the big blind with K♦ J♦, calls.

* The flop is: 2♦ 7♣ A♦

* Violette bets 100,000. (pot = 315,000).
* Elezra raises all-in to 475,000. (pot = 790,000).

So, it's 375,000 more to Violette. She folds.

Q: Based on expected value, should she have called?
Her chances must be at least 375,000 / (790,000 + 375,000) = 32%.

Violette has K♦ J♦.   The flop is: 2♦ 7♣ A♦.

Q: Based on expected value, should she have called?

Her chances must be at least 375,000 / (790,000 + 375,000) = 32%.

vs.	AQ: 38%.	AK: 37%	AA: 26%	77: 26%	A7: 31%
	A2: 34%	72: 34%	TT: 54%	T9: 87%	73: 50%

Harrington's principle: always assume at least a 10% chance that opponent is bluffing.

Bayesian approach: average all possibilities, weighting them by their likelihood.

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Reality: Elezra had 7♦ 3♥. Her chances were 51%. Bad fold.
What was her prob. of winning (given just her cards and Elezra's, and the flop)?
Of choose(45,2) = 990 combinations for the turn & river, how many give her the win?
First, how many outs did she have? eight ♦s + 3 kings + 3 jacks = 14.
She wins with (out, out) or (out, nonout) or (non-♦ Q, non-♦ T)
choose(14,2) + 14 x 31 + 3 * 3 = 534
but not (k or j, 7 or non-♦ 3) and not (3♦ , 7 or non-♦ 3)
- 6 * 4 - 1 * 4 = 506.
So the answer is 506 / 990 = 51.1%.

P(flop 4 of a kind).

Suppose you're all in next hand, no matter what cards you get.

P(flop 4 of a kind) = 13*48 / choose(52,5) = 0.024% = 1 in **4165**.

P(flop 4 of a kind | pocket pair)?

No matter which pocket pair you have, there are choose(50,3) possible flops, each equally likely, and how many of them give you 4-of-a-kind?

48. (e.g. if you have 7♠ 7♥, then need to flop 7♦ 7♣ x, & there are 48 choices for x)

So P(flop 4-of-a-kind | pp) = 48/choose(50,3) = 0.245% = 1 in **408**.

Variance and SD.

Expected Value: $E(X) = \mu = \sum k\ P(X=k)$.

Variance: $V(X) = \sigma^2 = E[(X-\mu)^2]$. Turns out this $= E(X^2) - \mu^2$.

Standard deviation $= \sigma = \sqrt{V(X)}$. Indicates how far an observation would *typically* deviate from μ.

Examples:

<u>Game 1.</u> Say X = \$4 if red card, X = \$-5 if black.

$E(X) = (\$4)(0.5) + (\$-5)(0.5) = -\$0.50$.

$E(X^2) = (\$4^2)(0.5) + (\$-5^2)(0.5) = (\$16)(0.5) + (\$25)(0.5) = \$20.5$.

So $\sigma^2 = E(X^2) - \mu^2 = \$20.5 - \$-0.50^2 = \$20.25$. $\boldsymbol{\sigma = \$4.50}$.

<u>Game 2.</u> Say X = \$1 if red card, X = \$-2 if black.

$E(X) = (\$1)(0.5) + (\$-2)(0.5) = -\$0.50$.

$E(X^2) = (\$1^2)(0.5) + (\$-2^2)(0.5) = (\$1)(0.5) + (\$4)(0.5) = \$2.50$.

So $\sigma^2 = E(X^2) - \mu^2 = \$2.50 - \$-0.50^2 = \$2.25$. $\boldsymbol{\sigma = \$1.50}$.