Stat 100a, introduction to Probability.

Outline for the day:

1. Variance and sd.
2. teams, emails, bruin.
3. Midterm 1 and hw2.
4. Markov and Chebyshev inequalities.
5. Luck and skill in poker.
6. Lederer and Minieri.

HW1 is due Oct14 in the first 5 min of class.

Exam1 is Oct26.

Read through chapter 5. ♠ ♣ ♥ ♦

♠ ♣ ♥ ♦

Variance and SD.

Expected Value: $E(X) = \mu = \sum k\, P(X=k)$.

Variance: $V(X) = \sigma^2 = E[(X-\mu)^2]$. Turns out this $= E(X^2) - \mu^2$.

Standard deviation $= \sigma = \sqrt{V(X)}$. Indicates how far an observation would *typically* deviate from μ.

Examples:

<u>Game 1.</u> Say X = \$4 if red card, X = \$-5 if black.

$E(X) = (\$4)(0.5) + (\$-5)(0.5) = -\$0.50$.

$E(X^2) = (\$4^2)(0.5) + (\$-5^2)(0.5) = (\$16)(0.5) + (\$25)(0.5) = \$20.5$.

So $\sigma^2 = E(X^2) - \mu^2 = \$20.5 - \$-0.50^2 = \$20.25$. $\boldsymbol{\sigma = \$4.50}$.

<u>Game 2.</u> Say X = \$1 if red card, X = \$-2 if black.

$E(X) = (\$1)(0.5) + (\$-2)(0.5) = -\$0.50$.

$E(X^2) = (\$1^2)(0.5) + (\$-2^2)(0.5) = (\$1)(0.5) + (\$4)(0.5) = \$2.50$.

So $\sigma^2 = E(X^2) - \mu^2 = \$2.50 - \$-0.50^2 = \$2.25$. $\boldsymbol{\sigma = \$1.50}$.

2. teams, emails, bruin.

The teams will be posted tonight at 9pm in teams.txt on the course website.

I will post your email addresses on the course website in namesandemails.txt until Thursday night. If you do not want yours listed for privacy reasons, please let me know by 9pm tonight and I will not list yours.

R project. teams, emails, and bruin.

The project is problem 8.2, page 249.

You need to write code to go all in or fold. In R, try:

```
install.packages(holdem)
library(holdem)
library(help="holdem")
```

gravity, timemachine, tommy, ursula, vera, william, and xena are examples.

cards[1,1] is your higher card (2-14).

cards[2,1] is your lower card (2-14).

cards[1,2] and cards[2,2] are suits of your higher card & lower card.

```
help(tommy)
tommy
```

```
function (numattable, cards, board, round, currentbet, mychips,
  pot, roundbets, blinds, chips, ind, dealer, tablesleft)
{ a = 0
  if (cards[1, 1] == cards[2, 1])
     a = mychips
  a
}
```

```
help(vera)
# All in with a pair, any suited cards, or if the smaller card is at least 9.
function (numattable, cards, board, round, currentbet, mychips,
    pot, roundbets, blinds, chips, ind, dealer, tablesleft) {
  a = 0
  if ((cards[1, 1] == cards[2, 1]) || (cards[1, 2] == cards[2,2]) ||
      (cards[2, 1] > 8.5)) a = mychips
    a
}
```

You need to email me your function, to frederic@stat.ucla.edu, by Sun Nov28, 8pm. It should be written (or cut and pasted) simply into the body of the email. If you write it in Word, save as text first, and then paste it into the email. Just submit one email per team.

For instance, if your function is named "bruin", you might do:

```
bruin = function (numattable, cards, board, round, currentbet,
     mychips, pot, roundbets, blinds, chips, ind, dealer, tablesleft) {
## all in with any pair higher than 7s, or if lower card is J or higher,
## or if you have less than 3 times the big blind
a = 0
if ((cards[1, 1] == cards[2, 1]) && (cards[1, 1] > 6.5)) a = mychips
if (cards[2,1] > 10.5) a = mychips
if(mychips < 3*blinds) a = mychips
a1
} ## end of bruin
```

Markov and Chebyshev inequalities. Ch 4.6.

The Markov inequality states

If X takes only non-negative values, and c is any number > 0, then

$P(X \ge c) \le E(X)/c$.

Proof. The discrete case is given on page 123.

If X is discrete and nonnegative, then

$$
\begin{aligned}
E(X) &= \textstyle\sum_b b\, P(X = b) \\
&= \textstyle\sum_{b<c} b\, P(X = b) \quad + \quad \sum_{b\ge c} b\, P(X = b) \\
&\ge \textstyle\sum_{b\ge c} b\, P(X = b) \\
&\ge \textstyle\sum_{b\ge c} c\, P(X = b) \\
&= c \textstyle\sum_{b\ge c} P(X = b) \\
&= c\, P(X \ge c).
\end{aligned}
$$

Here is a proof for the case where X is continuous with pdf f(y).

$E(X) = \int y\, f(y)\, dy$

$= \int_0^c yf(y)dy + \int_c^\infty yf(y)dy$

$\ge \int_c^\infty yf(y)dy$

$\ge \int_c^\infty cf(y)dy$

$= c \int_c^\infty f(y)dy$

$= c\, P(X \ge c)$.

Thus, $P(X \ge c) \le E(X) / c$.

Markov and Chebyshev inequalities. Ch 4.6.

The Markov inequality states

If X takes only non-negative values, and c is any number > 0, then

$P(X \geq c) \leq E(X)/c$.

The Chebyshev inequality states

For any random variable Y with expected value μ and variance σ^2, and any real number $a > 0$,

$P(|Y - \mu| \geq a) \leq \sigma^2 / a^2$.

Proof. The Chebyshev inequality follows directly from the Markov equality by letting $c = a^2$ and $X = (Y-\mu)^2$.

Examples of the use of the Markov and Chebyshev inequalities are on p83.

Luck and skill in poker. ♠ ♣ ♥ ♦

Let equity = your expected portion of the pot after the hand, assuming no future betting.
= your expected number of chips after the hand - chips you had before the hand
= pot * p, where p = your probability of winning if nobody folds.
I define luck as the expected profit gained during the dealing of the cards,
= equity gained during the dealing of the cards.
Skill = expected profit gained during the betting rounds.

Example.
You have Q♣ Q♦. I have 10♠ 9♠. Board is 10♦ 8♣ 7♣ 4♣. Pot is $5.
The river is 2♦, you bet $3, and I call.
On the river, how much expected profit did you gain by luck and how much by skill?

Expected profit by luck on river = your equity after 2♦ is exposed – your equity just pre-2♦
= 100% ($5) - 35/44 ($5) = $1.02.
Why 35/44? I can win with a 10, 9, 6, or J that is not a club. There are 1 + 2 + 3 + 3 = 9 of these cards, so the remaining 35 cards give you the win.

Expected profit by skill on river
= increase in pot on river * P(you win) - your cost
= $6 * 100% - $3 = $3.

Luck and skill in poker, continued. ♠ ♣ ♥ ♦

Example.

You have Q♣ Q♦. I have 10♠ 9♠. Board is 10♦ 8♣ 7♣ 4♣. Pot is \$5.

The river is 2♦, you bet \$3, and I call.

On the river, how much expected profit did you gain by luck and how much by skill?

Alternatively, let x = the number of chips you have after your \$3 bet on the river.

Before this bet, you had x + \$3 chips.

Expected profit gained by skill on river = your equity after all the betting is over - your equity when the 2♦ is dealt

= your expected number of chips after all the betting is over – your expected number of chips when the 2♦ is dealt

= (100%)(x + \$11) – (100%)(x + \$3 + \$5)

= \$3.

Lederer and Minieri.

I define luck as the expected profit gained during the dealing of the cards.
Skill = expected profit gained during the betting rounds.

Are there any problems with these definitions?

Bluffing. Ivey and Booth.