Stat 100a, Introduction to Probability.

Outline for the day:

1. Computation of luck and skill, Booth and Ivey.
2. Midterm 1 and hw2.
3. Facts about expected value, exp number trick, cond. prob.
4. Bernoulli random variables.
5. Binomial random variables.
6. Geometric random variables.
7. Negative binomial random variables.
8. Moment generating functions.

HW1 is due today in the first 5 min of class.

Exam1 is Oct26.

Read through chapter 5. ♠ ♣ ♥ ♦

Homework 2 is on the course website. It is now due Thu Nov4.

Your emails are on the course website, but I will take the emails off tonight. http://www.stat.ucla.edu/~frederic/100A/F21

Luck and skill in poker. ♠ ♣ ♥ ♦

Let equity = your expected portion of the pot after the hand, assuming no future betting.
= your expected number of chips after the hand - chips you had before the hand
= pot * p, where p = your probability of winning if nobody folds.
I define luck as the expected profit gained during the dealing of the cards,
= equity gained during the dealing of the cards.
Skill = expected profit gained during the betting rounds.

Example.
You have Q♣ Q♦. I have 10♠ 9♠. Board is 10♦ 8♣ 7♣ 4♣. Pot is $5.
The river is 2♦, you bet $3, and I call.
On the river, how much expected profit did you gain by luck and how much by skill?

Expected profit by luck on river = your equity after 2♦ is exposed – your equity just pre-2♦
= 100% ($5) - 35/44 ($5) = $1.02.
Why 35/44? I can win with a 10, 9, 6, or J that is not a club. There are 1 + 2 + 3 + 3 = 9 of these cards, so the remaining 35 cards give you the win.

Expected profit by skill on river
= increase in pot on river * P(you win) - your cost
= $6 * 100% - $3 = $3.

Luck and skill in poker, continued. ♠ ♣ ♥ ♦

Example.
You have Q♣ Q♦. I have 10♠ 9♠. Board is 10♦ 8♣ 7♣ 4♣. Pot is $5.
The river is 2♦, you bet $3, and I call.
On the river, how much expected profit did you gain by luck and how much by skill?

Alternatively, let x = the number of chips you have after your $3 bet on the river.
Before this bet, you had x + $3 chips.

Expected profit gained by skill on river = your equity after all the betting is over - your equity when the 2♦ is dealt
= your expected number of chips after all the betting is over – your expected number of chips when the 2♦ is dealt
= (100%)(x + $11) – (100%)(x + $3 + $5)
= $3.

Lederer and Minieri.

I define luck as the expected profit gained during the dealing of the cards.
Skill = expected profit gained during the betting rounds.

Are there any problems with these definitions?

Bluffing. Ivey and Booth.

Midterm 1 and homework 2.

Midterm 1 is one hour and 15 min, on Tue Oct26.

Around 14 multiple choice questions all worth the same amount.

You can use any books and notes you want, but no computers, tablet, ipads, phones, or anything that can surf the net or do email.

Bring a calculator and a pen or pencil.

None of the above is an option but it is hardly ever the answer.

Answers are rounded to 2 decimal places.

Homework 2 is problems 4.6, 4.8, 4.26 and 5.2. 4.26 is only in the 2nd edition, but it is also in hw2.pdf. 5.2 is tricky.

On problem 5.2, let Z = the time until you have been dealt a pocket pair and you have also been dealt two black cards.

Consider $P(Z > k)$, and $P(Z > k-1)$. These are actually easier to derive in this case than $P(Z = k)$. Can you get $P(Z = k)$ in terms of these?

Mike Cloud raised to 15,000 with A♣ A♠, Hellmuth called with A♥ K♠, Daniel Negreanu called from the big blind with 6♦ 4♥, and the flop came K♣ 8♥ K♥. Before the flop, the pot was 57,000 chips.

After the flop, all three players checked, the turn was the J♥, Negreanu checked, Cloud bet 15,000, Hellmuth called, and Negreanu folded.

The river was the 7♠, Cloud checked, Hellmuth bet 37,000, and Cloud called. How much expected profit did Hellmuth gain due to luck and how much due to skill on the river?

Answer—When the turn was dealt, Hellmuth's probability of winning in a showdown was 41/42 ~ 97.62%. After the betting on the turn was over, the pot was 87,000 chips. When the 7♠ was revealed on the river, Hellmuth's equity increased from 97.62% × 87,000 = 84,929.4 to 100% × 87,000, for an increase of 2070.6 chips due to luck.

Hellmuth's expected profit gained due to skill on the river is simply 37,000 chips: the pot size increased by 74,000 while Hellmuth had a 100% chance of winning, but the cost to Hellmuth was 37,000, so his profit was 37,000.

Facts about expected value.

For any random variable X and any constants a and b,

$E(aX + b) = aE(X) + b$.

Also, $E(X+Y) = E(X) + E(Y)$,

unless $E(X) = \infty$ and $E(Y) = -\infty$, in which case $E(X)+E(Y)$ is undefined.

Thus $\sigma^2 = E[(X-\mu)^2]$

$$= E[(X^2 - 2\mu X + \mu^2)]$$

$$= E(X^2) - 2\mu E(X) + \mu^2$$

$$= E(X^2) - 2\mu^2 + \mu^2$$

$$= E(X^2) - \mu^2.$$

Expected number trick.

The board consists of 5 cards. Find the expected number of clubs on the board.

Let $X_1 = 1$ if the 1st card is a club, and 0 otherwise.

Let $X_2 = 1$ if the 2nd card is a club, and 0 otherwise.

etc.

$X = X_1 + X_2 + X_3 + X_4 + X_5$.

So $E(X) = E(X_1) + E(X_2) + E(X_3) + E(X_4) + E(X_5)$

$= [¼ (1) + ¾ (0)] \times 5 = 1.25$.

Even though X_1, X_2, X_3, X_4, and X_5 are not independent, nevertheless

$E(X_1 + X_2 + X_3 + X_4 + X_5) = E(X_1)+E(X_2)+E(X_3)+E(X_4)+E(X_5)$.

Conditional probability.

When A and B are different outcomes on different collections of cards or different hands, then $P(B|A)$ can often be found directly.

But when A and B are outcomes on the same event, or same card, then sometimes it is helpful to use the definition $P(B|A) = P(AB)/P(A)$.

For example, let A = the event your hole cards are black, and let B = the event your hole cards are clubs.

$P(B|A) = P(AB)/P(A) = C(13,2)/C(52,2) / [C(26,2)/C(52,2)]$.

However, if A is the event your hole cards are black and B is the event the flop cards are all black, then $P(B|A) = C(24,3)/C(50,3)$ directly.

Bernoulli Random Variables, ch. 5.1.

If X = 1 with probability p, and X = 0 otherwise, then X = *Bernoulli (p)*.

Probability mass function (pmf):

P(X = 1) = p

P(X = 0) = q, where p+q = 100%.

If X is Bernoulli (p), then μ = E(X) = p, and σ = $\sqrt{}$(pq).

For example, suppose X = 1 if you have a pocket pair next hand; X = 0 if not.

p = 5.88%. So, q = 94.12%.

[Two ways to figure out p:

(a) Out of choose(52,2) combinations for your two cards, 13 * choose(4,2) are pairs.

13 * choose(4,2) / choose(52,2) = 5.88%.

(b) Imagine *ordering* your 2 cards. No matter what your 1st card is, there are 51 equally likely choices for your 2nd card, and 3 of them give you a pocket pair. 3/51 = 5.88%.]

μ = E(X) = .0588. SD = σ = $\sqrt{}$(.0588 * 0.9412) = 0.235.

Binomial Random Variables, ch. 5.2.

Suppose now X = # of times something with prob. p occurs, out of n independent trials

Then X = *Binomial (n.p)*.

e.g. the number of pocket pairs, out of 10 hands.

Now X could = 0, 1, 2, 3, …, or n.

pmf: P(X = k) = choose(n, k) * $p^k q^{n-k}$.

e.g. say n=10, k=3: P(X = 3) = choose(10,3) * $p^3 q^7$.

Why? Could have 1 1 1 0 0 0 0 0 0 0, or 1 0 1 1 0 0 0 0 0 0, etc.

choose(10, 3) choices of places to put the 1's, and for each the prob. is $p^3 q^7$.

Key idea: $X = Y_1 + Y_2 + \ldots + Y_n$, where the Y_i are independent and *Bernoulli* (p).

If X is Bernoulli (p), then μ = p, and $\sigma = \sqrt{(pq)}$.

If X is Binomial (n,p), then μ = np, and $\sigma = \sqrt{(npq)}$.

4. Binomial Random Variables, continued.

Suppose X = the number of pocket pairs you get in the next 100 hands.

<u>What's P(X = 4)? What's E(X)? σ?</u> X = Binomial (100, 5.88%).

$P(X = k) = \text{choose}(n, k) * p^k q^{n-k}$.

So, $P(X = 4) = \text{choose}(100, 4) * 0.0588^4 * 0.9412^{96} = 13.9\%$, or 1 in **7.2.**

$E(X) = np = 100 * 0.0588 =$ **5.88**. $\sigma = \sqrt{(100 * 0.0588 * 0.9412)} =$ **2.35**.

So, out of 100 hands, you'd *typically* get about 5.88 pocket pairs, +/- around 2.35.