Stat 100a, Introduction to Probability.

Outline for the day:

1. Conditional probability.
2. Bernoulli random variables.
3. Binomial random variables.
4. Variance of sum.
5. Harman/Negreanu and running it twice.
6. Geometric random variables.
7. Negative binomial random variables.

Exam1 is Oct26.

Read through chapter 5. ♠ ♣ ♥ ♦

Homework 2 is on the course website. It is now due Nov9.

There is no lecture Thu Nov4 because of the faculty meeting.

http://www.stat.ucla.edu/~frederic/100A/F21 ♠ ♣ ♥ ♦

Conditional probability.

When A and B are different outcomes on different collections of cards or different hands, then P(B|A) can often be found directly.

But when A and B are outcomes on the same event, or same card, then sometimes it is helpful to use the definition P(B|A) = P(AB)/P(A).

For example, let A = the event your hole cards are black, and let B = the event your hole cards are clubs.

P(B|A) = P(AB)/P(A) = C(13,2)/C(52,2) / [C(26,2)/C(52,2)].

However, if A is the event your hole cards are black and B is the event the flop cards are all black, then P(B|A) = C(24,3)/C(50,3) directly.

Bernoulli Random Variables, ch. 5.1.

If X = 1 with probability p, and X = 0 otherwise, then X = *Bernoulli (p)*.

Probability mass function (pmf):

$P(X = 1) = p$

$P(X = 0) = q$, where $p+q = 100\%$.

If X is Bernoulli (p), then μ = E(X) = p, and $\sigma = \sqrt{(pq)}$.

For example, suppose X = 1 if you have a pocket pair next hand; X = 0 if not.

p = 5.88%. So, q = 94.12%.

[Two ways to figure out p:

(a) Out of choose(52,2) combinations for your two cards, 13 * choose(4,2) are pairs.

13 * choose(4,2) / choose(52,2) = 5.88%.

(b) Imagine *ordering* your 2 cards. No matter what your 1st card is, there are 51 equally likely choices for your 2nd card, and 3 of them give you a pocket pair. 3/51 = 5.88%.]

μ = E(X) = .0588. SD = $\sigma = \sqrt{(.0588 * 0.9412)} = 0.235$.

Binomial Random Variables, ch. 5.2.

Suppose now X = # of times something with prob. p occurs, out of n independent trials

Then X = *Binomial (n.p)*.

e.g. the number of pocket pairs, out of 10 hands.

Now X could = 0, 1, 2, 3, …, or n.

pmf: P(X = k) = choose(n, k) * $p^k q^{n-k}$.

e.g. say n=10, k=3: P(X = 3) = choose(10,3) * $p^3 q^7$.

Why? Could have 1 1 1 0 0 0 0 0 0 0, or 1 0 1 1 0 0 0 0 0 0, etc.

choose(10, 3) choices of places to put the 1's, and for each the prob. is $p^3 q^7$.

Key idea: $X = Y_1 + Y_2 + \dots + Y_n$, where the Y_i are independent and *Bernoulli* (p).

If X is Bernoulli (p), then μ = p, and $\sigma = \sqrt{(pq)}$.

If X is Binomial (n,p), then μ = np, and $\sigma = \sqrt{(npq)}$.

Binomial Random Variables, continued.

Suppose X = the number of pocket pairs you get in the next 100 hands.

What's P(X = 4)? What's E(X)? σ? X = Binomial (100, 5.88%).

$P(X = k) = \text{choose}(n, k) * p^k q^{n-k}$.

So, $P(X = 4) = \text{choose}(100, 4) * 0.0588^4 * 0.9412^{96} = 13.9\%$, or 1 in **7.2.**

E(X) = np = 100 * 0.0588 = **5.88**. σ = √(100 * 0.0588 * 0.9412) = **2.35**.

So, out of 100 hands, you'd *typically* get about 5.88 pocket pairs, +/- around 2.35.

Variance of sums and binomial random variables, ch. 5.2.

If X = # of times something with prob. p occurs, out of n independent trials, then X = *Binomial (n.p)*.

For example, the number of pocket pairs out of 10 hands is binomial(10, 5.88%).

When X is binomial(n,p), $X = Y_1 + Y_2 + \ldots + Y_n$, where the Y_i are independent and *Bernoulli* (p).

Fact about variance. If X_i are independent, then $\mathrm{Var}(X_1+\ldots+X_n) = \mathrm{Var}(X_i) + \ldots + \mathrm{Var}(X_n)$.

If X is Bernoulli (p), then $\mu = p$, and var(X) = pq, so $\sigma = \sqrt{(pq)}$.

If X is Binomial (n,p), then $\mu = np$, and var(X) = npq, so $\sigma = \sqrt{(npq)}$.

Harman / Negreanu, and running it twice.

Harman has 10♠ 7♠ .   Negreanu has K♥ Q♥ . The flop is 10u 7♣ Ku .

Harman's all-in. $156,100 pot. P(Negreanu wins) = 28.69%. P(Harman wins) = 71.31%.

Let X = amount Harman has after the hand.

If they run it once, E(X) = $0 x 29% + $156,100 x 71.31% = **$111,314.90.**

<u>If they run it twice, what is E(X)?</u>

There's some probability p_1 that Harman wins both times ==> X = $156,100.

There's some probability p_2 that they each win one ==> X = $78,050.

There's some probability p_3 that Negreanu wins both ==> X = $0.

E(X) = $156,100 x p_1 + $78,050 x p_2 + $0 x p_3.

If the different runs were *independent*, then p_1 = P(Harman wins 1st run & 2nd run)

would = P(Harman wins 1st run) x P(Harman wins 2nd run) = 71.31% x 71.31% ~ 50.85%.

But, they're not quite independent! Very hard to compute p_1 and p_2.

However, you don't need p_1 and p_2 !

X = the amount Harman gets from the 1st run + amount she gets from 2nd run, so

E(X) = E(amount Harman gets from 1st run) + E(amount she gets from 2nd run)

= $78,050 x P(Harman wins 1st run) + $0 x P(Harman loses first run)

+ $78,050 x P(Harman wins 2nd run) + $0 x P(Harman loses 2nd run)

= $78,050 x 71.31% + $0 x 28.69% + $78,050 x 71.31% + $0 x 28.69% = **$111,314.90.**

HAND RECAP Harman 10♠ 7♠ Negreanu K♥ Q♥ The flop is 10u 7♣ Ku .

Harman's all-in. $156,100 pot.P(Negreanu wins) = 28.69%. P(Harman wins) = 71.31%.

The standard deviation (SD) changes a lot! **Say they run it once.** (see p127.)

$V(X) = E(X^2) - \mu^2$.

μ = $111,314.9, so $\mu^2 \sim$ $12.3 billion.

$E(X^2)$ = ($156,100^2)(71.31%) + ($0^2$)(28.69%) = $17.3 billion.

V(X) = $17.3 billion - $12.3 bill. = $5.09 billion. SD σ = sqrt($5.09 billion)~$71,400.

So if they run it once, Harman expects to get back about $111,314.9 **+/- $71,400**.

If they run it twice? Hard to compute, but approximately, if each run were independent, then $V(X_1+X_2) = V(X_1) + V(X_2)$,

so if X_1 = amount she gets back on 1st run, and X_2 = amount she gets from 2nd run,

then $V(X_1+X_2) \sim V(X_1) + V(X_2) \sim$ $1.25 billion + $1.25 billion = $2.5 billion,

The standard deviation σ = sqrt($2.5 billion) ~ $50,000.

So if they run it twice, Harman expects to get back about $111,314.9 **+/- $50,000.**

Geometric random variables, ch 5.3.

Suppose now X = # of trials until the *first* occurrence.

(Again, each trial is independent, and each time the probability of an occurrence is p.)

Then X = *Geometric (p)*.

e.g. the number of hands til you get your next pocket pair.

[*Including the hand where you get the pocket pair. If you get it right away, then X = 1.*]

Now X could be 1, 2, 3, …, up to ∞.

pmf: $P(X = k) = p^1 q^{k-1}$.

e.g. say k=5: $P(X = 5) = p^1 q^4$. Why? Must be 0 0 0 0 1. Prob. = q * q * q * q * p.

If X is Geometric (p), then μ = 1/p, and σ = (√q) ÷ p.

e.g. Suppose X = the number of hands til your next pocket pair. P(X = 12)? E(X)? σ?

X = Geometric (5.88%).

$P(X = 12) = p^1 q^{11}$ = 0.0588 * 0.9412 ^ 11 = **3.02%.**

E(X) = 1/p = **17.0**. σ = sqrt(0.9412) / 0.0588 = **16.5**.

So, you'd typically *expect* it to take 17 hands til your next pair, +/- around 16.5 hands.

Negative binomial random variables, ch5.4.

Recall: if each trial is independent, and each time the probability of an occurrence is p,

and X = # of trials until the <u>*first*</u> occurrence, then:

X is Geometric (p), $P(X = k) = p^1 q^{k-1}$, $\mu = 1/p$, $\sigma = (\sqrt{q}) \div p$.

Suppose now X = # of trials until the *rth* occurrence.

Then X = *negative binomial (r,p)*.

e.g. the number of hands you have to play til you've gotten r=3 pocket pairs.

Now X could be 3, 4, 5, …, up to ∞.

pmf: $P(X = k) = \text{choose}(k-1, r-1)\, p^r q^{k-r}$, for k = r, r+1, ….

e.g. say r=3 & k=7: $P(X = 7) = \text{choose}(6,2)\, p^3 q^4$.

Why? Out of the first 6 hands, there must be exactly r-1 = 2 pairs. Then pair on 7th.

P(exactly 2 pairs on first 6 hands) = $\text{choose}(6,2)\, p^2 q^4$. P(pair on 7th) = p.

If X is negative binomial (r,p), then $\mu = r/p$, and $\sigma = (\sqrt{rq}) \div p$.

<u>e.g. Suppose X = the number of hands til your 12th pocket pair. P(X = 100)? E(X)? **σ**?</u>

X = Neg. binomial (12, 5.88%).

$P(X = 100) = \text{choose}(99,11)\, p^{12} q^{88}$

= choose(99,11) * 0.0588 ^ 12 * 0.9412 ^ 88 = **0.104%.**

E(X) = r/p = 12/0.0588 ~ **204**. σ = sqrt(12*0.9412) / 0.0588 = **57.2**.

So, you'd typically *expect* it to take 204 hands til your 12th pair, +/- around 57.2 hands.