Stat 100a: Introduction to Probability.

Outline for the day:

1. Random walks.
2. Reflection principle.
3. Ballot theorem.
4. Exam 2.

Exam 2 is today from 1040 to 1150am.

HW3 is due Thu.

Computer project is due Jul 27.

1. Random walks, ch. 7.6**.**

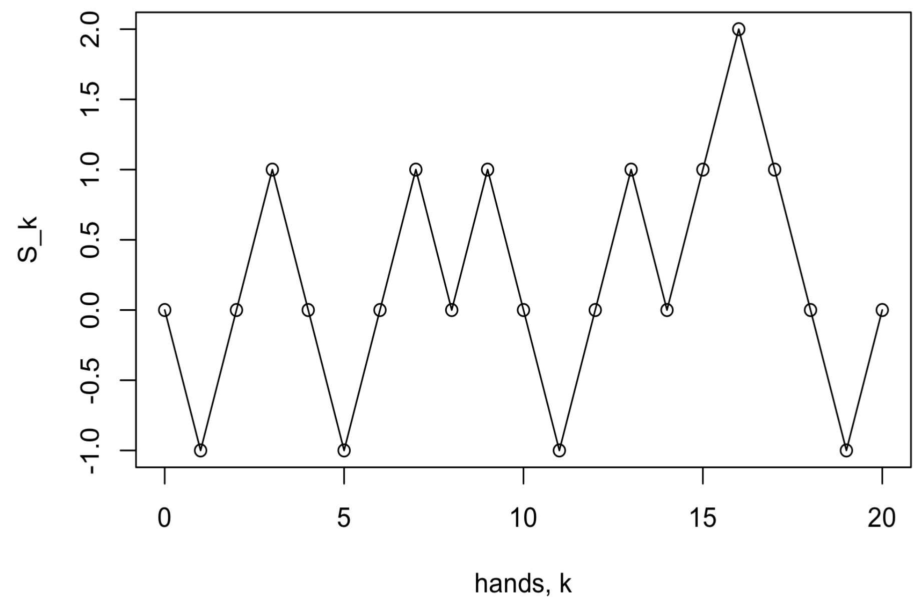
Suppose that $X_1, X_2, \ldots,$ are iid,

and $S_k = X_0 + X_1 + \ldots + X_k$ for $k = 0, 1, 2, \ldots$.

The totals $\{S_0, S_1, S_2, \ldots\}$ form a *random walk*.

The classical *(simple)* case is when each X_i is

1 or -1 with probability ½ each.



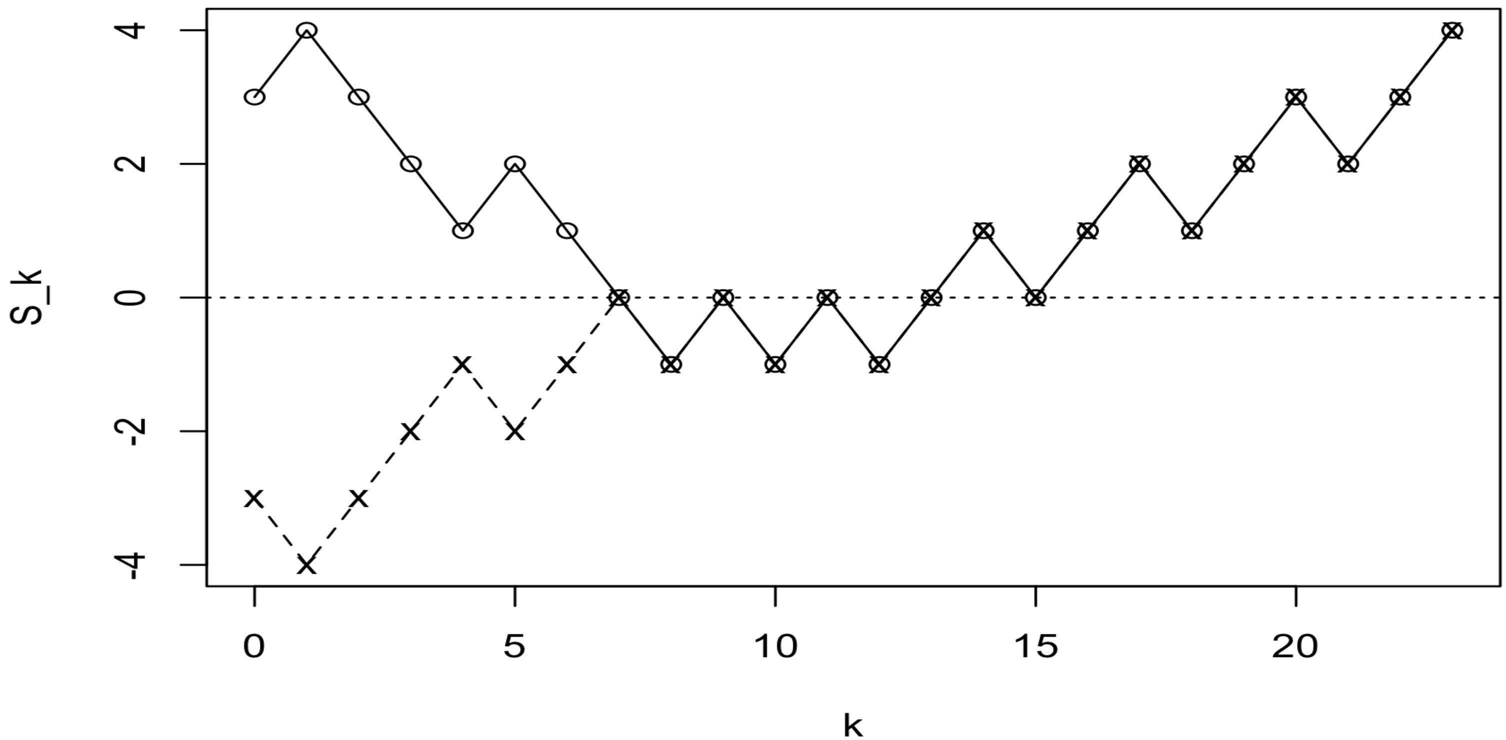
* *Reflection principle:* The number of paths from $(0,X_0)$ to (n,y) that touch the x-axis = the number of paths from $(0,-X_0)$ to (n,y), for any n,y, and $X_0 > 0$.

* *Ballot theorem:* In $n = a+b$ hands, if player A won a hands and B won b hands, where $a>b$, and if the hands are aired in random order, P(A won more hands than B *throughout* the telecast) $= (a-b)/n$.

[In an election, if candidate X gets x votes, and candidate Y gets y votes, where $x > y$, then the probability that X always leads Y throughout the counting is $(x-y) / (x+y)$.]

* For a simple random walk, $P(S_1 \neq 0, S_2 \neq 0, \ldots, S_n \neq 0) = P(S_n = 0)$, for any even n.

2. Reflection Principle. The number of paths from $(0,X_0)$ to (n,y) that touch the x-axis = the number of paths from $(0,-X_0)$ to (n,y), for any n,y, and $X_0 > 0$.



For each path from $(0,X_0)$ to (n,y) that touches the x-axis, you can reflect the first part til it touches the x-axis, to find a path from $(0,-X_0)$ to (n,y), and vice versa.

Total number of paths from $(0,-X_0)$ to (n,y) is easy to count: it's just $C(n,a)$, where you go up *a* times and down *b* times

[i.e. $a-b = y - (-X_0) = y + X_0$. $a+b=n$, so $b = n-a$, $2a-n=y+X_0$, $a=(n+y+X_0)/2$].

3. Ballot theorem. In n = a+b hands, if player A won a hands and B won b hands, where a>b, and if the hands are aired in random order, then P(A won more hands than B *throughout* the telecast) = (a-b)/n.

Proof: We know that, after n = a+b hands, the total difference in hands won is a-b.

Let x = a-b.

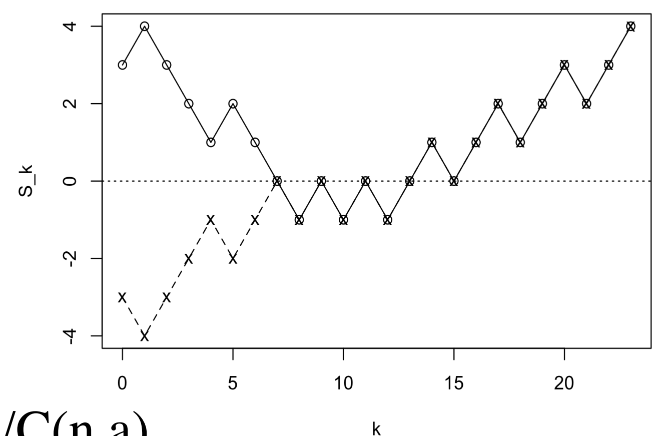
We want to count the number of paths from (1,1) to (n,x) that do not touch the x-axis. By the reflection principle, the number of paths from (1,1) to (n,x) that **do** touch the x-axis equals the total number of paths from (1,-1) to (n,x).
So the number of paths from (1,1) to (n,x) that **do not** touch the x-axis equals the number of paths from (1,1) to (n,x) minus the number of paths from (1,-1) to (n,x)

= C(n-1,a-1) – C(n-1,a)

= (n-1)! / [(a-1)! (n-a)!] – (n-1)! / [a! (n-a-1)!]

= {n! / [a! (n-a)!]} [(a/n) - (n-a)/n]

= C(n,a) (a-b)/n.



And each path is equally likely, and has probability 1/C(n,a).

So, P(going from (0,0) to (n,a) without touching the x-axis = (a-b)/n.

Statistics 100a Midterm 1

Rick Paik Schoenberg, 7/21/15, 10:40am-11:50am.

PRINT YOUR NAME:

SIGN YOUR NAME:

Do not turn the page and start the exam until you are told to do so.

You may use the textbook, a calculator, a pen or pencil, and 2 pages double sided of notes.

There are 10 multiple choice questions worth 12.5 points each.

Final answers are rounded to 3 significant digits.

No partial credit is given for multiple choice questions. Circle one answer only. If your answer is none of the above, then indicate the correct answer.

Turning 4 of a kind means having 4 of a kind after the turn is revealed. The turn is the 4th community card.

For problems 1 and 3, assume that you are guaranteed to be all-in next hand, no matter what. Also, assume that all of these questions relate specifically to the *next hand only*. In other words, when I ask what the probability is that you will be dealt a certain type of hand, I don't mean *eventually*. I mean on the very next hand.

7s means the seven of spades. Qh means the queen of hearts, etc.

$e \approx 2.718$.