Stat 100a: Introduction to Probability.

Outline for the day:

1. Review list.
2. Random walk example.
3. Bayes' rule example.
4. Conditional probability examples.
5. Projects.

The final will be open book, plus 2 pages of notes.

Bring a calculator and a pen or pencil.

Submit your reviews of the course online via my.ucla.edu.

There's no R stuff on the final.

Suggested problems from ch 4-7 to look at are 4.5, 4.6, 4.7, 4.8, 4.9, 4.13, 4.14, 4.16, 5.1, 5.2, 5.5, 5.6, 6.2, 6.4, 6.9, 6.10, 6.11, 6.12, 7.1, 7.2, 7.3, 7.4, 7.5, 7.8, 7.13, 7.14, 7.15.

1. Review list.

1) Basic principles of counting.
2) Axioms of probability, and addition rule.
3) Permutations & combinations.
4) Conditional probability.
5) Independence.
6) Multiplication rules. P(AB) = P(A) P(B|A) [= P(A)P(B) if ind.]
7) Odds ratios.
8) Random variables (RVs).
9) Discrete RVs, and probability mass function (pmf).
10) Expected value.
11) Pot odds calculations.
12) Luck, skill, and deal-making.
13) Variance and SD.
14) Bernoulli RV. [0-1. $\mu = p$, $\sigma = \sqrt{(pq)}$.]
15) Binomial RV. [# of successes, out of n tries. $\mu = np$, $\sigma = \sqrt{(npq)}$.]
16) Geometric RV. [# of tries til 1st success. $\mu = 1/p$, $\sigma = (\sqrt{q}) / p$.]
17) Negative binomial RV. [# of tries til rth success. $\mu = r/p$, $\sigma = (\sqrt{rq}) / p$.]
18) Poisson RV [# of successes in some time interval. [$\mu = \lambda$, $\sigma = \sqrt{\lambda}$.]
19) E(X+Y), V(X+Y) (ch. 7.1).
20) Bayes's rule (ch. 3.4).
20) Continuous RVs
21) Probability density function (pdf)
22) Uniform RV
23) Normal RV
24) Exponential RV
25) Moment generating functions
26) Markov and Chebyshev inequalities
25) Law of Large Numbers (LLN)
26) Central Limit Theorem (CLT)
27) Conditional expectation.
28) Confidence intervals for the sample mean.
29) Fundamental theorem of poker
30) Random Walks, Reflection Principle, Ballot Theorem, avoiding zero
31) Chip proportions and induction.

Basically, we've done all of ch. 1-7 except 6.7.

2. Another random walk example.

Suppose that a \$10 winner-take-all tournament has $32 = 2^5$ players. So, you need to double up 5 times to win. Winner gets \$320.

Suppose that on each hand of the tournament, you have probability $p = 0.7$ to double up, and with probability $q = 0.3$ you will be eliminated. What is your expected profit in the tournament?

Your expected *return* = (\$320) x P(win the tournament) + (\$0) x P(you don't win)

= (\$320) x 0.7^5 = \$53.78. But it costs \$10.

So expected *profit* = \$53.78 - \$10 = \$43.78.

3. Bayes' rule example.

Your opponent raises all-in before the flop. Suppose you think she would do that *80%* of the time with AA, KK, or QQ, and she would do that *30%* of the time with AK or AQ, and 1% of the time with anything else.

Given only this, and not even your cards, what's P(she has AK)?

Given nothing, P(AK) = 16/C(52,2) = 16/1326. P(AA) = C(4,2)/C(52,2) = 6/1326.

Using Bayes' rule,

$$P(AK \mid \text{all-in}) = \frac{P(\text{all-in} \mid AK) * P(AK)}{P(\text{all-in}|AK)P(AK) + P(\text{all-in}|AA)P(AA) + P(\text{all-in}|KK)P(KK) + \ldots},$$

$$= \frac{30\% \times 16/1326}{[30\%\text{x}16/1326] + [80\%\text{x}6/1326] + [80\%\text{x}6/1326] + [80\%\text{x}6/1326] + [30\%\text{x}16/1326] + [1\%\ (1326-16-6-6-6-16)/1326)]}.$$

(AK) (AA) (KK) (QQ) (AQ) (anything else)

= **13.06%**. Compare with 16/1326 ~ 1.21%.

4. Conditional prob. examples.

Approximate P(SOMEONE has AA, given you have KK)? Out of your 8 opponents?

Note that given that you have KK,

P(player 2 has AA & player 3 has AA)

= P(player 2 has AA) x P(player 3 has AA | player 2 has AA)

= choose(4,2) / choose(50,2) x 1/choose(48,2)

= 0.0000043, or 1 in 230,000.

So, very little overlap! Given you have KK,

P(someone has AA) = P(player2 has AA or player3 has AA or … or pl.9 has AA)

~ P(player2 has AA) + P(player3 has AA) + … + P(player9 has AA)

= 8 x choose(4,2) / choose(50,2) = 3.9%, or 1 in **26.**

What is **exactly** P(SOMEONE has an Ace | you have KK)? (8 opponents)

(or more than one ace)

Given that you have KK, P(SOMEONE has an Ace) = 100% - P(nobody has an Ace).

And P(nobody has an Ace) = choose(46,16)/choose(50,16)

= **20.1%.**

So P(SOMEONE has an Ace) = **79.9%.**

4. More conditional probability examples.

P(You have AK | you have exactly one ace)?

= P(You have AK and exactly one ace) / P(exactly one ace)

= P(AK) / P(exactly one ace)

= (16/C(52,2)) ÷ (4x48/C(52,2))

= 4/48 = 8.33%.

P(You have AK | you have at least one ace)?

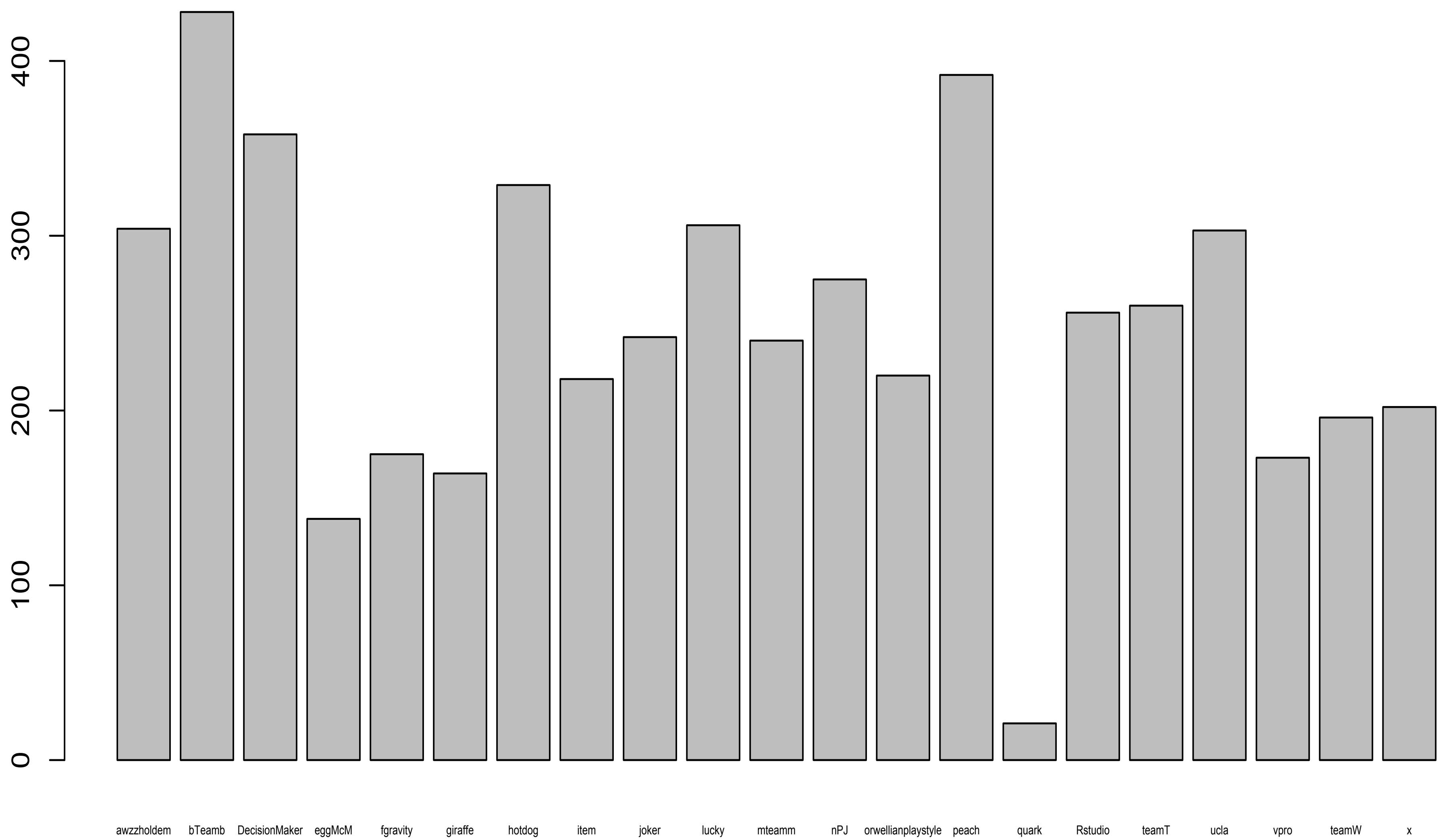
= P(You have AK and at least one ace) / P(at least one ace)

= P(AK) / P(at least one ace)

= (16/C(52,2)) ÷ (((4x48 + C(4,2))/C(52,2)) ~ 8.08%.

P(You have AK | your FIRST card is an ace)?

= 4/51 = 7.84%.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