

# Stat 100a, Introduction to Probability. Rick Paik Schoenberg

## Outline for the day:

1. Addiction.
2. Syllabus, etc.
3. Wasicka/Gold/Binger example.
4. Meaning of probability.
5. Axioms of probability.
6. Hw1 terms.
7. Basic principle of counting.
8. Permutations and combinations.
9. Ly vs. Negreanu (flush draw) example
10. R.
11. A♠ vs 2♣ after first ace.

## 2. Syllabus, etc.

For this week:

- (i) Learn the rules of Texas Hold'em.  
(see <http://www.fulltiltpoker.net/holdem.php>  
and <http://www.fulltiltpoker.net/handRankHigh.php> )
- (ii) Read addiction handout, addiction1.pdf, on the course website,  
<http://www.stat.ucla.edu/~frederic/100a/W15> .
- (iii) Download R and try it out.  
( <http://cran.stat.ucla.edu> )
- (iv) Read ch. 1-2 of the textbook.

Note that the CCLE website for this course is not maintained. The course website is <http://www.stat.ucla.edu/~frederic/100a/W15> .

I do not give hw hints in office hours. Conceptual questions only.

If you have taken Stat 35 before, please see me after class.

## Wasicka/Gold/Binger Example

Blinds: \$200,000-\$400,000 with \$50,000 antes.

### **Chip Counts:**

Jamie Gold	\$60,000,000
Paul Wasicka	\$18,000,000
Michael Binger	\$11,000,000

**Payouts:** 3rd place: \$4,123,310.    2nd place: \$6,102,499.    1st place: \$12,000,000.

Day 7, Hand 229.    Gold: 4s 3c.    Binger: Ah 10h.    Wasicka: 8s 7s.

An example of the type of questions we will be addressing in this class is on the next slide. Don't worry about all the details yet.

## Wasicka/Gold/Binger Example, Continued

Gold: 4♠ 3♣.    Binger: A♥ 10♥.    Wasicka: 8♠ 7♠.  
Flop: 10♣ 6♠ 5♠.                      (Turn: 7♣. River: Q♠.)

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Wasicka folded?!?

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He had 8♠ 7♠ and the flop was 10♣ 6♠ 5♠. Worst case scenario: suppose he were up against 9♠ 4♠ and 9♥ 9♦. How could Wasicka win?

88            (3: 8♣ 8♦, 8♣ 8♥, 8♦ 8♥)

77            (3)

44            (3)

[Let “X” = non-49, “Y” = A2378JQK, and “n” = non-♠.]

4n Xn    (3 x 32)

9♣ 4n    (3)

9♣ Yn    (24).    **Total: 132 out of 903 = 14.62%.**

## 4. Meaning of Probability.

Notation: “ $P(A) = 60\%$ ”.  $A$  is an *event*.  
Not “ $P(60\%)$ ”.

Definition of probability:

Frequentist: If repeated independently under the same conditions millions and millions of times,  $A$  would happen 60% of the times.

Bayesian: Subjective feeling about how likely something seems.

$P(A \text{ or } B)$  means  $P(A \text{ or } B \text{ or both })$

*Mutually exclusive*:  $P(A \text{ and } B) = 0$ .

*Independent*:  $P(A \text{ given } B)$  [written “ $P(A|B)$ ”] =  $P(A)$ .

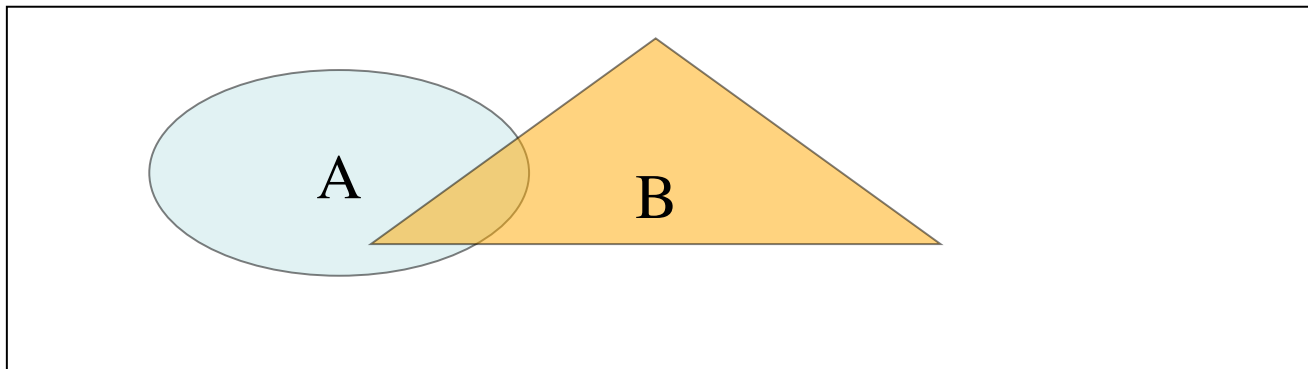
$P(A^c)$  means  $P(\text{not } A)$ .

## 5. Axioms (initial assumptions/rules) of probability:

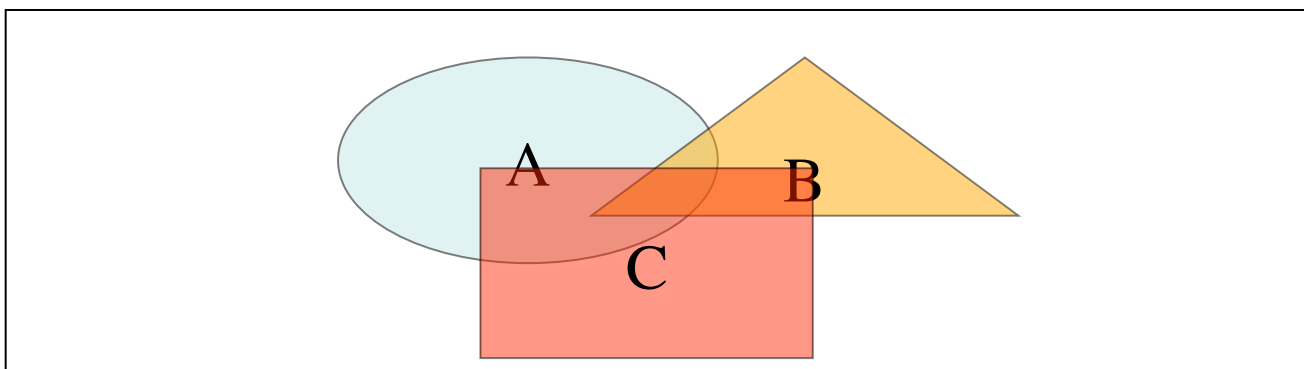
- 1)  $P(A) \geq 0$ .
- 2)  $P(A) + P(A^c) = 1$ .
- 3) If  $A_1, A_2, A_3, \dots$  are mutually exclusive, then  
 $P(A_1 \text{ or } A_2 \text{ or } A_3 \text{ or } \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$

(#3 is sometimes called the *addition rule*)

Probability  $\Leftrightarrow$  Area. Measure theory, Venn diagrams



$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$



Fact:  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$

$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC).$

Fact: If  $A_1, A_2, \dots, A_n$  are equally likely & mutually exclusive,  
and if  $P(A_1 \text{ or } A_2 \text{ or } \dots \text{ or } A_n) = 1,$   
then  $P(A_k) = 1/n.$

[So, you can *count*:  $P(A_1 \text{ or } A_2 \text{ or } \dots \text{ or } A_k) = k/n.$ ]

Ex. You have 76, and the board is KQ54.  $P(\text{straight})?$

[52-2-4=46.]  $P(\text{straight}) = P(8 \text{ on river OR } 3 \text{ on river})$

$= P(8 \text{ on river}) + P(3 \text{ on river}) = 4/46 + 4/46.$

**6. Hw1 terms.**      2 pair rules, the nuts, the unbreakable nuts.

### **7. Basic Principle of Counting.**

If there are  $a_1$  distinct possible outcomes on experiment #1, and for each of them, there are  $a_2$  distinct possible outcomes on experiment #2, then there are  $a_1 \times a_2$  distinct possible *ordered* outcomes on both.

e.g. you get 1 card, opp. gets 1 card. # of distinct possibilities?  
 $52 \times 51$ . [ordered:  $(A\clubsuit, K\heartsuit) \neq (K\heartsuit, A\clubsuit)$  .]

In general, with  $j$  experiments, each with  $a_i$  possibilities,  
the # of distinct outcomes *where order matters* is  $a_1 \times a_2 \times \dots \times a_j$  .



## 8. Permutations and Combinations.

e.g. you get 1 card, opp. gets 1 card.

# of distinct possibilities?

52 x 51. [ordered: (A♣ , K♥) ≠ (K♥ , A♣) .]

Each such outcome, where order matters, is called a *permutation*.

Number of permutations of the deck?  $52 \times 51 \times \dots \times 1 = 52!$

$$\sim 8.1 \times 10^{67}$$

A combination is a collection of outcomes, where order *doesn't* matter.

e.g. in hold'em, how many *distinct* 2-card hands are possible?

52 x 51 if order matters, but then you'd be double-counting each

[ since now (A♣ , K♥) = (K♥ , A♣) .]

So, the number of *distinct* hands where *order doesn't matter* is

$$52 \times 51 / 2.$$

In general, with n distinct objects, the # of ways to choose k *different* ones, *where order doesn't matter*, is

$$\text{"n choose k"} = \text{choose}(n,k) = \frac{n!}{k! (n-k)!} .$$

$k! = 1 \times 2 \times \dots \times k$ . [convention:  $0! = 1$ .]

$$\text{choose}(n,k) = \binom{n}{k} = \frac{n!}{k! (n-k)!}.$$

### 9. Ly vs. Negreanu, p66.

Ex. Suppose you have 2 ♣s, and there are exactly 2 ♣s on the flop.  
Given this info, what is  $P(\text{at least one more } \clubsuit \text{ on turn or river})$ ?

Answer:  $52-5 = 47$  cards left (9 ♣s, 38 others).

So  $n = \text{choose}(47,2) = 1081$  combinations for next 2 cards.

Each equally likely (and obviously mutually exclusive).

Two-♣ combos:  $\text{choose}(9,2) = 36$ . One-♣ combos:  $9 \times 38 = 342$ .

Total = 378. So answer is  $378/1081 = 35.0\%$ .

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Answer #2: Use the addition rule...

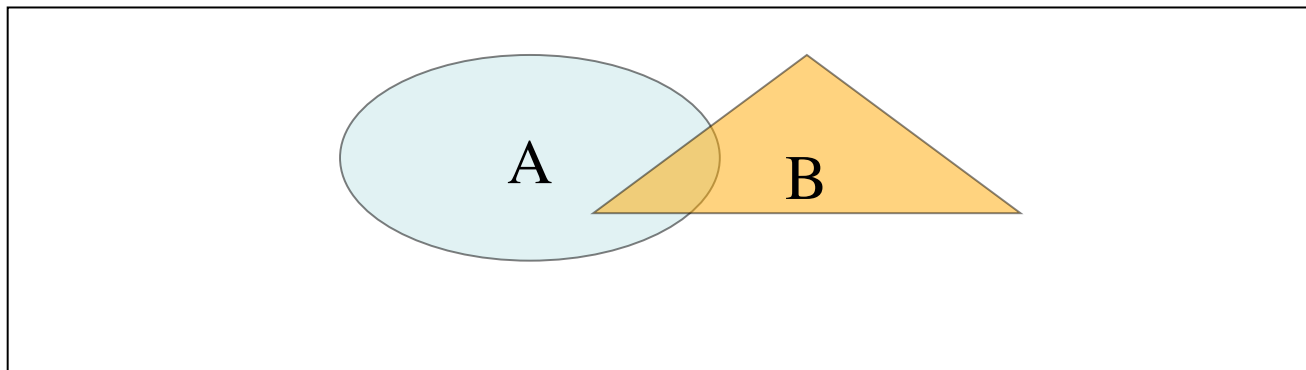
## ADDITION RULE, revisited.....

Axioms (initial assumptions/rules) of probability:

- 1)  $P(A) \geq 0$ .
- 2)  $P(A) + P(A^c) = 1$ .
- 3) Addition rule:

If  $A_1, A_2, A_3, \dots$  are mutually exclusive, then

$$P(A_1 \text{ or } A_2 \text{ or } A_3 \text{ or } \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$$



As a result, even if A and B might not be mutually exclusive,  
 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ . (p6 of book)

Ex. You have 2 ♣s, and there are exactly 2 ♣s on the flop. Given this info, what is  $P(\text{at least one more } \clubsuit \text{ on turn or river})$ ?

Answer #1:  $52-5 = 47$  cards left (9 ♣s, 38 others).

So  $n = \text{choose}(47,2) = 1081$  combinations for next 2 cards.

Each equally likely (and obviously mutually exclusive).

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Answer #2: Use the addition rule.

$$\begin{aligned} P(\geq 1 \text{ more } \clubsuit) &= P(\clubsuit \text{ on turn OR river}) \\ &= P(\clubsuit \text{ on turn}) + P(\clubsuit \text{ on river}) - P(\text{both}) \\ &= 9/47 + 9/47 - \text{choose}(9,2)/\text{choose}(47,2) \\ &= 19.15\% + 19.15\% - 3.3\% = 35.0\%. \end{aligned}$$

Ex. You have AK. Given this, what is

P(at least one A or K comes on board of 5 cards)?

Wrong Answer:

P(A or K on 1st card) + P(A or K on 2nd card) + ...

=  $6/50 \times 5 = 60.0\%$ .

No: these events are NOT Mutually Exclusive!!!

Right Answer:

choose(50,5) = 2,118,760 boards possible.

How many have exactly one A or K?  $6 \times \text{choose}(44,4) = 814,506$

# with exactly 2 aces or kings?  $\text{choose}(6,2) \times \text{choose}(44,3) = 198,660$

# with exactly 3 aces or kings?  $\text{choose}(6,3) \times \text{choose}(44,2) = 18,920 \dots$

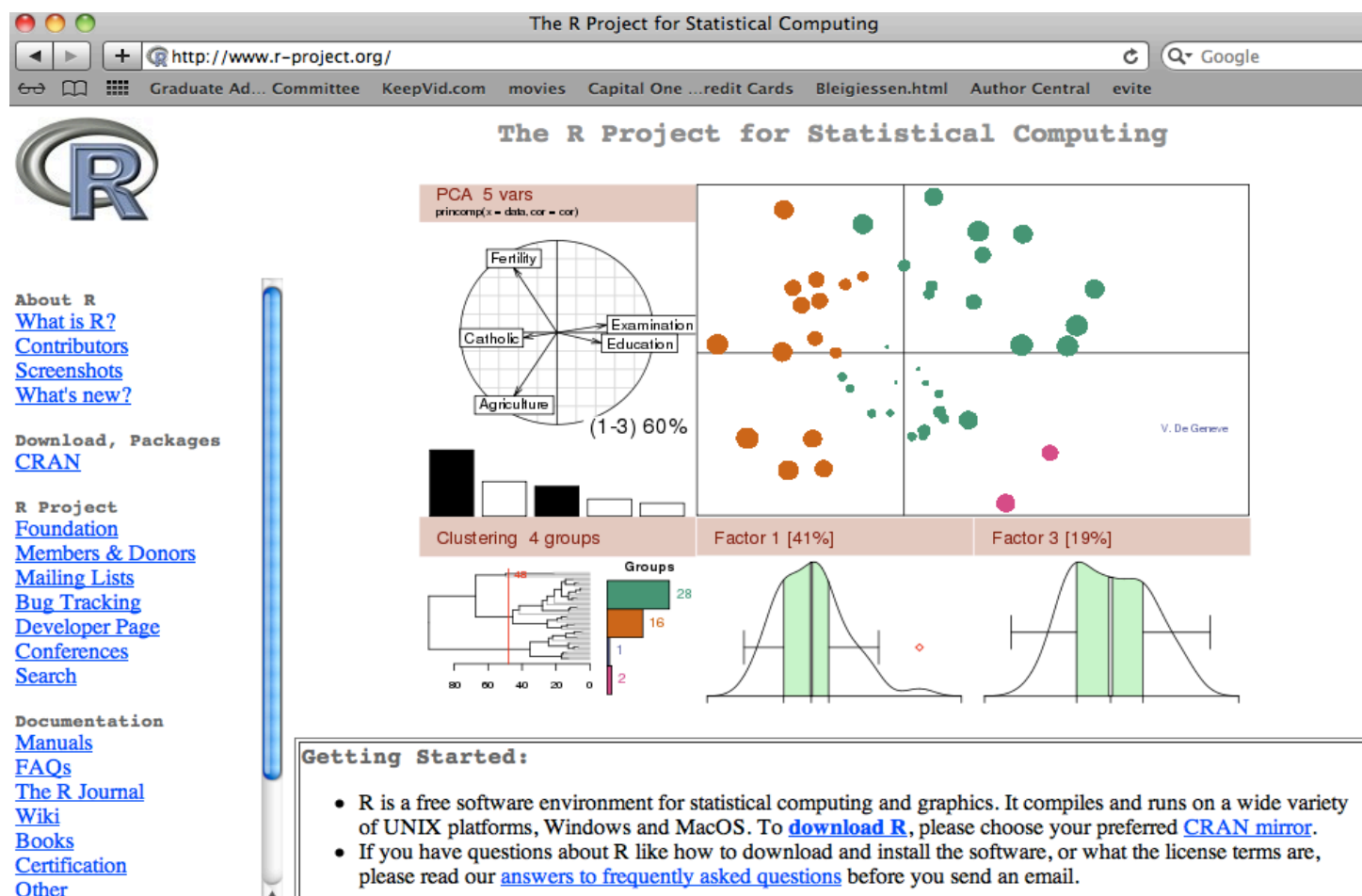
... altogether, 1,032,752 boards have at least one A or K,

So it's  $1,032,752 / 2,118,760 = 48.7\%$ .

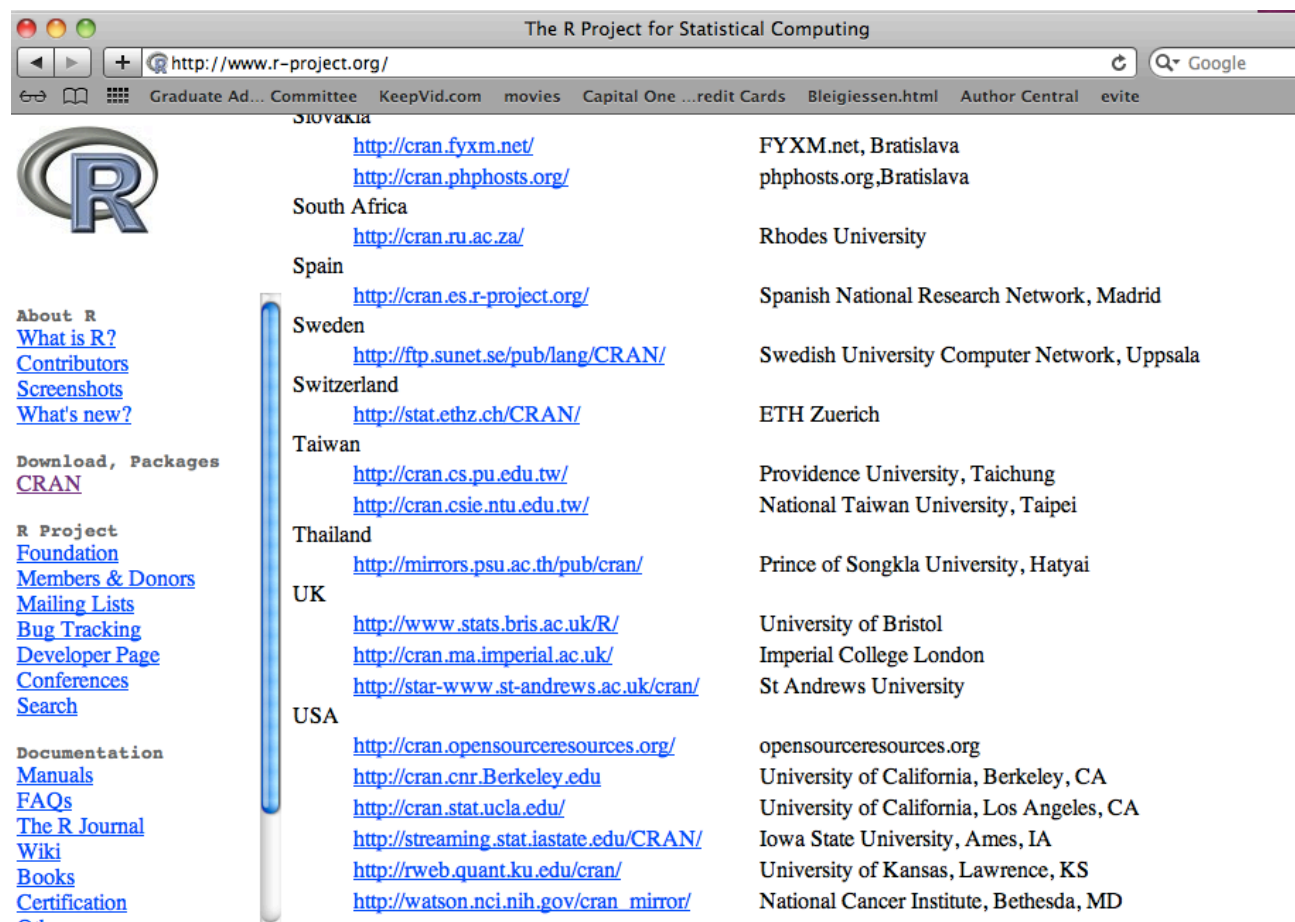
Easier way: P(no A and no K) =  $\text{choose}(44,5)/\text{choose}(50,5)$

=  $1086008 / 2118760 = 51.3\%$ , so answer =  $100\% - 51.3\% = 48.7\%$

**10. R.** To download and install *R*, go directly to [cran.stat.ucla.edu](http://cran.stat.ucla.edu), or as it says in the book at the bottom of p157, you can start at [www.r-project.org](http://www.r-project.org), in which case you click on “download *R*”, scroll down to UCLA, and click on [cran.stat.ucla.edu](http://cran.stat.ucla.edu). From there, click on “download *R* for ...”, and then get the latest version.

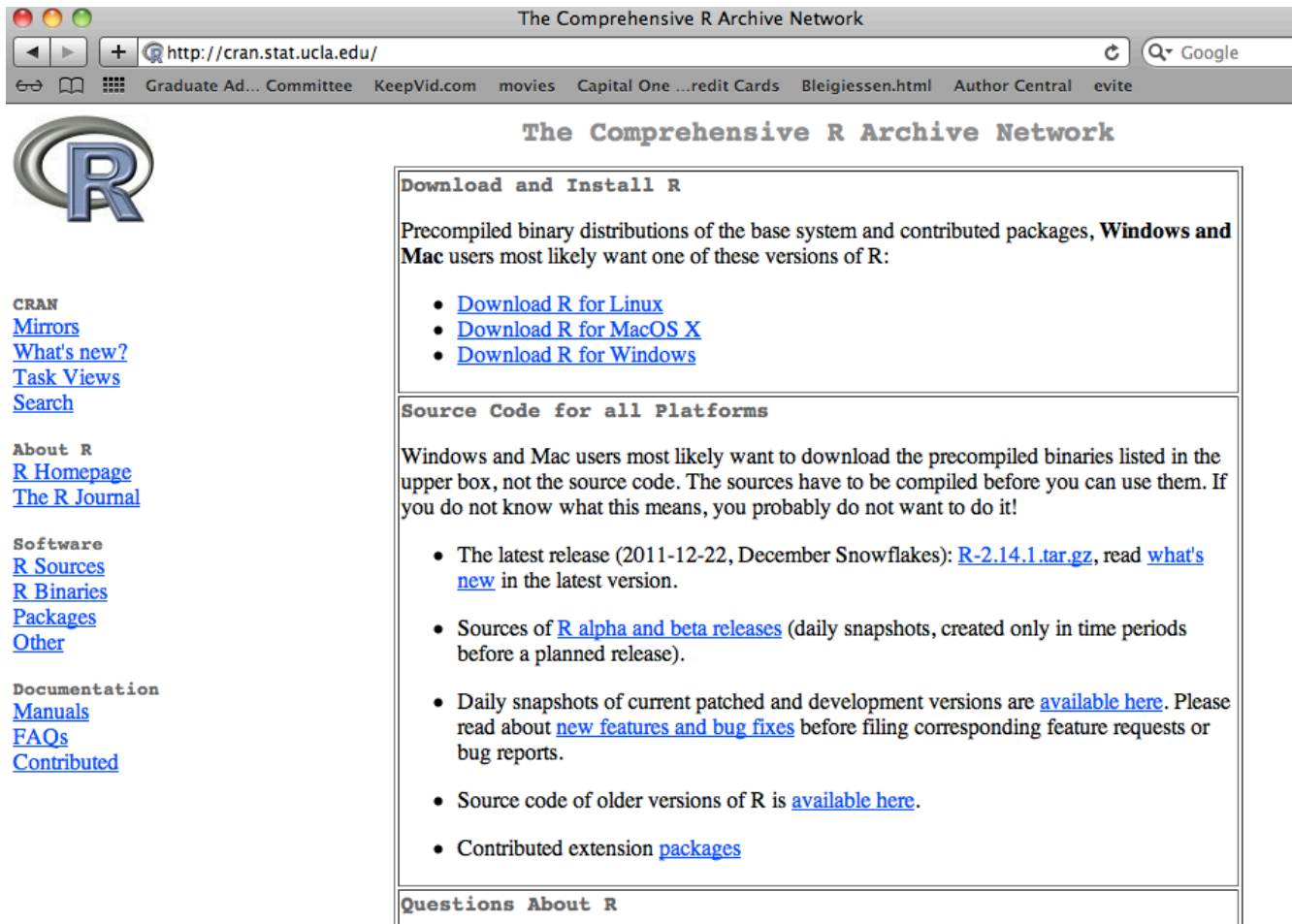


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**Download and Install R**

Precompiled binary distributions of the base system and contributed packages, **Windows and Mac** users most likely want one of these versions of R:

- [Download R for Linux](#)
- [Download R for MacOS X](#)
- [Download R for Windows](#)

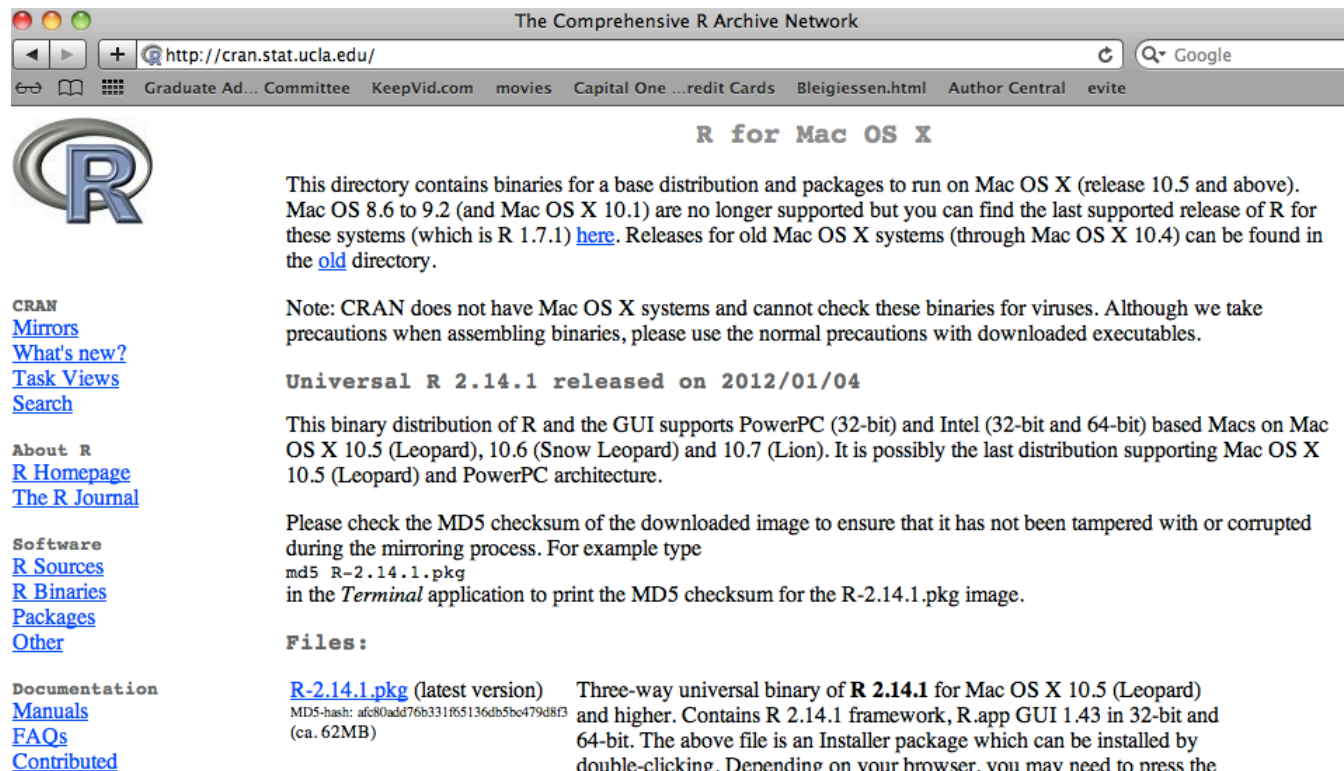
**Source Code for all Platforms**

Windows and Mac users most likely want to download the precompiled binaries listed in the upper box, not the source code. The sources have to be compiled before you can use them. If you do not know what this means, you probably do not want to do it!

- The latest release (2011-12-22, December Snowflakes): [R-2.14.1.tar.gz](#), read [what's new](#) in the latest version.
- Sources of [R alpha and beta releases](#) (daily snapshots, created only in time periods before a planned release).
- Daily snapshots of current patched and development versions are [available here](#). Please read about [new features and bug fixes](#) before filing corresponding feature requests or bug reports.
- Source code of older versions of R is [available here](#).
- Contributed extension [packages](#)

**Questions About R**

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The screenshot shows a web browser window titled "The Comprehensive R Archive Network". The address bar shows <http://cran.stat.ucla.edu/>. The page content is for "R for Mac OS X". It includes a sidebar with links like "CRAN Mirrors", "What's new?", "Task Views", "Search", "About R", "R Homepage", "The R Journal", "Software", "R Sources", "R Binaries", "Packages", "Other", "Documentation", "Manuals", "FAQs", and "Contributed". The main content area has a heading "R for Mac OS X" and a paragraph explaining that the directory contains binaries for Mac OS X (release 10.5 and above). It also includes a note about CRAN not having Mac OS X systems, a release date "Universal R 2.14.1 released on 2012/01/04", and instructions on how to check the MD5 checksum of the downloaded image. At the bottom, there is a section for "Files:" with a link to "R-2.14.1.pkg (latest version)" and a detailed description of the package contents and installation instructions.

**R for Mac OS X**

This directory contains binaries for a base distribution and packages to run on Mac OS X (release 10.5 and above). Mac OS 8.6 to 9.2 (and Mac OS X 10.1) are no longer supported but you can find the last supported release of R for these systems (which is R 1.7.1) [here](#). Releases for old Mac OS X systems (through Mac OS X 10.4) can be found in the [old](#) directory.

Note: CRAN does not have Mac OS X systems and cannot check these binaries for viruses. Although we take precautions when assembling binaries, please use the normal precautions with downloaded executables.

**Universal R 2.14.1 released on 2012/01/04**

This binary distribution of R and the GUI supports PowerPC (32-bit) and Intel (32-bit and 64-bit) based Macs on Mac OS X 10.5 (Leopard), 10.6 (Snow Leopard) and 10.7 (Lion). It is possibly the last distribution supporting Mac OS X 10.5 (Leopard) and PowerPC architecture.

Please check the MD5 checksum of the downloaded image to ensure that it has not been tampered with or corrupted during the mirroring process. For example type

```
md5 R-2.14.1.pkg
```

in the *Terminal* application to print the MD5 checksum for the R-2.14.1.pkg image.

**Files:**

[R-2.14.1.pkg](#) (latest version) Three-way universal binary of **R 2.14.1** for Mac OS X 10.5 (Leopard) and higher. Contains R 2.14.1 framework, R.app GUI 1.43 in 32-bit and 64-bit. The above file is an Installer package which can be installed by double-clicking. Depending on your browser, you may need to press the control key and click on this link to download the file.

MD5-hash: `afc80add76b331f65136db5bc479d8f3`  
(ca. 62MB)

This package **only** contains the R framework, 32-bit GUI (R.app) and 64-bit GUI (R64.app). For Tcl/Tk libraries (needed if you want to use `tcltk`) and GNU Fortran (needed if you want to compile packages from sources that contain FORTRAN code) please see [the tools](#)

**11.** Deal til first ace appears. Let  $X$  = the *next* card after the ace.

$P(X = A\spadesuit)$ ?  $P(X = 2\clubsuit)$ ?