Stat 100a: Introduction to Probability.

Outline for the day:

1. Ly vs. Negreanu flush draw example.
2. A or K on the board example.
3. Addiction and poker legality.
4. Kohlberg and Murphy example.
5. A♠ vs 2♣ after first ace.
6. Conditional probability, independence, and multiplication rule.
7. Independence and dependence situations.
8. Negreanu and Elezra example.

♣ ♥ ◆ ♠

On problem 2.4, use the convention that a royal flush is an example of a straight flush.

Note that the answer to 2.9 is in the back of the book.

1. **Ly vs. Negreanu, p66.**

Ex. Suppose you have 2 ♣s, and there are exactly 2 ♣s on the flop. Given this info, what is P(at least one more ♣ on turn or river)?

Answer: 52-5 = 47 cards left (9 ♣s, 38 others).

So n = choose(47,2) = 1081 combinations for next 2 cards.

Each equally likely (and obviously mutually exclusive).

Two-♣ combos: choose(9,2) = 36. One-♣ combos: 9 x 38 = 342. Total = 378. So answer is 378/1081 = 35.0%.

Answer #2: Use the addition rule…

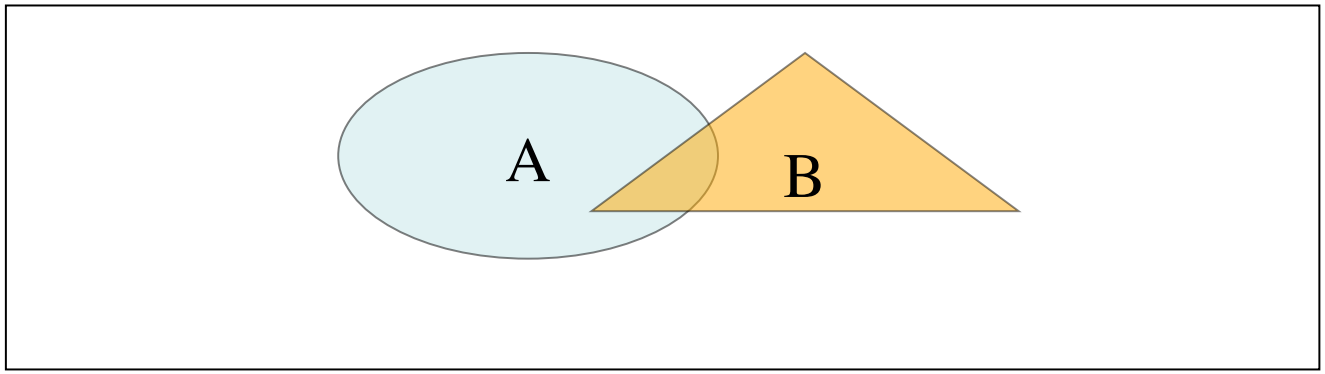
ADDITION RULE, revisited…..

Axioms (initial assumptions/rules) of probability:

1) $P(A) \geq 0$.
2) $P(A) + P(A^c) = 1$.
3) Addition rule:

   If $A_1, A_2, A_3, \ldots$ are mutually exclusive, then

   $P(A_1 \text{ or } A_2 \text{ or } A_3 \text{ or } \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$



As a result, even if A and B might not be mutually exclusive,

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. (p6 of book)

Ex. You have 2 ♣s, and there are exactly 2 ♣s on the flop. Given this info, what is P(at least one more ♣ on turn or river)?

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Answer #2: Use the addition rule.

P(≥ 1 more ♣) = P(♣ on turn OR river)

= P(♣ on turn) + P(♣ on river) - P(both)

= 9/47 + 9/47 - choose(9,2)/choose(47,2)

= 19.15% + 19.15% - 3.3% = 35.0%.

2. A or K on the board.

Ex. You have AK. Given this, what is
P(at least one A or K comes on board of 5 cards)?

Wrong Answer:
P(A or K on 1st card) + P(A or K on 2nd card) + …
= 6/50 x 5 = **60.0%.**
No: these events are NOT Mutually Exclusive!!!

Right Answer:
choose(50,5) = 2,118,760 boards possible.
How many have exactly one A or K? 6 x choose(44,4) = 814,506
with exactly 2 aces or kings? choose(6,2) x choose(44,3) = 198,660
with exactly 3 aces or kings? choose(6,3) x choose(44,2) = 18,920 …
… altogether, 1,032,752 boards have at least one A or K,
So it's 1,032,752 / 2,118,760 = **48.7%.**

Easier way: P(no A and no K) = choose(44,5)/choose(50,5)
= 1086008 / 2118760 = 51.3%, so answer = 100% - 51.3% = **48.7%**
3. Addiction and legality of poker.

4. Example: *Poker Royale: Comedians vs. Poker Pros*, Fri 9/23/05.

Linda Johnson	$543,000	Kathy Kolberg	$300,000
Phil Laak	$475,000	Sue Murphy	$155,000
Tammy Pescatelli	$377,000	Mark Curry	$0.

No small blind. Johnson in big blind for $8000.

Murphy (8♥ 8♠). Calls $8,000.

Kolberg. (9♣ 9♦). Raises to $38,000.

Pescatelli (K♥ 3♠) folds, Laak (9♥ 3♥) folds, Johnson (J♥ 6♦) folds.

Murphy calls.

TV Screen: Kolberg. **(9♣ 9♦)** 81% Murphy **(8♥ 8♠)** 19%

Flop: **8♣ 10♦ 10♠.**

Murphy quickly goes all in. Kolberg thinks for 2 min, then calls.

Laak (to Murphy): "You're 92% to take it down."

TV Screen: Kolberg. **(9♣ 9♦)** 17% Murphy **(8♥ 8♠)** 83%

Who's right?

(Turn **9♠** river **A♦**), so Murphy is eliminated. Laak went on to win.

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Cardplayer.com: 16.8% 83.2%

Laak (about Kolberg): "She has two outs twice."

P(9 on the turn or river, given just their 2 hands and the flop)?

= P(9 on turn) + P(9 on river) - P(9 on both)

= 2/45 + 2/45 - 1/choose(45,2) = 8.8%

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Given just their 2 hands and the flop, what is

P(9 or 10 on the turn or river, but not 98 or 10 8)?

P(9 or 10 on the turn) + P(9 or 10 on river) - P(9 10) – [P(98) + P(10 8)]

= 4/45 + 4/45 - [choose(4,2) + 2 + 2]/choose(45,2) = 16.77%

5. Deal til first ace appears. Let X = the *next* card after the ace.

P(X = A♠)? P(X = 2♣)?

(a) How many permutations of the 52 cards are there?

52!

(b) How many of these perms. have A♠ right after the 1st ace?

(i) How many perms of the *other* 51 cards are there?

51!

(ii) For *each* of these, imagine putting the A♠ right after the 1st ace.

1:1 correspondence between permutations of the other 51 cards & permutations of 52 cards such that A♠ is right after 1st ace.

So, the answer to question (b) is 51!.

Answer to the overall question is 51! / 52! = 1/52.

Obviously, same goes for 2♣.

3. Negreanu and Elezra.

6. Conditional Probability, Independence, & Mult. Rule.

P(A & B) is often written "P(AB)".

"P(A U B)" means P(A or B [or both]).

Conditional Probability:

P(A given B) [written"P(A|B)"] = P(AB) / P(B).

Independent: A and B are "independent" if P(A|B) = P(A).

Fact (*multiplication rule for independent events*):

If A and B are independent, then P(AB) = P(A) x P(B)

Fact (general *multiplication rule*):

P(AB) = P(A) P(B|A)

P(ABC…) = P(A) x P(B|A) x P(C|A&B) …

7. Independence and Dependence Examples

Independence: $P(A \mid B) = P(A)$ [and $P(B|A) = P(B)$].

So, when independent, $P(A\&B) = P(A)P(B|A) = P(A)P(B)$.

Reasonable to assume the following are independent:

a) Outcomes on different rolls of a die.

b) Outcomes on different flips of a coin.

c) Outcomes on different spins of a spinner.

d) Outcomes on different poker hands.

e) Outcomes when sampling from a large population.

Ex: P(you get AA on 1st hand and I get AA on 2nd hand)

= P(you get AA on 1st) x P(I get AA on 2nd)

= 1/221 x 1/221 = 1/48841.

P(you get AA on 1st hand and I get AA on 1st hand)

= P(you get AA) x P(I get AA | you have AA)

= 1/221 x 1/(50 choose 2) = 1/221 x 1/1225 = 1/270725.

8. Example: High Stakes Poker, 1/8/07, Negreanu vs. Elezra.

Greenstein folds, Todd Brunson folds, Harman folds. Elezra calls $600, Farha raises to $2600, Sheikhan folds. Negreanu calls, Elezra calls. Pot is $8,800.

Flop: 6♠ 10♠ 8♥.

Negreanu bets $5000. Elezra raises to $15000. Farha folds.
Negreanu thinks for 2 minutes….. then goes all-in for another $96,000.
Elezra: 8♣ 6♣. (Elezra calls. Pot is $214,800.)
Negreanu: A♦ 10♥.

At this point, the odds on tv show 73% for Elezra and 25% for Negreanu.
They "run it twice". First: 2♠ 4♥. Second time? A♥ 8♦!

P(Negreanu hits an A or 10 on turn & still loses)?

Given both their hands, and the flop, and the first “run”, what is P(Negreanu hits an A or 10 on the turn & loses)?

Since he can’t lose if he hits a 10 on the turn, it’s:

P(A on turn & Negreanu loses)

= P(A on turn) x P(Negreanu loses | A on the turn)

= 3/43 x 4/42

= 0.66% (1 in 150.5)

Note: this is very different from:

P(A or 10 on turn) x P(Negreanu loses),

which would be about 5/43 x 73% = 8.49% (1 in 12)