Stat 100a: Introduction to Probability.

Outline for the day:

1. Negative binomial random variables.
2. Moment generating functions of different distributions.
3. Survivor functions.
4. Violette/Elezra pot odds example.
5. P(4 of a kind) examples.
6. Heads up with KK.
7. Project examples.

HW1 is due this Thu in class.
Put your name and section letter on your hw.
Hand in your hw in piles according to section.
Mid 1 is Thu Feb 5.
For project teams see teamsresults.txt .
Project is due Tue Mar 3 8pm by email.

1. Negative Binomial Random Variables, ch 5.4.

Recall: if each trial is independent, and each time the probability of an occurrence is p, and X = # of trials until the *first* occurrence, then:

X is Geometric (p), $P(X = k) = p^1 q^{k-1}$, $\mu = 1/p$, $\sigma = (\sqrt{q}) \div p$.

Suppose now X = # of trials until the *rth* occurrence.

Then X = *negative binomial (r,p)*.

e.g. the number of hands you have to play til you've gotten r=3 pocket pairs.

Now X could be 3, 4, 5, …, up to ∞.

pmf: $P(X = k) = \text{choose}(k-1, r-1)\, p^r q^{k-r}$, for k = r, r+1, ….

e.g. say r=3 & k=7: $P(X = 7) = \text{choose}(6,2)\, p^3 q^4$.

Why? Out of the first 6 hands, there must be exactly r-1 = 2 pairs. Then pair on 7th.

P(exactly 2 pairs on first 6 hands) = $\text{choose}(6,2)\, p^2 q^4$. P(pair on 7th) = p.

If X is negative binomial (r,p), then $\mu = r/p$, and $\sigma = (\sqrt{rq}) \div p$.

e.g. Suppose X = the number of hands til your 12th pocket pair. P(X = 100)? E(X)? σ?

X = Neg. binomial (12, 5.88%).

$P(X = 100) = \text{choose}(99,11)\, p^{12} q^{88}$

= choose(99,11) * 0.0588 ^ 12 * 0.9412 ^ 88 = **0.104%.**

E(X) = r/p = 12/0.0588 ~ **204**. σ = sqrt(12*0.9412) / 0.0588 = **57.2**.

So, you'd typically *expect* it to take 204 hands til your 12th pair, +/- around 57.2 hands.

2. Moment generating functions of some random variables.

Bernoulli(p). $\phi_X(t) = pe^t + q$.

Binomial(n,p). $\phi_X(t) = (pe^t + q)^n$. p94.

Geometric(p). $\phi_X(t) = pe^t/(1 - qe^t)$.

Neg. binomial (r,p). $\phi_X(t) = [pe^t/(1 - qe^t)]^r$. p97.

Poisson(λ). $\phi_X(t) = e^{\{\lambda e^t - \lambda\}}$. p100.

Uniform (a,b). $\phi_X(t) = (e^{tb} - e^{ta})/[t(b-a)]$. p108.

Exponential (λ). $\phi_X(t) = \lambda/(\lambda - t)$. p123.

Normal. $\phi_X(t) = e^{\{t\mu + t^2\sigma^2/2\}}$.

Note t is missing in neg. binomial one on p97.

3. Survivor functions. p96 and 115.

Recall the cdf $F(b) = P(X \leq b)$.
The survivor function is $S(b) = P(X > b) = 1 – F(b)$.
Some random variables have really simple survivor functions and it can be convenient to work with them.
If X is geometric, then $S(b) = P(X > b) = q^b$, for $b = 0,1,2,....$
For instance, let $b=2$. $X > 2$ means the 1st two were misses,
i.e. $P(X>2) = q^2$.
For exponential X, $F(b) = 1 - exp(-\lambda b)$, so $S(b) = exp(-\lambda b)$.

An interesting fact is that, if X takes on only values in $\{0,1,2,3,...\}$,
then $E(X) = S(0) + S(1) + S(2) + ...$
Proof. See p96.

$$
\begin{aligned}
S(0) &= P(X=1) + P(X=2) + P(X=3) + P(X=4) + ... \\
S(1) &= \qquad\qquad\quad\; P(X=2) + P(X=3) + P(X=4) + ... \\
S(2) &= \qquad\qquad\qquad\qquad\qquad\;\; P(X=3) + P(X=4) + ... \\
S(3) &= \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\; P(X=4) + ...
\end{aligned}
$$

Add these up and you get
$0\,P(X=0) + 1P(X=1) + 2P(X=2) + 3P(X=3) + 4P(X=4) + ...$
$= \sum kP(X=k) = E(X)$.

4. Pot odds ex. *Poker Superstars Invitational Tournament*, FSN, October 2005.

Ted Forrest: 1 million chips
Freddy Deeb: 825,000 Blinds: 15,000 / 30,000
Cindy Violette: 650,000
Eli Elezra: 575,000

* Elezra raises to 100,000
* Forrest folds.
* Deeb, the small blind, folds.
* Violette, the big blind with K♦ J♦, calls.

* The flop is: 2♦ 7♣ A♦

* Violette bets 100,000. (pot = 315,000).
* Elezra raises all-in to 475,000. (pot = 790,000).

So, it's 375,000 more to Violette. She folds.

Q: Based on expected value, should she have called?
Her chances must be at least 375,000 / (790,000 + 375,000) = 32%.

Violette has K♦ J♦. The flop is: 2♦ 7♣ A♦.

Q: Based on expected value, should she have called?

Her chances must be at least 375,000 / (790,000 + 375,000) = 32%.

vs.	AQ: 38%.	AK: 37%	AA: 26%	77: 26%	A7: 31%
	A2: 34%	72: 34%	TT: 54%	T9: 87%	73: 50%

Harrington's principle: always assume at least a 10% chance that opponent is bluffing.

Bayesian approach: average all possibilities, weighting them by their likelihood.

Violette has K♦ J♦.   The flop is: 2♦ 7♣ A♦.

Q: Based on expected value, should she have called?

Her chances must be at least 375,000 / (790,000 + 375,000) = 32%.

vs.	AQ: 38%.	AK: 37%	AA: 26%	77: 26%	A7: 31%
	A2: 34%	72: 34%	TT: 54%	T9: 87%	73: 50%

Harrington's principle: always assume at least a 10% chance that opponent is bluffing.
Bayesian approach: average all possibilities, weighting them by their likelihood.

Reality: Elezra had 7♦ 3♥. Her chances were 51%. Bad fold.
What was her prob. of winning (given just her cards and Elezra's, and the flop)?
Of choose(45,2) = 990 combinations for the turn & river, how many give her the win?
First, how many outs did she have? eight ♦s + 3 kings + 3 jacks = 14.
She wins with (out, out) or (out, nonout) or (non-♦ Q, non-♦ T)
choose(14,2) + 14 x 31 + 3 * 3 = 534
but not (k or j, 7 or non-♦ 3) and not (3♦ , 7 or non-♦ 3)
- 6 * 4 - 1 * 4 = 506.
So the answer is 506 / 990 = 51.1%.

5. 4 of a kind examples.
Suppose you're all in next hand, no matter what cards you get.

P(eventually make 4-of-a-kind)? [including case where all 4 are on board]
Trick: just forget card order, and consider all collections of 7 cards.
Out of choose(52,7) different combinations, each equally likely, how many of them involve 4-of-a-kind?
13 choices for the 4-of-a-kind.
For each such choice, there are choose(48,3) possibilities for the other 3 cards.
So, P(4-of-a-kind) = 13 * choose(48,3) / choose(52,7) ~ 0.168%, or 1 in **595.**

P(flop 4-of-a-kind) =
13*48 / choose(52,5) = 0.024% = 1 in **4165**.

P(flop 4-of-a-kind | pocket pair)?
No matter which pocket pair you have, there are choose(50,3) possible flops, each equally likely, and how many of them give you 4-of-a-kind?
48. (e.g. if you have 7♣ 7♥, then need to flop 7♦ 7♠ x, & there are 48 choices for x)
So P(flop 4-of-a-kind | pp) = 48/choose(50,3) = 0.245% = 1 in **408**.

6. Heads up with KK?

Recall Dan Harrington says that,
"with a hand like AA, you really want to be one-on-one."
It's questionable though.
Do you want multiple callers with KK?

a) You have \$100 and KK and are all-in against TT. You're 81% to double up, so your expected number of chips after the hand is

0.81 x \$200 = **\$162**.

b) You have \$100 and KK and are all-in against A9 and TT. You're 58% to have \$300, so your expected value is **\$174**.

- So, if you have KK and an opponent with TT has already called you, and another who has A9 is thinking about whether to call you too, you actually want A9 to call!
- Given this, one may question Harrington's suggested strategy of raising huge in order to isolate yourself against one player.

7. Computer projects.

The project is problem 8.2, p165, code to go all in or fold. In R, try:

```
install.packages(holdem)
library(holdem)
library(help="holdem")
```

timemachine, tommy, ursula, vera, william, and xena are examples.

crds1[1,1] is your higher card (2-14).

crds1[2,1] is your lower card (2-14).

crds1[1,2] and crds1[2,2] are suits of your higher card & lower card.

```
help(tommy)
tommy
```

```
tommy = function (numattable1, crds1, board1, round1, currentbet,
    mychips1, pot1, roundbets, blinds1, chips1, ind1, dealer1, tablesleft)
{ a1 = 0
  if (crds1[1, 1] == crds1[2, 1]) a1 = mychips1
  a1
}
```

help(vera)

All in with a pair, any suited cards, or if the smaller card is at least 9.

```
vera = function (numattable1, crds1, board1, round1, currentbet,
     mychips1,
   pot1, roundbets, blinds1, chips1, ind1, dealer1, tablesleft)
{a1 = 0
 if ((crds1[1, 1] == crds1[2, 1]) || (crds1[1, 2] == crds1[2,2]) ||
     (crds1[2, 1] > 8.5)) a1 = mychips1
   a1
}
```

You need to email me your function, to frederic@stat.ucla.edu. It should be written (or cut and pasted) simply into the body of the email. If you write it in Word, save as text first, and then paste it into the email. Just submit one email per team.

For instance, you might do:

```
bruin = function (numattable1, crds1, board1, round1, currentbet, mychips1,
    pot1, roundbets, blinds1, chips1, ind1, dealer1, tablesleft) {
## all in with any pair higher than 7s, or if lower card is J or higher
a1 = 0
if ((crds1[1, 1] == crds1[2, 1]) && (crds1[1, 1] > 6.5)) a1 = mychips1
if (crds1[2,1] > 10.5) a1 = mychips1
a1
} ## end of bruin
```