Stat 100a: Introduction to Probability.

Outline for the day:

1. Exam notes.
2. Luck, skill and equity, ch. 4.4.
3. Review list.
4. Random walk example.
5. Bayes' rule example.
6. Equity gained example.
7. Conditional probability examples.

Exam 2 is Tue Mar 3.
The project is due Tue Mar 3 8pm by email to me.
NO CLASS or OH Tue Mar 10.
Hw3 is due Mar 12.
Exam 3 is Thu Mar 12.

1. Exam notes.

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Both exams are scantron.

They are open book, open note. Bring a calculator and a #2 pencil.

There's no R stuff on the exams.

Suggested problems from ch 4-7 to look at are 4.5, 4.6, 4.7, 4.8, 4.9, 4.13, 4.14, 4.16, 5.1, 5.2, 5.5, 5.6, 6.2, 6.4, 6.9, 6.10, 6.11, 6.12, 7.1, 7.2, 7.3, 7.4, 7.5, 7.8, 7.13, 7.14, 7.15.

2. Luck, skill, and equity.

Is poker luck or skill?

Who cares? The courts do, for one.

How would you define luck or skill?

Let equity = your expected holdings after the hand,

i.e. your chips + pot x p, where p = your probability of winning the hand in a showdown, i.e. assuming no future betting.

According to the book's definitions,

luck = equity gained during the dealing of the cards.

skill = equity gained during betting.

For example,

Lederer vs. Minieri, hand number 7. Consider the turn.

Hand 7. Minieri 10♦ 7♠, Lederer Q♣ 2♥. Minieri 43.57%, Lederer 56.43%. Minieri raises to 3200, Lederer calls 1600.

Flop 8♠ 2♠ Q♥. Minieri 7.27%, Lederer 92.73%. Lederer checks, Minieri bets 3200, Lederer calls.

Turn 4♦. Minieri 0%, Lederer 100%. Lederer checks, Minieri bets 10,000, Lederer calls.

River A♥. Lederer checks, Minieri checks.

Luck -205.76 - 2323.20 **- 930.56** = -3459.52.

Skill -205.76 - 2734.72 **- 10000** = -12940.48.

Let x = Minieri's number of chips after the hand. The pot size was 6400+6400 = 12800 before the turn.

Minieri's gain due to luck on the turn = Minieri's equity right after the turn was dealt – equity before turn dealt

= [x + 10,000 + (12,800)(0%)] – [x + 10,000 + (12,800)(7.27%)] = -930.56.

Minieri's gain due to skill on the turn = his equity after turn betting - equity right before turn betting

= [x + (12,800)(0%)] – [x + 10,000 + (12,800)(0%)] = -$10,000.

Were there some key hands where one of the two players gained a lot due to either luck or skill? Which ones?

Do the definitions seem to fit?

When would they not fit?

3. Review list.

1) Basic principles of counting.
2) Axioms of probability, and addition rule.
3) Permutations & combinations.
4) Conditional probability.
5) Independence.
6) Multiplication rules. P(AB) = P(A) P(B|A) [= P(A)P(B) if ind.]
7) Odds ratios.
8) Random variables (RVs).
9) Discrete RVs, and probability mass function (pmf).
10) Expected value.
11) Pot odds calculations.
12) Luck, skill, and deal-making.
13) Variance and SD.
14) Bernoulli RV. [0-1. $\mu = p$, $\sigma = \sqrt{(pq)}$.]
15) Binomial RV. [# of successes, out of n tries. $\mu = np$, $\sigma = \sqrt{(npq)}$.]
16) Geometric RV. [# of tries til 1st success. $\mu = 1/p$, $\sigma = (\sqrt{q}) / p$.]
17) Negative binomial RV. [# of tries til rth success. $\mu = r/p$, $\sigma = (\sqrt{rq}) / p$.]
18) Poisson RV [# of successes in some time interval. [$\mu = \lambda$, $\sigma = \sqrt{\lambda}$.]
19) E(X+Y), V(X+Y) (ch. 7.1).
20) Bayes's rule (ch. 3.4).
20) Continuous RVs
21) Probability density function (pdf)
22) Uniform RV
23) Normal RV
24) Exponential and Pareto RVs
25) Moment generating functions
26) Markov and Chebyshev inequalities
25) Law of Large Numbers (LLN)
26) Central Limit Theorem (CLT)
27) Conditional expectation.
28) Confidence intervals for the sample mean.
29) Fundamental theorem of poker
30) Random Walks, Reflection Principle, Ballot Theorem, avoiding zero
31) Chip proportions and induction.

Basically, we've done all of ch. 1-7 except 6.7.

4. Another random walk example.

Suppose that a $10 winner-take-all tournament has $32 = 2^5$ players. So, you need to double up 5 times to win. Winner gets $320.

Suppose that on each hand of the tournament, you have probability $p = 0.7$ to double up, and with probability $q = 0.3$ you will be eliminated. What is your expected profit in the tournament?

Your expected *return* = ($320) x P(win the tournament) + ($0) x P(you don't win)

= ($320) x 0.7^5 = $53.78. But it costs $10.

So expected *profit* = $53.78 - $10 = $43.78.

Alternatively, your profit could = $310 or $-10, so

expected profit = ($310)($0.7^5$) + (-$10)(1 - 0.7^5)

= $52.10 - $10 + $1.68

= $43.78.

5. Bayes' rule example.

Your opponent raises all-in before the flop. Suppose you think she would do that *80%* of the time with AA, KK, or QQ, and she would do that *30%* of the time with AK or AQ, and 1% of the time with anything else.

Given only this, and not even your cards, what's P(she has AK)?

Given nothing, P(AK) = 16/C(52,2) = 16/1326. P(AA) = C(4,2)/C(52,2) = 6/1326.

Using Bayes' rule,

$$P(\text{AK} \mid \text{all-in}) = \frac{P(\text{all-in} \mid \text{AK}) * P(\text{AK})}{P(\text{all-in}|\text{AK})P(\text{AK}) + P(\text{all-in}|\text{AA})P(\text{AA}) + P(\text{all-in}|\text{KK})P(\text{KK}) + \ldots},$$

$$= \frac{30\% \times 16/1326}{[30\%\text{x}16/1326] + [80\%\text{x}6/1326] + [80\%\text{x}6/1326] + [80\%\text{x}6/1326] + [30\%\text{x}16/1326] + [1\%\ (1326-16-6-6-6-16)/1326)]}.$$

(AK) (AA) (KK) (QQ) (AQ) (anything else)

= **13.06%.** Compare with 16/1326 ~ 1.21%.

6. Equity gained example.

You have Qc Qd. I have 10s 9s. Board is 10d 8c 7c 4c. Pot is \$5.
The river is 2d, you bet \$3, and I call.
On the river, how much equity did you gain by luck and how much equity did you gain by skill?

Equity gained by luck on river = your equity when 2d is exposed – your equity pre-2d
= 100% (\$5) - 35/44 (\$5) = \$1.02.
Why 35/44? Before the river, I can win with a J, 10, 9, 6, that's not a club. There are 3 + 1 + 2 + 3 = 9 of these cards, so the remaining 35 cards give you the win.

Let x = the number of chips you had after betting the \$3, i.e. the number of chips you would have had if I called you and you lost.
Your equity gained by skill on the river
= your equity after river betting - your equity right before river betting
= [x + \$11 (100%)] – [x + \$3 + \$5(100%)]
= x + \$11 - x - \$8
= \$3.

alternatively it = increase in pot on river * P(you win) – your cost on the river
= \$6 * 100% - \$3
= \$3.

7. Conditional prob. examples.

Approximate P(SOMEONE has AA, given you have KK)? Out of your 8 opponents?

Note that given that you have KK,

P(player 2 has AA & player 3 has AA)

= P(player 2 has AA) x P(player 3 has AA | player 2 has AA)

= choose(4,2) / choose(50,2) x 1/choose(48,2)

= 0.0000043, or 1 in 230,000.

So, very little overlap! Given you have KK,

P(someone has AA) = P(player2 has AA or player3 has AA or … or pl.9 has AA)

~ P(player2 has AA) + P(player3 has AA) + … + P(player9 has AA)

= 8 x choose(4,2) / choose(50,2) = 3.9%, or 1 in **26.**

What is **exactly** P(SOMEONE has an Ace | you have KK)? (8 opponents)

(or more than one ace)

Given that you have KK,

P(SOMEONE has an Ace) = 100% - P(nobody has an Ace).

And P(nobody has an Ace) = choose(46,16)/choose(50,16)

= **20.1%.**

So P(SOMEONE has an Ace) = **79.9%.**

Here are some more conditional probability examples.

P(You have AK | you have exactly one ace)?

= P(You have AK and exactly one ace) / P(exactly one ace)

= P(AK) / P(exactly one ace)

= (16/C(52,2)) ÷ (4x48/C(52,2))

= 4/48 = 8.33%.

P(You have AK | you have at least one ace)?

= P(You have AK and at least one ace) / P(at least one ace)

= P(AK) / P(at least one ace)

= 16/C(52,2) ÷ {4x48 + C(4,2)}/C(52,2) ~ 8.08%.

P(You have AK | your FIRST card is an ace)?

= 4/51 = 7.84%.