Stat 100a, Introduction to Probability.

Outline for the day:

1. Daniel vs. Gus.
2. P(flop 3 of a kind).
3. P(eventually make 4 of a kind).
4. P(A♠ after first ace).
5. Bayes' rule.

Finish chapters 1-3 and start on ch4.

For problem 2.4, consider a royal flush an example of a straight flush. That is, calculate P(straight flush or royal flush). ♠ ♣ ♥ ♦

1. High Stakes Poker, Daniel vs. Gus.

Which is more likely, given no info about your cards:

* flopping 3 of a kind,

or

* eventually making 4 of a kind?

2. P(flop 3 of a kind)?

[including case where all 3 are on board, and *not including full houses*]

Key idea: forget order! Consider all combinations of your 2 cards and the flop.

Sets of 5 cards. Any such combo is equally likely! choose(52,5) different ones.

P(flop 3 of a kind) = # of different 3 of a kinds / choose(52,5)

How many different 3 of a kind combinations are possible?

13 * choose(4,3) different choices for the triple.

For each such choice, there are choose(12,2) choices left for the numbers on the other 2 cards, and for each of these numbers, there are 4 possibilities for its suit.

So, P(flop 3 of a kind) = 13 * choose(4,3) * choose(12,2) * 4 * 4 / choose(52,5)

~ 2.11%, or 1 in 47.3.

P(flop 3 of a kind or a full house) = 13 * choose(4,3) * choose(48,2) / choose(52,5)

~ 2.26%, or 1 in 44.3.

3. P(eventually make 4 of a kind)? [including case where all 4 are on board]

Again, just forget card order, and consider all collections of 7 cards.

Out of choose(52,7) different combinations, each equally likely, how many of them involve 4-of-a-kind?

13 choices for the 4-of-a-kind.

For each such choice, there are choose(48,3) possibilities for the other 3 cards.

So, P(4-of-a-kind) = 13 * choose(48,3) / choose(52,7) ~ 0.168%, or 1 in 595.

4. Deal til first ace appears. Let X = the *next* card after the ace.

P(X = A♠)? P(X = 2♣)?

(a) How many permutations of the 52 cards are there?

52!

(b) How many of these perms. have A♠ right after the 1st ace?

(i) How many perms of the *other* 51 cards are there?

51!

(ii) For *each* of these, imagine putting the A♠ right after the 1st ace.

1:1 correspondence between permutations of the other 51 cards & permutations of 52 cards such that A♠ is right after 1st ace.

So, the answer to question (b) is 51!.

Answer to the overall question is 51! / 52! = 1/52.

Obviously, same goes for 2♣.

5. Bayes's rule.

Suppose that $B_1, B_2, \ldots, B_n$ are disjoint events and that exactly one of them must occur.

Suppose you want $P(B_1 \mid A)$, but you only know $P(A \mid B_1)$, $P(A \mid B_2)$, etc., and you also know $P(B_1), P(B_2), \ldots, P(B_n)$.

Bayes' Rule: If $B_1, \ldots, B_n$ are disjoint events with $P(B_1 \text{ or } \ldots \text{ or } B_n) = 1$, then

$$P(B_i \mid A) = P(A \mid B_i) * P(B_i) \div [\sum P(A \mid B_j)P(B_j)].$$

Why? Recall: $P(X \mid Y) = P(X \,\&\, Y) \div P(Y)$. So $P(X \,\&\, Y) = P(X \mid Y) * P(Y)$.

$$P(B_1 \mid A) = P(A \,\&\, B_1) \div P(A)$$

$$= P(A \,\&\, B_1) \div [\, P(A \,\&\, B_1) + P(A \,\&\, B_2) + \ldots + P(A \,\&\, B_n)\,]$$

$$= P(A \mid B_1) * P(B_1) \div [\, P(A \mid B_1)P(B_1) + P(A \mid B_2)P(B_2) + \ldots + P(A \mid B_n)P(B_n)\,].$$

Bayes's rule, continued.

Bayes's rule: If $B_1, \ldots, B_n$ are disjoint events with $P(B_1 \text{ or } \ldots \text{ or } B_n) = 1$, then

$P(B_i \mid A) = P(A \mid B_i) * P(B_i) \div [\sum P(A \mid B_j)P(B_j)]$.

See example 3.4.1, p50. If a test is 95% accurate and 1% of the pop. has a condition, then given a random person from the population,

P(she has the condition | she tests positive)

= P(cond | +)

= P(+ | cond) P(cond) $\div$ [P(+ | cond) P(cond) + P(+ | no cond) P(no cond)]

= 95% x 1% $\div$ [95% x 1% + 5% x 99%]

~ 16.1%.

Tests for rare conditions must be extremely accurate.

Bayes' rule example.

Your opponent makes a huge bet of 10 times the big blind.

Suppose she'd only do that with AA, or 54, or AK.

Suppose P(huge bet | AA) = 5%. P(huge bet | 54) = 70%. P(huge bet | AK) = 25%.

What is P(AA | huge bet)?

$$P(AA \mid \text{huge bet}) = \frac{P(\text{huge bet} \mid AA) * P(AA)}{P(\text{huge bet} \mid AA) * P(AA) + P(\text{huge bet} \mid 54) * P(54) + P(\text{huge bet} \mid AK) * P(AK)}$$

$$= \frac{5\% * C(4,2)/C(52,2)}{5\% * C(4,2)/C(52,2) + 70\% * 16/C(52,2) + 25\% * 16/C(52,2)}$$

$$= 1.94\%.$$