Stat 100a: Introduction to Probability.

Outline for the day:

1. P(flop 4 of a kind).
2. Variance and SD.
3. Markov and Chebyshev inequalities.
4. Luck and skill in poker.

I will assign you to teams for the projects next week.
HW1 is due today, Wed Jan19, 2pm, by email to
STAT100AW22@stat.ucla.edu .
Exam1 is Mon Jan31. It is expected to be in person, in class.
Hw2 is due Mon Feb14, 2pm, again by email to
STAT100AW22@stat.ucla.edu.
Read through chapter 5. ♠ ♣ ♥ ♦

P(flop 4 of a kind).

Suppose you're all in next hand, no matter what cards you get.

P(flop 4 of a kind) = 13*48 / choose(52,5) = 0.024% = 1 in **4165**.

P(flop 4 of a kind | pocket pair)?

No matter which pocket pair you have, there are choose(50,3) possible flops, each equally likely, and how many of them give you 4-of-a-kind?

48. (e.g. if you have 7♠ 7♥, then need to flop 7♦ 7♣ x, & there are 48 choices for x)

So P(flop 4-of-a-kind | pp) = 48/choose(50,3) = 0.245% = 1 in **408**.

Variance and SD.

Expected Value: $E(X) = \mu = \sum k \; P(X=k)$.

Variance: $V(X) = \sigma^2 = E[(X- \mu)^2]$. Turns out this $= E(X^2) - \mu^2$.

Standard deviation $= \sigma = \sqrt{V(X)}$. Indicates how far an observation would *typically* deviate from μ.

Examples:

<u>Game 1.</u> Say X = \$4 if red card, X = \$-5 if black.

E(X) = (\$4)(0.5) + (\$-5)(0.5) = -\$0.50.

$E(X^2)$ = (\$$4^2$)(0.5) + (\$$-5^2$)(0.5) = (\$16)(0.5) + (\$25)(0.5) = \$20.5.

So $\sigma^2 = E(X^2) - \mu^2$ = \$20.5 - \$$-0.50^2$ = \$20.25. **σ = \$4.50**.

<u>Game 2.</u> Say X = \$1 if red card, X = \$-2 if black.

E(X) = (\$1)(0.5) + (\$-2)(0.5) = -\$0.50.

$E(X^2)$ = (\$$1^2$)(0.5) + (\$$-2^2$)(0.5) = (\$1)(0.5) + (\$4)(0.5) = \$2.50.

So $\sigma^2 = E(X^2) - \mu^2$ = \$2.50 - \$$-0.50^2$ = \$2.25. **σ = \$1.50**.

Markov and Chebyshev inequalities. Ch 4.6.

The Markov inequality states

If X takes only non-negative values, and c is any number > 0, then

$P(X \geq c) \leq E(X)/c$.

Proof. The discrete case is given on page 123.

If X is discrete and nonnegative, then

$$
\begin{aligned}
E(X) &= \sum_b b\, P(X = b) \\
&= \sum_{b<c} b\, P(X = b) \quad + \quad \sum_{b\geq c} b\, P(X = b) \\
&\geq \sum_{b\geq c} b\, P(X = b) \\
&\geq \sum_{b\geq c} c\, P(X = b) \\
&= c \sum_{b\geq c} P(X = b) \\
&= c\, P(X \geq c).
\end{aligned}
$$

Here is a proof for the case where X is continuous with pdf f(y).

$E(X) = \int y\, f(y)\, dy$

$= \int_0^c yf(y)dy + \int_c^\infty yf(y)dy$

$\geq \int_c^\infty yf(y)dy$

$\geq \int_c^\infty cf(y)dy$

$= c \int_c^\infty f(y)dy$

$= c\, P(X\geq c)$.

Thus, $P(X \geq c) \leq E(X) / c$.

Markov and Chebyshev inequalities. Ch 4.6.

The Markov inequality states

If X takes only non-negative values, and c is any number > 0, then

$P(X \geq c) \leq E(X)/c$.

The Chebyshev inequality states

For any random variable Y with expected value μ and variance σ^2, and any real number $a > 0$,
$P(| Y - \mu | \geq a) \leq \sigma^2 / a^2$.

Proof. The Chebyshev inequality follows directly from the Markov equality by letting $c = a^2$ and $X = (Y-\mu)^2$.

Example. Suppose the average time until a player is eliminated in a given tournament is 7 hours. Take a player at random. What does the Markov inequality say about the probability that the player lasts longer than 21 hours?

Let X = time til the player is eliminated. $X \geq 0$, so $P(X \geq 21) \leq 7/21 = 1/3$.

More examples of the use of the Markov and Chebyshev inequalities are on p83.

Luck and skill in poker. ♠ ♣ ♥ ♦

Let equity = your expected portion of the pot after the hand, assuming no future betting.
= your expected number of chips after the hand - chips you had before the hand
= pot * p, where p = your probability of winning if nobody folds.
I define luck as the expected profit gained during the dealing of the cards,
= equity gained during the dealing of the cards.
Skill = expected *profit* gained during the betting rounds.

Example.
You have Q♣ Q♦. I have 10♠ 9♠. Board is 10♦ 8♣ 7♣ 4♣. Pot is $5.
The river is 2♦, you bet $3, and I call.
On the river, how much expected profit did you gain by luck and how much by skill?

Expected profit by luck on river = your equity after 2♦ is exposed – your equity just pre-2♦
= 100% ($5) - 35/44 ($5) = $1.02.
Why 35/44? I can win with a 10, 9, 6, or J that is not a club. There are 1 + 2 + 3 + 3 = 9 of these cards, so the remaining 35 cards give you the win.

Expected profit gained by skill on river (Let x = the number of chips you have after your $3 bet on the river. Before this bet, you had x + $3 chips.)
= your expected number of chips after all the betting is over – your expected number of chips when the 2♦ is dealt
= (100%)(x + $11) – (100%)(x + $3 + $5) = $3.

Lederer and Minieri.

I define luck as the expected profit gained during the dealing of the cards.
Skill = expected profit gained during the betting rounds.

Are there any problems with these definitions?

Bluffing. Ivey and Booth.