

Stat 100a, Introduction to Probability. Rick Paik Schoenberg

Outline for the day:

1. Addiction.
2. Syllabus, etc.
3. Wasicka/Gold/Binger Example.
4. Meaning of Probability, axioms.
5. Axioms of probability.
6. Hw1 terms.
7. Ly vs. Negreanu (flush draw) example
8. Basic principle of counting.
9. Permutations and combinations.
10. R.
11. $A\spadesuit$ vs $2\clubsuit$ after first ace.

2. Syllabus, etc.

For next class:

- (i) Learn the rules of Texas Hold'em.
(see <http://www.fulltiltpoker.net/holdem.php>
and <http://www.fulltiltpoker.net/handRankHigh.php>)
- (ii) Read addiction handout, addiction1.pdf, on the course website,
<http://www.stat.ucla.edu/~frederic/100a/sum14> .
- (iii) Download R and try it out.
(<http://cran.stat.ucla.edu>)
- (iv) Read ch. 1-2 of the textbook.

If you are the **first** to find an error in the book not on the error website, email me and if you're right you'll get +1 pt on the final exam for a rel. major error (or $\frac{1}{4}$ for a very minor error), but you get $-\frac{1}{4}$ if you're wrong, for either type of error.

If you have taken Stat 35 before, please see me during the break.

Wasicka/Gold/Binger Example

Blinds: \$200,000-\$400,000 with \$50,000 antes.

Chip Counts:

Jamie Gold	\$60,000,000
Paul Wasicka	\$18,000,000
Michael Binger	\$11,000,000

Payouts: 3rd place: \$4,123,310. 2nd place: \$6,102,499. 1st place: \$12,000,000.

Day 7, Hand 229. Gold: 4s 3c. Binger: Ah 10h. Wasicka: 8s 7s.

Wasicka/Gold/Binger Example, Continued

Gold: 4♠ 3♣. Binger: A♥ 10♥. Wasicka: 8♠ 7♠.
Flop: 10♣ 6♠ 5♠. (Turn: 7♣. River: Q♠.)

Wasicka folded?!?

He had 8♠ 7♠ and the flop was 10♣ 6♠ 5♠. Worst case scenario: suppose he were up against 9♠ 4♠ and 9♥ 9♦. How could Wasicka win?

88 (3: 8♣ 8♦, 8♣ 8♥, 8♦ 8♥)

77 (3)

44 (3)

[Let “X” = non-49, “Y” = A2378JQK, and “n” = non-♠.]

4n Xn (3 x 32)

9♣ 4n (3)

9♣ Yn (24). **Total: 132 out of 903 = 14.62%.**

4. Meaning of Probability.

Notation: “ $P(A) = 60\%$ ”. A is an *event*.
Not “ $P(60\%)$ ”.

Definition of probability:

Frequentist: If repeated independently under the same conditions millions and millions of times, A would happen 60% of the times.

Bayesian: Subjective feeling about how likely something seems.

$P(A \text{ or } B)$ means $P(A \text{ or } B \text{ or both })$

Mutually exclusive: $P(A \text{ and } B) = 0$.

Independent: $P(A \text{ given } B)$ [written “ $P(A|B)$ ”] = $P(A)$.

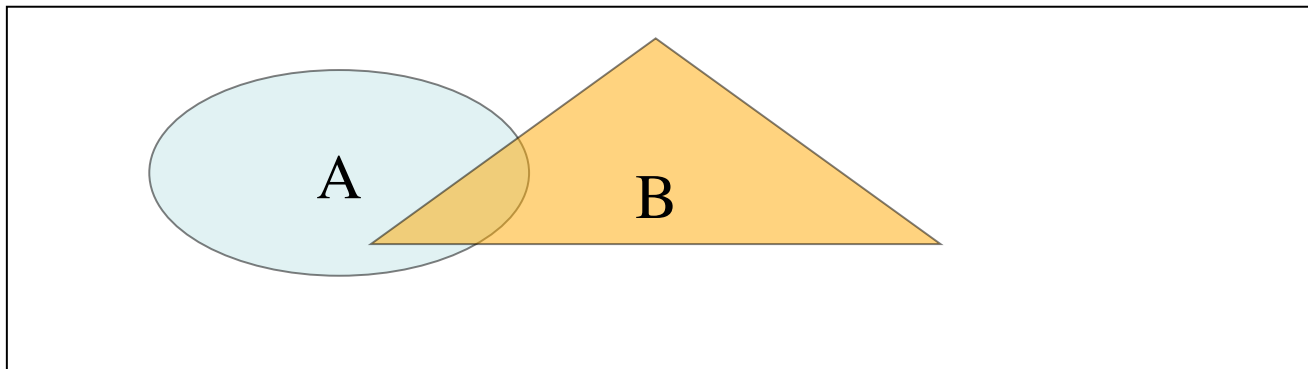
$P(A^c)$ means $P(\text{not } A)$.

5. Axioms (initial assumptions/rules) of probability:

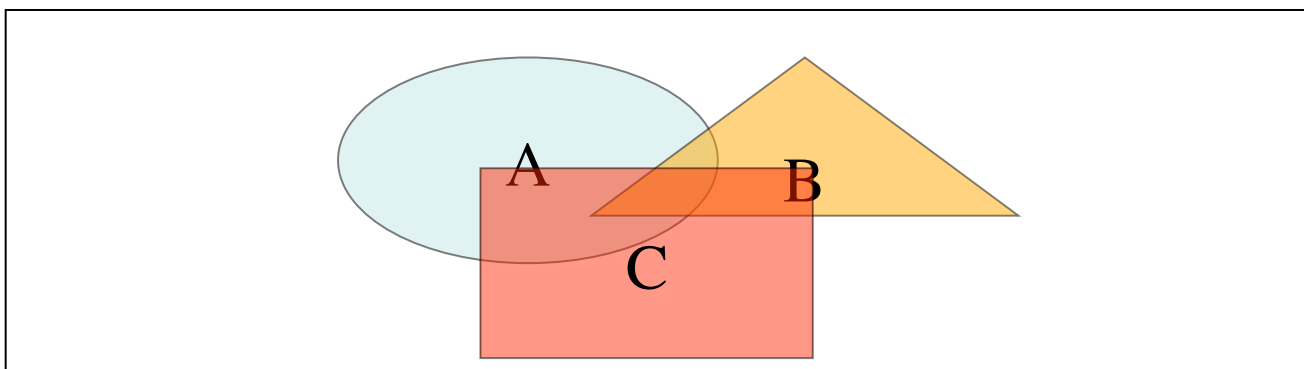
- 1) $P(A) \geq 0$.
- 2) $P(A) + P(A^c) = 1$.
- 3) If A_1, A_2, A_3, \dots are mutually exclusive, then
 $P(A_1 \text{ or } A_2 \text{ or } A_3 \text{ or } \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$

(#3 is sometimes called the *addition rule*)

Probability \Leftrightarrow Area. Measure theory, Venn diagrams



$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$



Fact: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$

$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC).$

Fact: If A_1, A_2, \dots, A_n are equally likely & mutually exclusive,
and if $P(A_1 \text{ or } A_2 \text{ or } \dots \text{ or } A_n) = 1,$
then $P(A_k) = 1/n.$

[So, you can *count*: $P(A_1 \text{ or } A_2 \text{ or } \dots \text{ or } A_k) = k/n.$]

Ex. You have 76, and the board is KQ54. $P(\text{straight})?$

[52-2-4=46.] $P(\text{straight}) = P(8 \text{ on river OR } 3 \text{ on river})$

$= P(8 \text{ on river}) + P(3 \text{ on river}) = 4/46 + 4/46.$

6. Hw1 terms. 2 pair tiebreaker, the nuts, the unbreakable nuts.

7. Ly vs. Negreanu, p66.

8. Basic Principle of Counting.

If there are a_1 distinct possible outcomes on experiment #1, and for each of them, there are a_2 distinct possible outcomes on experiment #2, then there are $a_1 \times a_2$ distinct possible *ordered* outcomes on both.

e.g. you get 1 card, opp. gets 1 card. # of distinct possibilities?
 52×51 . [ordered: $(A\clubsuit, K\heartsuit) \neq (K\heartsuit, A\clubsuit)$.]

In general, with j experiments, each with a_i possibilities, the # of distinct outcomes *where order matters* is $a_1 \times a_2 \times \dots \times a_j$.

9. Permutations and Combinations.

e.g. you get 1 card, opp. gets 1 card.

of distinct possibilities?

52 x 51. [ordered: (A♣ , K♥) ≠ (K♥ , A♣) .]

Each such outcome, where order matters, is called a *permutation*.

Number of permutations of the deck? $52 \times 51 \times \dots \times 1 = 52!$

$$\sim 8.1 \times 10^{67}$$

A combination is a collection of outcomes, where order *doesn't* matter.

e.g. in hold'em, how many *distinct* 2-card hands are possible?

52 x 51 if order matters, but then you'd be double-counting each

[since now (A♣ , K♥) = (K♥ , A♣) .]

So, the number of *distinct* hands where *order doesn't matter* is

$$52 \times 51 / 2.$$

In general, with n distinct objects, the # of ways to choose k *different* ones, *where order doesn't matter*, is

$$\text{"n choose k"} = \text{choose}(n,k) = \frac{n!}{k! (n-k)!} .$$

$k! = 1 \times 2 \times \dots \times k$. [convention: $0! = 1$.]

$$\text{choose}(n,k) = \binom{n}{k} = \frac{n!}{k! (n-k)!}.$$

Ex. You have 2 ♣s, and there are exactly 2 ♣s on the flop. Given this info, what is $P(\text{at least one more } \clubsuit \text{ on turn or river})$?

Answer: $52-5 = 47$ cards left (9 ♣s, 38 others).

So $n = \text{choose}(47,2) = 1081$ combinations for next 2 cards.

Each equally likely (and obviously mutually exclusive).

Two-♣ combos: $\text{choose}(9,2) = 36$. One-♣ combos: $9 \times 38 = 342$.

Total = 378. So answer is $378/1081 = 35.0\%$.

Answer #2: Use the addition rule...

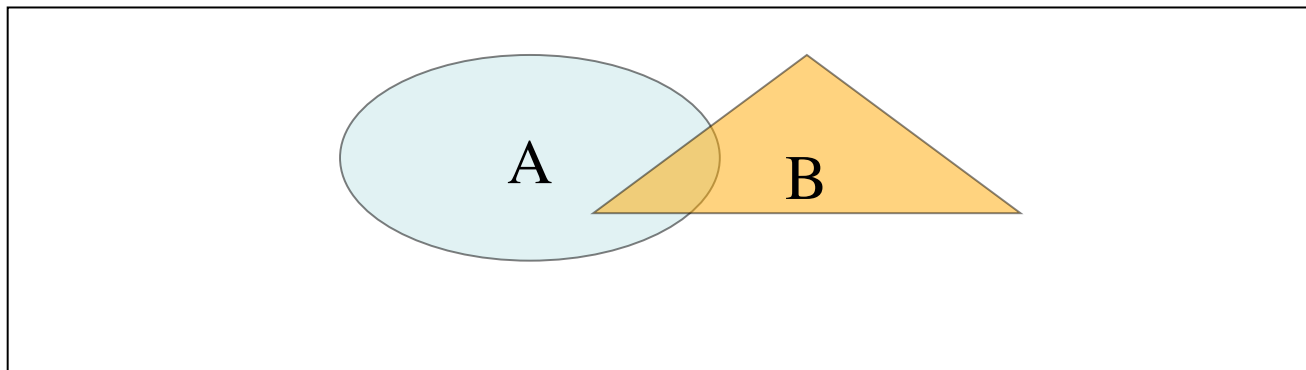
ADDITION RULE, revisited.....

Axioms (initial assumptions/rules) of probability:

- 1) $P(A) \geq 0$.
- 2) $P(A) + P(A^c) = 1$.
- 3) Addition rule:

If A_1, A_2, A_3, \dots are mutually exclusive, then

$$P(A_1 \text{ or } A_2 \text{ or } A_3 \text{ or } \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$$



As a result, even if A and B might not be mutually exclusive,
 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. (p6 of book)

Ex. You have 2 ♣s, and there are exactly 2 ♣s on the flop. Given this info, what is $P(\text{at least one more } \clubsuit \text{ on turn or river})$?

Answer #1: $52-5 = 47$ cards left (9 ♣s, 38 others).

So $n = \text{choose}(47,2) = 1081$ combinations for next 2 cards.

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Answer #2: Use the addition rule.

$$\begin{aligned} P(\geq 1 \text{ more } \clubsuit) &= P(\clubsuit \text{ on turn OR river}) \\ &= P(\clubsuit \text{ on turn}) + P(\clubsuit \text{ on river}) - P(\text{both}) \\ &= 9/47 + 9/47 - \text{choose}(9,2)/\text{choose}(47,2) \\ &= 19.15\% + 19.15\% - 3.3\% = 35.0\%. \end{aligned}$$

Ex. You have AK. Given this, what is

P(at least one A or K comes on board of 5 cards)?

Wrong Answer:

P(A or K on 1st card) + P(A or K on 2nd card) + ...

= $6/50 \times 5 = 60.0\%$.

No: these events are NOT Mutually Exclusive!!!

Right Answer:

choose(50,5) = 2,118,760 boards possible.

How many have exactly one A or K? $6 \times \text{choose}(44,4) = 814,506$

with exactly 2 aces or kings? $\text{choose}(6,2) \times \text{choose}(44,3) = 198,660$

with exactly 3 aces or kings? $\text{choose}(6,3) \times \text{choose}(44,2) = 18,920 \dots$

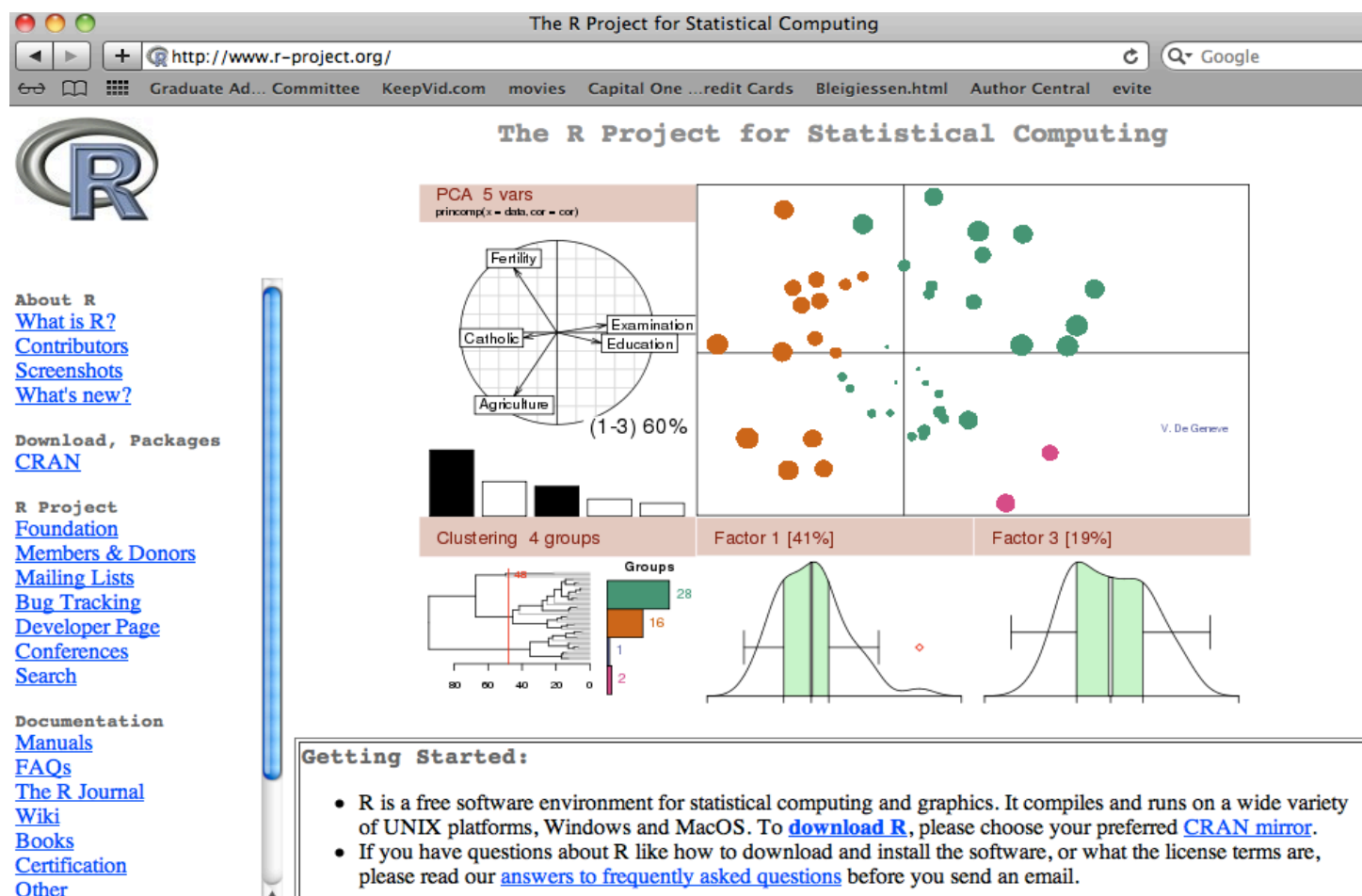
... altogether, 1,032,752 boards have at least one A or K,

So it's $1,032,752 / 2,118,760 = 48.7\%$.

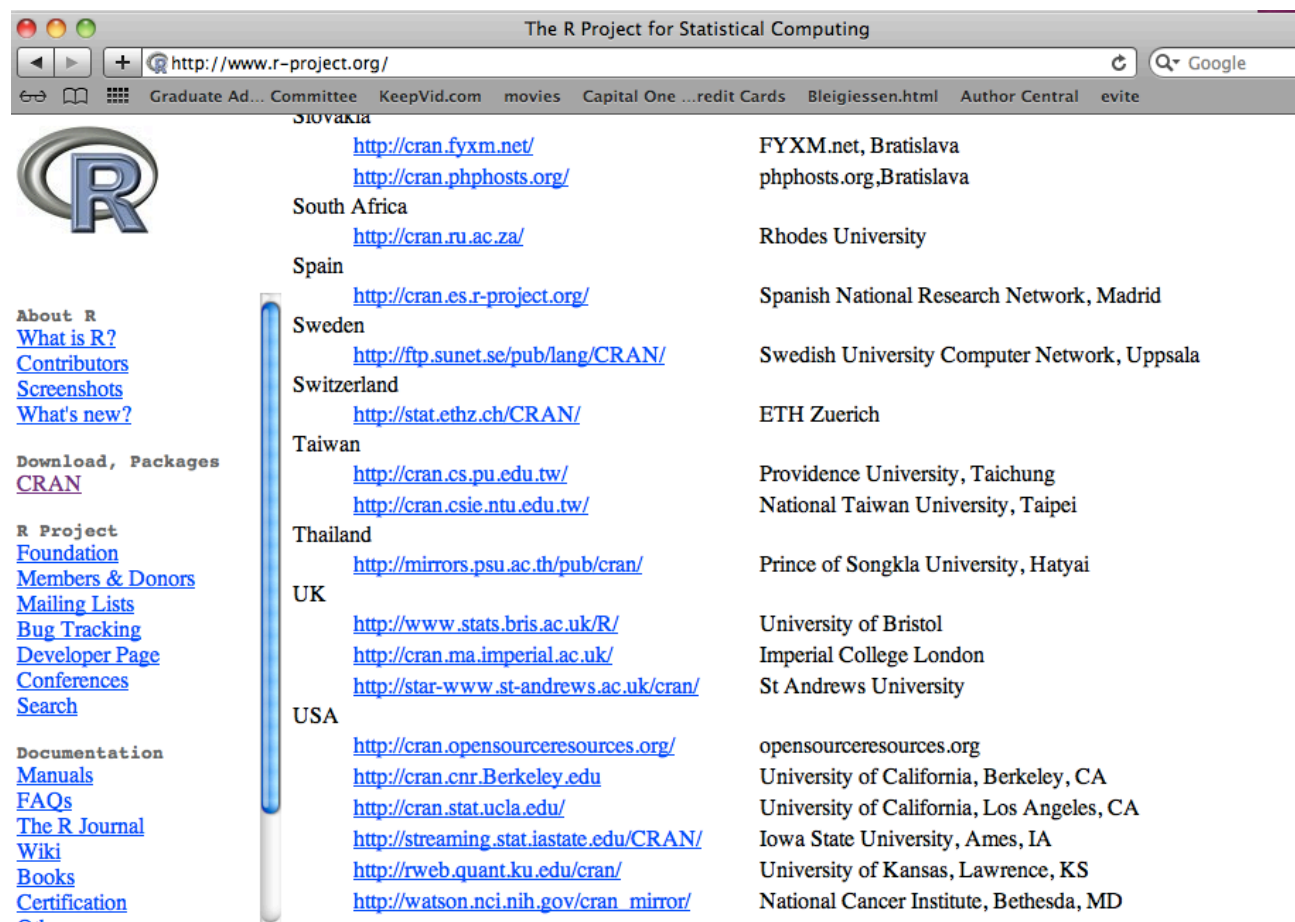
Easier way: P(no A and no K) = $\text{choose}(44,5)/\text{choose}(50,5)$

= $1086008 / 2118760 = 51.3\%$, so answer = $100\% - 51.3\% = 48.7\%$

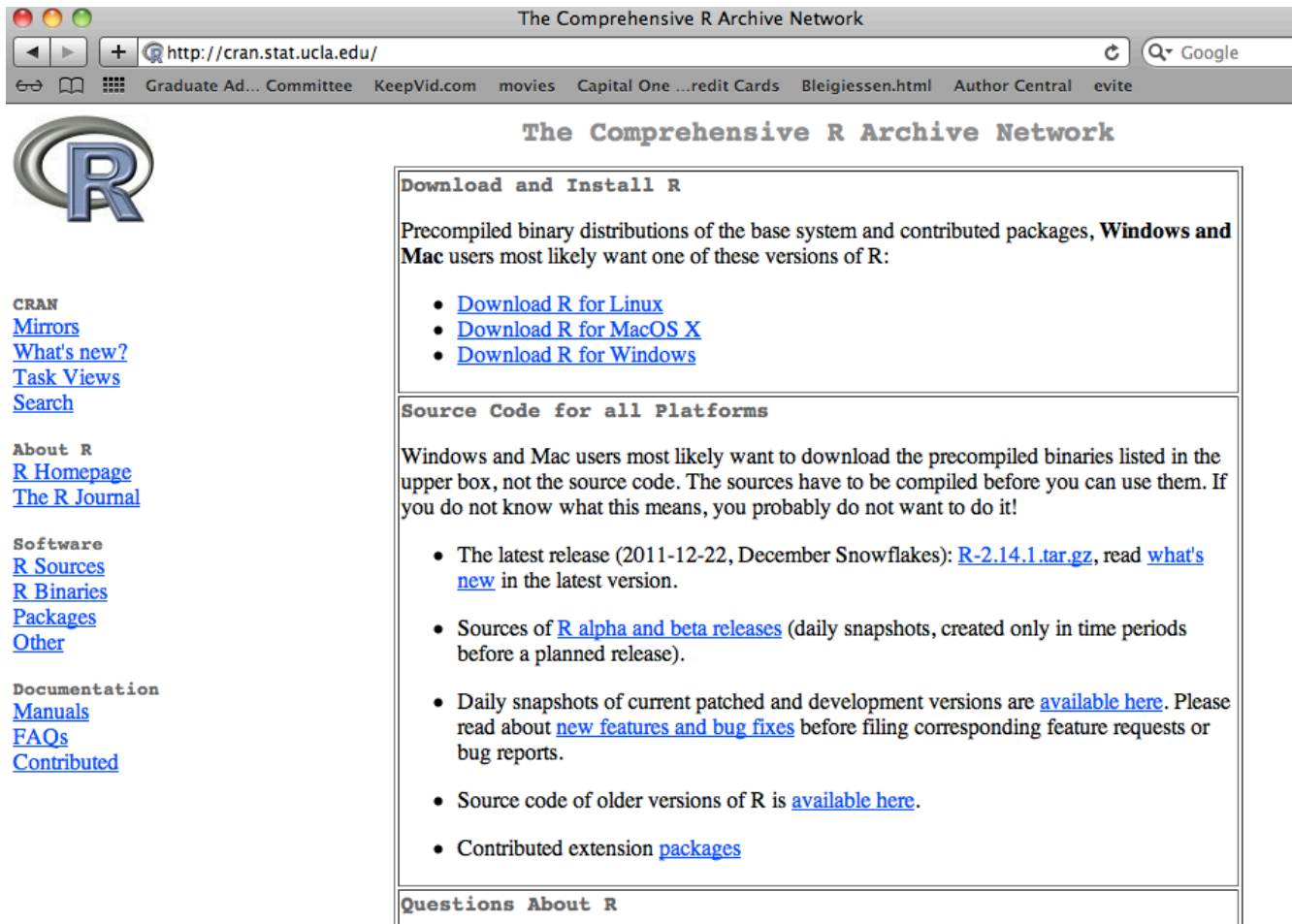
10. R. To download and install *R*, go directly to cran.stat.ucla.edu, or as it says in the book at the bottom of p157, you can start at www.r-project.org, in which case you click on “download *R*”, scroll down to UCLA, and click on cran.stat.ucla.edu. From there, click on “download *R* for ...”, and then get the latest version.



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The Comprehensive R Archive Network

Download and Install R

Precompiled binary distributions of the base system and contributed packages, **Windows and Mac** users most likely want one of these versions of R:

- [Download R for Linux](#)
- [Download R for MacOS X](#)
- [Download R for Windows](#)

Source Code for all Platforms

Windows and Mac users most likely want to download the precompiled binaries listed in the upper box, not the source code. The sources have to be compiled before you can use them. If you do not know what this means, you probably do not want to do it!

- The latest release (2011-12-22, December Snowflakes): [R-2.14.1.tar.gz](#), read [what's new](#) in the latest version.
- Sources of [R alpha and beta releases](#) (daily snapshots, created only in time periods before a planned release).
- Daily snapshots of current patched and development versions are [available here](#). Please read about [new features and bug fixes](#) before filing corresponding feature requests or bug reports.
- Source code of older versions of R is [available here](#).
- Contributed extension [packages](#)

Questions About R

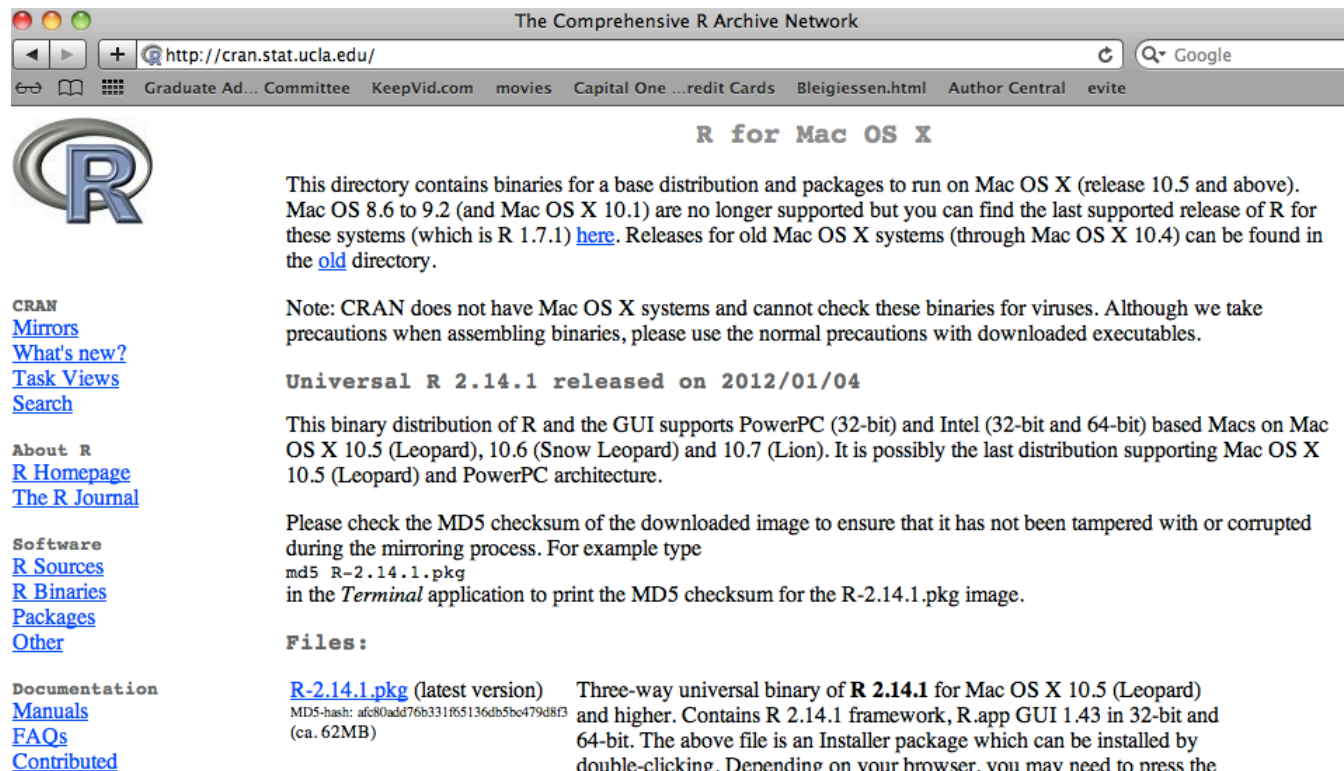
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The screenshot shows a web browser window titled "The Comprehensive R Archive Network". The address bar shows <http://cran.stat.ucla.edu/>. The browser's bookmark bar includes links like "Graduate Ad...", "Committee", "KeepVid.com", "movies", "Capital One ...redit Cards", "Bleigiessen.html", "Author Central", and "evite". The main content area is titled "R for Mac OS X" and features the R logo. The text explains that the directory contains binaries for Mac OS X (release 10.5 and above) and provides a note about CRAN's limitations. It announces the release of "Universal R 2.14.1" on 2012/01/04. A section for "Files:" lists the "R-2.14.1.pkg" (latest version) with its MD5 hash and size. A sidebar on the left contains links for "CRAN" (Mirrors, What's new?, Task Views, Search), "About R" (R Homepage, The R Journal), "Software" (R Sources, R Binaries, Packages, Other), and "Documentation" (Manuals, FAQs, Contributed).

R for Mac OS X

This directory contains binaries for a base distribution and packages to run on Mac OS X (release 10.5 and above). Mac OS 8.6 to 9.2 (and Mac OS X 10.1) are no longer supported but you can find the last supported release of R for these systems (which is R 1.7.1) [here](#). Releases for old Mac OS X systems (through Mac OS X 10.4) can be found in the [old](#) directory.

Note: CRAN does not have Mac OS X systems and cannot check these binaries for viruses. Although we take precautions when assembling binaries, please use the normal precautions with downloaded executables.

Universal R 2.14.1 released on 2012/01/04

This binary distribution of R and the GUI supports PowerPC (32-bit) and Intel (32-bit and 64-bit) based Macs on Mac OS X 10.5 (Leopard), 10.6 (Snow Leopard) and 10.7 (Lion). It is possibly the last distribution supporting Mac OS X 10.5 (Leopard) and PowerPC architecture.

Please check the MD5 checksum of the downloaded image to ensure that it has not been tampered with or corrupted during the mirroring process. For example type

```
md5 R-2.14.1.pkg
```

in the *Terminal* application to print the MD5 checksum for the R-2.14.1.pkg image.

Files:

[R-2.14.1.pkg](#) (latest version) Three-way universal binary of **R 2.14.1** for Mac OS X 10.5 (Leopard) and higher. Contains R 2.14.1 framework, R.app GUI 1.43 in 32-bit and 64-bit. The above file is an Installer package which can be installed by double-clicking. Depending on your browser, you may need to press the control key and click on this link to download the file.

MD5-hash: `afc80add76b331f65136db5bc479d8f3`
(ca. 62MB)

This package **only** contains the R framework, 32-bit GUI (R.app) and 64-bit GUI (R64.app). For Tcl/Tk libraries (needed if you want to use tcltk) and GNU Fortran (needed if you want to compile packages from sources that contain FORTRAN code) please see [the tools](#)

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11. Deal til first ace appears. Let X = the *next* card after the ace.

$P(X = A\spadesuit)$? $P(X = 2\clubsuit)$?