Stat 100a: Introduction to Probability.

Outline for the day:

1. Luck and skill in poker.
2. Ivey and Booth, bluffing and expected value.
3. Facts about expected value.
4. Gradegrubbing procedure,
5. Notes on midterm 1.
6. Assign teams.
7. Review list.
8. Practice problems.
9. Bernoulli random variables.
10. Binomial random variables.
11. Geometric random variables.
12. R projects.
13. Moment generating functions.

♠ ♣ ♥ ♦

1. Luck and skill in poker. pp 71-79.

Let equity = your expected portion of the pot after the hand, assuming no future betting.
= your expected number of chips after the hand - chips you had before the hand
= pot * p, where p = your probability of winning if nobody folds.
I define luck as the equity gained during the dealing of the cards.
Skill = equity gained during the betting rounds.

Are there any problems with these definitions?

2. Bluffing. Ivey and Booth.

3. Facts about expected value.

For any random variable X and any constants a and b,

$E(aX + b) = aE(X) + b$.

Also, $E(X+Y) = E(X) + E(Y)$,

unless $E(X) = \infty$ and $E(Y) = -\infty$, in which case $E(X)+E(Y)$ is undefined.

Thus $\sigma^2 = E[(X-\mu)^2]$

$= E[(X^2 - 2\mu X + \mu^2)]$

$= E(X^2) - 2\mu E(X) + \mu^2$

$= E(X^2) - 2\mu^2 + \mu^2$

$= E(X^2) - \mu^2$.

4. Gradegrubbing.

Grade complaint policy, both for exams and hws.

step 1. Show your exam to your Section TA.

step 2. If your TA agrees you deserve extra points, then the TA will bring it to me,

I will consider it, give it back to the TA, and then the TA will give it to you.

5. Midterm 1.

One hour, the first hour of class on Mon Aug 15.

Around 10 multiple choice questions all worth the same amount.

No books for this exam. The others will be open book and open note though.

You can use 1 page, doublesided, of 8.5 x 11 paper as notes for this exam.

Bring a calculator and a pen or pencil.

None of the above, and answers rounded to 2 decimal places.

6. Assign teams of 3.

7. Review List

1) Basic principles of counting.
2) Axioms of probability, and addition rule.
3) Permutations & combinations.
4) Conditional probability.
5) Independence.
6) Multiplication rules. $P(AB) = P(A)\ P(B|A)$ [= $P(A)P(B)$ if ind.]
7) Odds ratios.
8) Random variables (RVs).
9) Discrete RVs and probability mass function (pmf).
10) Expected value.
11) Pot odds calculations.
12) Luck, skill, and deal-making.
13) Variance and SD.
14) $E(aX+b)$ and $E(X+Y)$.
15) Bayes's rule.
16) Markov and Chebyshev inequalities.

We have basically done all of ch. 1-4, except 4.7.

8. Example problems.

What is the probability that you will be dealt a king and another card of the same suit as the king?

4 * 12 / C(52,2) = 3.62%.

P(flop an ace high flush)? [where the ace might be on the board]

-- 4 suits

-- one of the cards must be an ace. choose(12,4) possibilities for the others.

So P(flop ace high flush) = 4 * choose(12,4) / choose(52,5)

= 0.0762%, or 1 in **1313**.

P(flop a straight flush)?

-- 4 suits

-- 10 different straight-flushes in each suit. (5 high, 6 high, …, Ace high)

So P(flop straight flush) = 4 * 10 / choose(52,5)

= 0.00154%, or 1 in **64974**.

P(flop two pairs).

If you're sure to be all-in next hand, what is P(you will flop two pairs)?

This is a tricky one. Don't double-count (4♠ 4♦ 9♠ 9♦ Q♦) and (9♠ 9♦ 4♠ 4♦ Q♦)!

There are choose(13,2) possibilities for the NUMBERS of the two pairs.

For each such choice (such as 4 & 9),

there are choose(4,2) choices for the suits of the lower pair,

and the same for the suits of the higher pair.

So, choose(13,2) * choose(4,2) * choose(4,2) different possibilities for the two pairs.

For each such choice, there are 44 [52 - 8 = 44] different possibilities for your fifth card, so that it's not a full house but simply two pairs. So,

P(flop two pairs) = choose(13,2) * choose(4,2) * choose(4,2) * 44 / choose(52,5)

~ 4.75%, or 1 in **21**.

9. Bernoulli Random Variables, ch. 5.1.

If X = 1 with probability p, and X = 0 otherwise, then X = *Bernoulli (p)*.

Probability mass function (pmf):

$P(X = 1) = p$

$P(X = 0) = q$, where p+q = 100%.

If X is Bernoulli (p), then μ = E(X) = p, and $\sigma = \sqrt{(pq)}$.

For example, suppose X = 1 if you have a pocket pair next hand; X = 0 if not.

p = 5.88%. So, q = 94.12%.

[Two ways to figure out p:

(a) Out of choose(52,2) combinations for your two cards, 13 * choose(4,2) are pairs.

13 * choose(4,2) / choose(52,2) = 5.88%.

(b) Imagine *ordering* your 2 cards. No matter what your 1st card is, there are 51 equally likely choices for your 2nd card, and 3 of them give you a pocket pair. 3/51 = 5.88%.]

μ = E(X) = .0588. SD = $\sigma = \sqrt{(.0588 * 0.9412)} = 0.235$.

10. Binomial Random Variables, ch. 5.2.

Suppose now X = # of times something with prob. p occurs, out of n independent trials

Then X = *Binomial (n.p)*.

e.g. the number of pocket pairs, out of 10 hands.

Now X could = 0, 1, 2, 3, …, or n.

pmf: $P(X = k) = \text{choose}(n, k) * p^k q^{n-k}$.

e.g. say n=10, k=3: $P(X = 3) = \text{choose}(10,3) * p^3 q^7$.

Why? Could have 1 1 1 0 0 0 0 0 0 0, or 1 0 1 1 0 0 0 0 0 0, etc.

choose(10, 3) choices of places to put the 1's, and for each the prob. is $p^3 q^7$.

Key idea: $X = Y_1 + Y_2 + \ldots + Y_n$, where the Y_i are independent and *Bernoulli* (p).

If X is Bernoulli (p), then $\mu = p$, and $\sigma = \sqrt{(pq)}$.

If X is Binomial (n,p), then $\mu = np$, and $\sigma = \sqrt{(npq)}$.

Binomial Random Variables, continued.

Suppose X = the number of pocket pairs you get in the next 100 hands.

What's P(X = 4)? What's E(X)? σ? X = Binomial (100, 5.88%).

$P(X = k) = \text{choose}(n, k) * p^k q^{n-k}$.

So, P(X = 4) = choose(100, 4) * $0.0588^4 * 0.9412^{96}$ = 13.9%, or 1 in **7.2.**

E(X) = np = 100 * 0.0588 = **5.88**. σ = √(100 * 0.0588 * 0.9412) = **2.35**.

So, out of 100 hands, you'd *typically* get about 5.88 pocket pairs, +/- around 2.35.

11. Geometric Random Variables, ch 5.3.

Suppose now X = # of trials until the *first* occurrence.

(Again, each trial is independent, and each time the probability of an occurrence is p.)

Then X = *Geometric (p)*.

e.g. the number of hands til you get your next pocket pair.

[*Including the hand where you get the pocket pair. If you get it right away, then X = 1.*]

Now X could be 1, 2, 3, …, up to ∞.

pmf: $P(X = k) = p^1 q^{k-1}$.

e.g. say k=5: $P(X = 5) = p^1 q^4$. Why? Must be 0 0 0 0 1. Prob. = q * q * q * q * p.

If X is Geometric (p), then μ = 1/p, and σ = ($\sqrt{q}$) ÷ p.

e.g. Suppose X = the number of hands til your next pocket pair. P(X = 12)? E(X)? σ?

X = Geometric (5.88%).

$P(X = 12) = p^1 q^{11}$ = 0.0588 * 0.9412 ^ 11 = **3.02%.**

E(X) = 1/p = **17.0**. σ = sqrt(0.9412) / 0.0588 = **16.5**.

So, you'd typically *expect* it to take 17 hands til your next pair, +/- around 16.5 hands.

12. R project. The project is problem 8.2, p165.

You need to write code to go all in or fold. In R, try:

```
install.packages(holdem)
library(holdem)
library(help="holdem")
```

timemachine, tommy, ursula, vera, william, and xena are examples.

crds1[1,1] is your higher card (2-14).

crds1[2,1] is your lower card (2-14).

crds1[1,2] and crds1[2,2] are suits of your higher card & lower card.

```
help(tommy)
tommy
```

```
function (numattable1, crds1, board1, round1, currentbet, mychips1,
  pot1, roundbets, blinds1, chips1, ind1, dealer1, tablesleft)
{ a1 = 0
  if (crds1[1, 1] == crds1[2, 1])
     a1 = mychips1
  a1
}
```

help(vera)

All in with a pair, any suited cards, or if the smaller card is at least 9.

```
function (numattable1, crds1, board1, round1, currentbet, mychips1,
    pot1, roundbets, blinds1, chips1, ind1, dealer1, tablesleft)
{a1 = 0
 if ((crds1[1, 1] == crds1[2, 1]) || (crds1[1, 2] == crds1[2,2]) ||
      (crds1[2, 1] > 8.5)) a1 = mychips1
    a1
}
```

You need to email me your function, to frederic@stat.ucla.edu. It should be written (or cut and pasted) simply into the body of the email. If you write it in Word, save as text first, and then paste it into the email. Just submit one email per team.

For instance, if your letter is “b”, you might do:

```
bruin = function (numattable1, crds1, board1, round1, currentbet,
      mychips1, pot1, roundbets, blinds1, chips1, ind1, dealer1,
      tablesleft) {
## all in with any pair higher than 7s, or if lower card is J or higher
a1 = 0
if ((crds1[1, 1] == crds1[2, 1]) && (crds1[1, 1] > 6.5)) a1 = mychips1
if (crds1[2,1] > 10.5) a1 = mychips1
a1
} ## end of bruin
```

13. Moment generating functions, ch. 4.7

Suppose X is a random variable. E(X), E(X^2), E(X^3), etc. are the *moments* of X.

$ø_X(t) = E(e^{tX})$ is called the *moment generating function* of X.

Take derivatives with respect to t of $ø_X(t)$ and evaluate at t=0 to get moments of X.

1st derivative $(d/dt)\, e^{tX} = X\, e^{tX}$, $(d/dt)^2\, e^{tX} = X^2\, e^{tX}$, etc.

$(d/dt)^k\, E(e^{tX}) = E[(d/dt)^k\, e^{tX}] = E[X^k\, e^{tX}]$, (see p.84)

so $ø'_X(0) = E[X^1\, e^{0X}] = E(X)$,

$ø''_X(0) = E[X^2\, e^{0X}] = E(X^2)$, etc.

The moment gen. function $ø_X(t)$ uniquely characterizes the distribution of X.

So to show that X is, say, Poisson, you just need to show that it has the moment generating function of a Poisson random variable.

Also, if X_i are random variables with cdfs F_i, and $ø_{X_i}(t) \to ø(t)$, where $ø_X(t)$ is the moment generating function of X which has cdf F, then $X_i \to X$ in distribution, i.e. $F_i(y) \to F(y)$ for all y where F(y) is continuous, see p85.

Moment generating functions, continued.

$ø_X(t) = E(e^{tX})$ is called the *moment generating function* of X.

Suppose X is Bernoulli (0.4). What is $ø_X(t)$?

$E(e^{tX}) = (0.6)\ (e^{t(0)}) + (0.4)\ (e^{t(1)}) = 0.6 + 0.4\ e^t$.

Suppose X is Bernoulli (0.4) and Y is Bernoulli (0.7) and X and Y are independent.

What is the distribution of XY?

$ø_{XY}(t) = E(e^{tXY}) = P(XY=0)\ (e^{t(0)}) + P(XY=1)(e^{t(1)})$

$= P(X=0 \text{ or } Y=0)\ (1) + P(X=1 \text{ and } Y=1)e^t$

$= [1 - P(X=1)P(Y=1)] + P(X=1)P(Y=1)e^t$

$= [1 - 0.4 \text{ x } 0.7] + 0.4\text{x}0.7e^t$

$= 0.72 + 0.28e^t$, which is the moment generating function of a Bernoulli (0.28) random variable. Therefore XY is Bernoulli (0.28).

What about $Z = \min\{X,Y\}$?

Z = XY in this case, since X and Y are 0 or 1, so the answer is the same.