Stat 100a, Introduction to Probability. Rick Paik Schoenberg

Outline for the day:

1. Midterm 1.
2. Expected number trick.
3. P(flop 2 pairs).
4. Bernoulli random variables.
5. Binomial random variables.
6. Geometric random variables.

1. Midterm 1.

2. Expected number trick.

The board consists of 5 cards. Find the expected number of clubs on the board.

Let $X_1 = 1$ if the 1st card is a club, and 0 otherwise.

Let $X_2 = 1$ if the 2nd card is a club, and 0 otherwise.

etc.

$X = X_1 + X_2 + X_3 + X_4 + X_5$.

So $E(X) = E(X_1) + E(X_2) + E(X_3) + E(X_4) + E(X_5)$

$= [¼ (1) + ¾ (0)] \times 5 = 1.25$.

Even though X_1, X_2, X_3, X_4, and X_5 are not independent, nevertheless

$E(X_1 + X_2 + X_3 + X_4 + X_5) = E(X_1)+E(X_2)+E(X_3)+E(X_4)+E(X_5)$.

P(flop two pairs).

If you're sure to be all-in next hand, what is P(you will flop two pairs)?

This is a tricky one. Don't double-count (4♠ 4♦ 9♠ 9♦ Q♦) and (9♠ 9♦ 4♠ 4♦ Q♦).

There are choose(13,2) possibilities for the NUMBERS of the two pairs.

For each such choice (such as 4 & 9),

there are choose(4,2) choices for the suits of the lower pair,

and the same for the suits of the higher pair.

So, choose(13,2) * choose(4,2) * choose(4,2) different possibilities for the two pairs.

For each such choice, there are 44 [52 - 8 = 44] different possibilities for your fifth card, so that it's not a full house but simply two pairs. So,

P(flop two pairs) = C(13,2) * C(4,2) * C(4,2) * 44 / C(52,5)

~ 4.75%, or 1 in **21**.

P(flop two pairs).

Here is another way to do it. Find the mistake.

P(flop 2 pairs) = P(pocket pair and flop 2 pairs) + P(no pocket pair and flop 2 pairs)

= P(pocket pair) P(flop 2 pairs | pocket pair) + P(no pocket pair) P(flop 2 pairs | no pocket pair)

= P(pocket pair aa) * P(bbc | aa) + P(ab)*P(abc | ab)

= 13*C(4,2)/C(52,2) * 12*C(4,2)*44/C(50,3) + C(13,2)*4*4/C(52,2) * 3*3*44/C(50,3)

= 2.85%.

What is the problem here?

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= 2.85%.

What is the problem here?

P(flop 2 pairs | no pocket pair) ≠ P(ab)*P(abc | ab). If you have ab, it could come acc or bcc on the flop.

13*C(4,2)/C(52,2) * 12*C(4,2)*44/C(50,3) + C(13,2)*4*4/C(52,2) * **(3*3*44 + 6*11*C(4,2))** /C(50,3)

= 4.75%.

Bernoulli Random Variables, ch. 5.1.

If X = 1 with probability p, and X = 0 otherwise, then X = *Bernoulli (p)*.

Probability mass function (pmf):

P(X = 1) = p

P(X = 0) = q, where p+q = 100%.

If X is Bernoulli (p), then μ = E(X) = p, and σ = √(pq).

For example, suppose X = 1 if you have a pocket pair next hand; X = 0 if not.

p = 5.88%. So, q = 94.12%.

[Two ways to figure out p:

(a) Out of choose(52,2) combinations for your two cards, 13 * choose(4,2) are pairs.

13 * choose(4,2) / choose(52,2) = 5.88%.

(b) Imagine *ordering* your 2 cards. No matter what your 1st card is, there are 51 equally likely choices for your 2nd card, and 3 of them give you a pocket pair. 3/51 = 5.88%.]

μ = E(X) = .0588. SD = σ = √(.0588 * 0.9412) = 0.235.

Binomial Random Variables, ch. 5.2.

Suppose now X = # of times something with prob. p occurs, out of n independent trials

Then X = *Binomial (n.p)*.

e.g. the number of pocket pairs, out of 10 hands.

Now X could = 0, 1, 2, 3, …, or n.

pmf: $P(X = k) = \text{choose}(n, k) * p^k q^{n-k}$.

e.g. say n=10, k=3: $P(X = 3) = \text{choose}(10,3) * p^3 q^7$.

Why? Could have 1 1 1 0 0 0 0 0 0 0, or 1 0 1 1 0 0 0 0 0 0, etc.

choose(10, 3) choices of places to put the 1's, and for each the prob. is $p^3 q^7$.

Key idea: $X = Y_1 + Y_2 + \ldots + Y_n$, where the Y_i are independent and *Bernoulli* (p).

If X is Bernoulli (p), then $\mu = p$, and $\sigma = \sqrt{(pq)}$.

If X is Binomial (n,p), then $\mu = np$, and $\sigma = \sqrt{(npq)}$.

Binomial Random Variables, continued.

Suppose X = the number of pocket pairs you get in the next 100 hands.

What's P(X = 4)? What's E(X)? σ? X = Binomial (100, 5.88%).

$P(X = k) = choose(n, k) * p^k q^{n-k}$.

So, $P(X = 4) = choose(100, 4) * 0.0588^4 * 0.9412^{96} = 13.9\%$, or 1 in **7.2.**

E(X) = np = 100 * 0.0588 = **5.88**. σ = √(100 * 0.0588 * 0.9412) = **2.35**.

So, out of 100 hands, you'd *typically* get about 5.88 pocket pairs, +/- around 2.35.

6. Geometric random variables, ch 5.3.

Suppose now X = # of trials until the *first* occurrence.

(Again, each trial is independent, and each time the probability of an occurrence is p.)

Then X = *Geometric (p)*.

e.g. the number of hands til you get your next pocket pair.

[*Including the hand where you get the pocket pair. If you get it right away, then X = 1.*]

Now X could be 1, 2, 3, …, up to ∞.

pmf: $P(X = k) = p^1 q^{k-1}$.

e.g. say k=5: $P(X = 5) = p^1 q^4$. Why? Must be 0 0 0 0 1. Prob. = q * q * q * q * p.

If X is Geometric (p), then $\mu = 1/p$, and $\sigma = (\sqrt{q}) \div p$.

e.g. Suppose X = the number of hands til your next pocket pair. P(X = 12)? E(X)? σ?

X = Geometric (5.88%).

$P(X = 12) = p^1 q^{11}$ = 0.0588 * 0.9412 ^ 11 = **3.02%.**

E(X) = 1/p = **17.0**. σ = sqrt(0.9412) / 0.0588 = **16.5**.

So, you'd typically *expect* it to take 17 hands til your next pair, +/- around 16.5 hands.