# Stat 100a, Introduction to Probability. Rick Paik Schoenberg

### Outline for the day:

- 1. Addiction.
- 2. Syllabus, etc.
- 3. Wasicka/Gold/Binger example.
- 4. Meaning of probability.
- 5. Axioms of probability.
- 6. Hw1 terms.
- 7. Basic principle of counting.
- 8. Permutations and combinations.
- 9. *R*. ♠ ♣ ♥ ♦

#### 2. Syllabus, etc.

For this week:

(i) Learn the rules of Texas Hold'em.

(see <a href="http://www.fulltiltpoker.net">http://www.fulltiltpoker.net</a> for example)

- (ii) Read addiction handout, addiction.pdf, on the course website, http://www.stat.ucla.edu/~frederic/100A/sum21.
- (iii) Download R and try it out.

( http://cran.stat.ucla.edu )

(iv) Read ch. 1-3 of the textbook.

Note that the CCLE website for this course is not maintained. The course website is http://www.stat.ucla.edu/~frederic/100A/sum21.

I do not give hw hints by email. Conceptual questions only.

Only one question is off limits, and it is "What did we do in class?"

If you have taken Stat 100A or Stat 35 before, please see me after class.

### Wasicka/Gold/Binger Example

Blinds: \$200,000-\$400,000 with \$50,000 antes.

#### **Chip Counts:**

Jamie Gold \$60,000,000 Paul Wasicka \$18,000,000 Michael Binger \$11,000,000

**Payouts:** 3rd place: \$4,123,310. 2nd place: \$6,102,499. 1st place: \$12,000,000.

Day 7, Hand 229. Gold: 4s 3c. Binger: Ah 10h. Wasicka: 8s 7s.

An example of the type of questions we will be addressing in this class is on the next slide. Don't worry about all the details yet.

### Wasicka/Gold/Binger Example, Continued

Gold: 4♠ 3♣. Binger: A♥ 10♥. Wasicka: 8♠ 7♠

Flop: 10 \$\diamonds 6 \diamonds 5 \diamonds. (Turn: 7 \diamonds. River: Q \diamonds)

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Wasicka folded?!?

\_\_\_\_\_

He had 8♠ 7♠ and the flop was 10♣ 6♠ 5♠ Worst case scenario?

# Wasicka/Gold/Binger Example, Continued

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Gold: 4♠ 3♣. Binger: A♥ 10♥. Wasicka: 8♠ 7♠
Flop: 10 $\diamonds 6 \diamonds 5 \diamonds. (Turn: 7 \diamonds. River: Q \diamonds)
          Wasicka folded?!?
He had 8 \spadesuit 7 \spadesuit and the flop was 10 \clubsuit 6 \spadesuit 5 \spadesuit.
Suppose he were up against 9 \spadesuit 4 \spadesuit and 9 \heartsuit 9 \spadesuit. How could
Wasicka win?
88
        (3: 8 \clubsuit 8 \blacklozenge, 8 \clubsuit 8 \blacktriangledown, 8 \blacklozenge 8 \blacktriangledown)
           (3)
77
            (3)
44
 [Let "X" = non-49, "Y" = A2378JQK, and "n" = non-\spadesuit.]
4n Xn (3 x 32)
9♣ 4n (3)
9 Yn (24). Total: 132 out of 903 = 14.62\%.
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# 4. Meaning of Probability.

Notation: "P(A) = 60%". A is an *event*. Not "P(60%)".

Definition of probability:

<u>Frequentist</u>: If repeated independently under the same conditions millions and millions of times, A would happen 60% of the times.

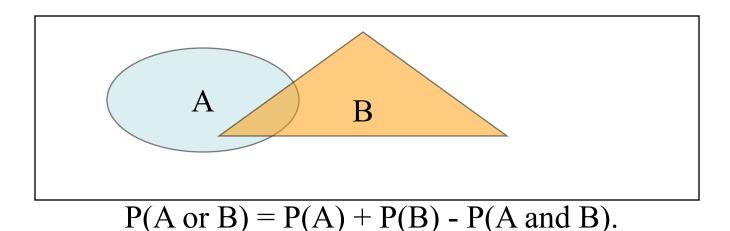
Bayesian: Subjective feeling about how likely something seems.

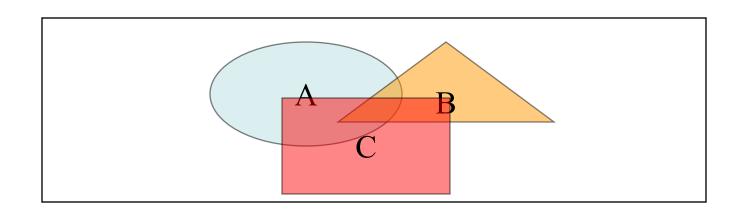
P(A or B) means P(A or B or both) Mutually exclusive: P(A and B) = 0. Independent: P(A given B) [written "P(A|B)"] = P(A).  $P(A^c)$  means P(not A).

### 5. Axioms (initial assumptions/rules) of probability:

- 1)  $P(A) \ge 0$ .
- 2)  $P(A) + P(A^c) = 1$ .
- 3) If  $A_1, A_2, A_3, \dots$  are mutually exclusive, then  $P(A_1 \text{ or } A_2 \text{ or } A_3 \text{ or } \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$

(#3 is sometimes called the *addition rule*)
Probability <=> Area. Measure theory, Venn diagrams





Fact: P(A or B) = P(A) + P(B) - P(A and B).

P(A or B or C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(AC

P(BC)+P(ABC).

Fact: If  $A_1, A_2, ..., A_n$  are equally likely & mutually exclusive, and if  $P(A_1 \text{ or } A_2 \text{ or } ... \text{ or } A_n) = 1$ , then  $P(A_k) = 1/n$ .

[So, you can *count*:  $P(A_1 \text{ or } A_2 \text{ or } \dots \text{ or } A_k) = k/n.$ ]

Ex. You have 76, and the board is KQ54. P(straight)?

[52-2-4=46.] P(straight) = P(8 on river OR 3 on river)

= P(8 on river) + P(3 on river) = 4/46 + 4/46.

#### 6. Hw1 terms.

**Assume you never fold.** I say this so one can't object "But I would never play  $7 \diamondsuit 5 \diamondsuit$ ." **flop a straight flush.** For example, you have  $7 \diamondsuit 5 \diamondsuit$  and the flop is  $4 \diamondsuit 8 \diamondsuit 6 \diamondsuit$ . **flopping 2 pairs.** For example, you have  $7 \diamondsuit 7 \heartsuit$  and the flop is  $3 \heartsuit 3 \diamondsuit 1 \heartsuit$ .

Or, you have  $7 \spadesuit 3 \heartsuit$  and the flop is  $7 \heartsuit 3 \spadesuit J \heartsuit$ .

**pocket pair.** When your two cards form a pair by themselves, like  $7 \blacklozenge 7 \blacktriangledown$ . **face cards.** K, Q, or J.

the nuts. Given the board, the best possible hand you could currently have in terms of the ranking order of poker hands, not in terms of probability of winning or improving in the future. For example, if the board is  $7 \lor 3 \spadesuit J \lor 8 \diamondsuit$ , then if you have  $10 \lor 9 \diamondsuit$ , then you have the nuts. If you have  $10 \lor 9 \lor$ , it would be slightly better in terms of probability of winning, but either way you have the nuts.

the unbreakable nuts. When you are guaranteed to win no matter what your opponent might have and no matter what board cards might come. In the above example where you have  $10 \checkmark 9 \checkmark$  and the board is  $7 \checkmark 3 \spadesuit J \checkmark 8 \diamondsuit$ , you do not have the unbreakable nuts because you could lose for instance if the river is  $9 \spadesuit$  and your opponent has  $Q \spadesuit 10 \diamondsuit$ . However, if the board is  $8 \checkmark 7 \checkmark 6 \checkmark$  and you have  $10 \checkmark 9 \checkmark$ , then you have the unbreakable nuts.

**in terms of.** 3.2b is not easy. Assuming A and B are independent, you have to express the odds against (AB) using only  $O_{A'}$  and  $O_{B'}$ . You can't use any other variables. In part a you expressed it in terms of P(A) and P(B), so just figure out how to convert P(A) into an expression of  $O_{A'}$ .

#### 7. Basic Principle of Counting.

If there are  $a_1$  distinct possible outcomes on trial #1, and for each of them, there are  $a_2$  distinct possible outcomes on trial #2, then there are  $a_1 \times a_2$  distinct possible *ordered* outcomes on both.

e.g. you get 1 card, opp. gets 1 card. # of distinct possibilities?  $52 \times 51$ . [ordered:  $(A \clubsuit, K \heartsuit) \neq (K \heartsuit, A \clubsuit)$ .]

In general, with j experiments, each with  $a_i$  possibilities, the # of distinct outcomes where order matters is  $a_1 \times a_2 \times ... \times a_i$ .

#### 8. Permutations and Combinations.

e.g. you get 1 card, opp. gets 1 card.

# of distinct possibilities?

52 x 51. [ordered: 
$$(A \clubsuit, K \heartsuit) \neq (K \heartsuit, A \clubsuit)$$
.]

Each such outcome, where order matters, is called a *permutation*.

Number of permutations of the deck?  $52 \times 51 \times ... \times 1 = 52!$ 

$$\sim 8.1 \times 10^{67}$$

A <u>combination</u> is a collection of outcomes, where order <u>doesn't</u> matter. e.g. in hold'em, how many <u>distinct</u> 2-card hands are possible?

52 x 51 if order matters, but then you'd be double-counting each

[ since now  $(A \clubsuit, K \checkmark) = (K \checkmark, A \clubsuit)$ .]

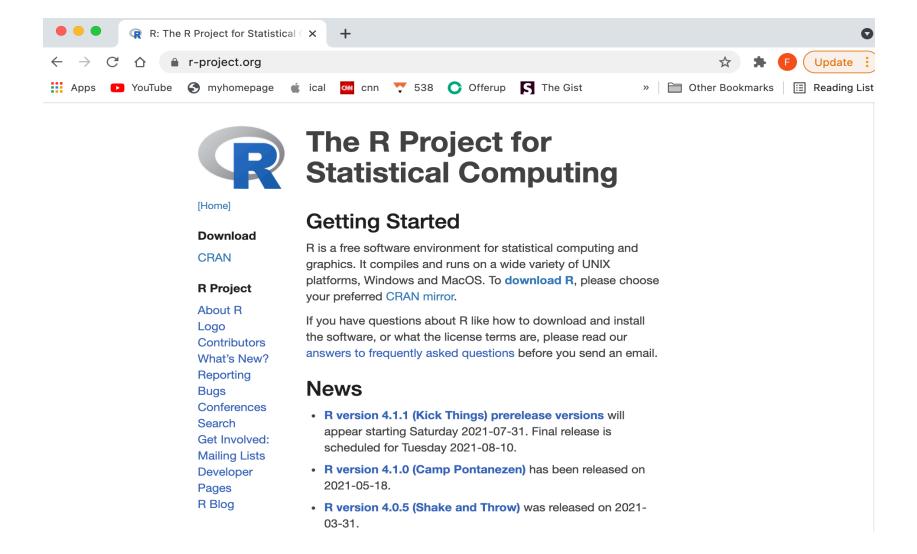
So, the number of *distinct* hands where *order doesn't matter* is  $52 \times 51 / 2$ .

In general, with n distinct objects, the # of ways to choose k *different* ones, *where order doesn't matter*, is

"n choose k" = choose(n,k) =  $\underline{n!}$ .

k! (n-k)!

**9. R.** To download and install R, go directly to cran.case.edu, or as it says in the book at the bottom of p157, you can start at <a href="www.r-project.org">www.r-project.org</a>, in which case you click on "download R", scroll down to USA, and click on cran.case.edu. From there, click on "download R for ...", and then get the latest version.



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#### **CRAN Mirrors**

The Comprehensive R Archive Network is available at the following URLs, please choose a location close to you. Some statistics on the status of the mirrors can be found here: <u>main page</u>, <u>windows release</u>, <u>windows old release</u>.

If you want to host a new mirror at your institution, please have a look at the <u>CRAN Mirror HOWTO</u>.

0-Cloud

https://cloud.r-project.org/
Automatic redirection to servers worldwide, currently sponsored by

Rstudio

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Belnet, the Belgian research and education network

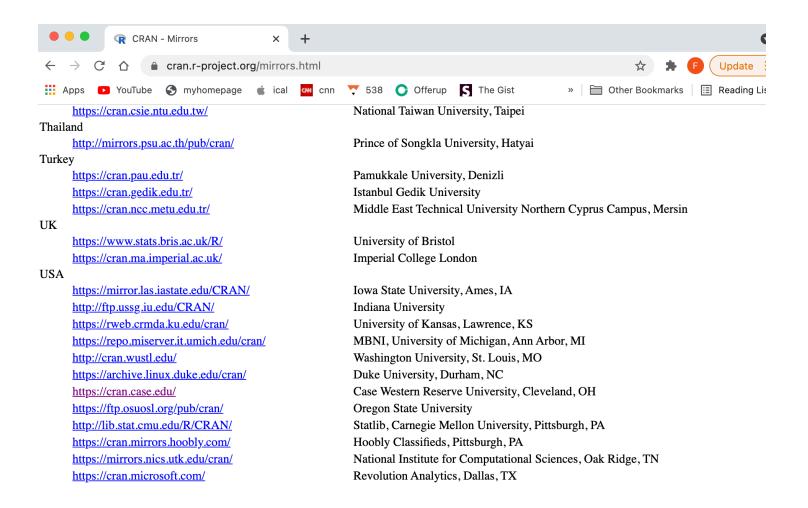
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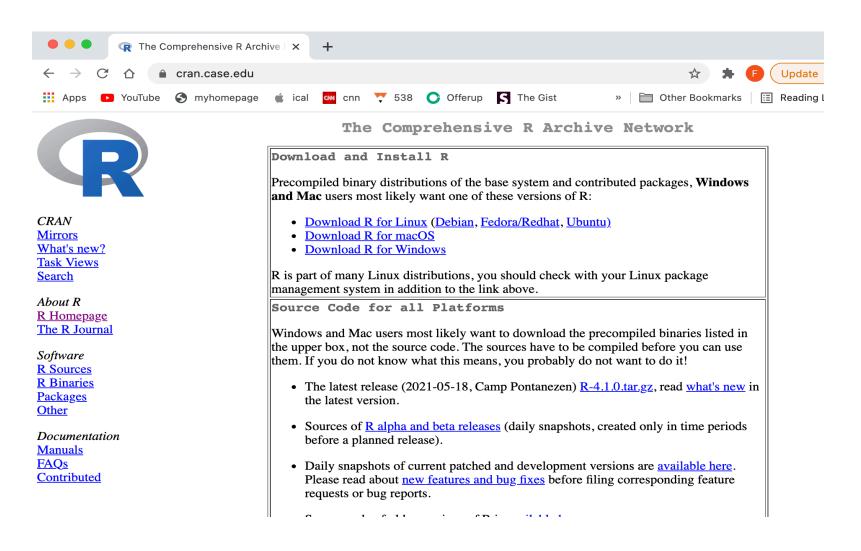
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Deal til first ace appears. Let  $X = the \ next$  card after the ace.

$$P(X = A \spadesuit)? P(X = 2 \clubsuit)?$$

We will discuss this next time.