Stat 100a, Introduction to Probability. Rick Paik Schoenberg

Outline for the day:

1. Midterm 1.
2. Expected number trick.
3. P(flop 2 pairs).
4. Bernoulli random variables.
5. Binomial random variables.
6. Geometric random variables.
7. Teams, emails, and bruin.

1. Midterm 1.

4. Let X = the number of clubs in your hand, so X must be 0, 1, or 2. What is $E(X^2)$?

a. 0.500. b. 0.618. c. 0.712. d. 0.891. e. None of the above.

X_i = # of clubs on card i. $E(X) = E(X_1) + E(X_2) = ¼ + ¼$.

$E(X^2) = 0*P(0 \text{ clubs}) + 1*13*39/C(52,2) + 4*C(13,2)/C(52,2)$.

11. Suppose your opponent would raise all in before the flop with 70% probability if she had AA, and with 40% probability if she had AK, and with any other hand she would never do it. Given only that she has raised all in before the flop, what is the probability that she has AA?

a. 33.0%. b. 39.6%. c. 50.0%. d. 70.0%. e. 81.9%. f. None of the above.

$.7*C(4,2)/C(52,2) \div (.7*C(4,2)/C(52,2) + .4*4*4/C(52,2))$

1. Midterm 1.

12. Suppose you are playing against only me. You have 2♣ 2♠, I have K♣ K♥, and the board is 3♥ 5♦ 8♠ 9♦. The pot is $97 when the 9♦ is revealed, and then I go all in, betting my last $3. You have at least $5 left. If you know what cards I have and want to maximize your expected number of chips, should you call?

c. Yes, because 2/44 > 3/103.

13. Continuing the above hand, suppose you do call. The pot is $103 when the river is revealed to be the 2♦. How much expected profit did you gain due to luck on the river?

a. $82.3. b. $87.8. c. $97.0. d. $98.3. e. None of the above.

$103 x 100% - $103 x 2/44.

14. Suppose X = 0 with probability 1/4, X = 1 with probability 1/2, and X = 2 with probability 1/4. What is the variance of X?

a. 0.500. b. 1.00. c. 1.55. d. 1.98. e. None of the above.

$E(X^2) = 0(1/4) + 1(1/2) + 4(1/4) = 1.5$. $E(X) = 0(1/4) + 1(1/2) + 2(1/4) = 1$.

So $E(X^2) – (E(X))^2 = 1.5 – 1^2$.

P(4 of a kind on the turn)? 13*C(48,2)/C(52,6)

2. Expected number trick.

The board consists of 5 cards. Find the expected number of clubs on the board.

Let X_1 = 1 if the 1st card is a club, and 0 otherwise.

Let X_2 = 1 if the 2nd card is a club, and 0 otherwise.

etc.

$X = X_1 + X_2 + X_3 + X_4 + X_5$.

So $E(X) = E(X_1) + E(X_2) + E(X_3) + E(X_4) + E(X_5)$

$= [¼ (1) + ¾ (0)] \times 5 = 1.25$.

Even though X_1, X_2, X_3, X_4, and X_5 are not independent, nevertheless $E(X_1 + X_2 + X_3 + X_4 + X_5) = E(X_1)+E(X_2)+E(X_3)+E(X_4)+E(X_5)$.

P(at least one club on board) = 1 – P(0 clubs) = 1 - C(39,5)/C(52,5)

P(flop two pairs).

If you're sure to be all-in next hand, what is P(you will flop two pairs)?

This is a tricky one. Don't double-count (4♠ 4♦ 9♠ 9♦ Q♦) and (9♠ 9♦ 4♠ 4♦ Q♦).

There are choose(13,2) possibilities for the NUMBERS of the two pairs.

For each such choice (such as 4 & 9),

there are choose(4,2) choices for the suits of the lower pair,

and the same for the suits of the higher pair.

So, choose(13,2) * choose(4,2) * choose(4,2) different possibilities for the two pairs.

For each such choice, there are 44 [52 - 8 = 44] different possibilities for your fifth card, so that it's not a full house but simply two pairs. So,

P(flop two pairs) = C(13,2) * C(4,2) * C(4,2) * 44 / C(52,5)

~ 4.75%, or 1 in **21**.

P(flop two pairs).

Here is another way to do it. Find the mistake.

P(flop 2 pairs) = P(pocket pair and flop 2 pairs) + P(no pocket pair and flop 2 pairs)

= P(pocket pair) P(flop 2 pairs | pocket pair) + P(no pocket pair) P(flop 2 pairs | no pocket pair)

= P(pocket pair aa) * P(bbc | aa) + P(ab)*P(abc | ab)

= 13*C(4,2)/C(52,2) * 12*C(4,2)*44/C(50,3) + C(13,2)*4*4/C(52,2) * 3*3*44/C(50,3)

= 2.85%.

What is the problem here?

P(flop two pairs).

Here is another way to do it. Find the mistake.

P(flop 2 pairs) = P(pocket pair and flop 2 pairs) + P(no pocket pair and flop 2 pairs)

= P(pocket pair) P(flop 2 pairs | pocket pair) + P(no pocket pair) P(flop 2 pairs | no pocket pair)

= P(pocket pair aa) * P(bbc | aa) + P(ab)*P(abc | ab)

= 13*C(4,2)/C(52,2) * 12*C(4,2)*44/C(50,3) + C(13,2)*4*4/C(52,2) * **3*3*44**/C(50,3)

= 2.85%.

What is the problem here?

P(flop 2 pairs | no pocket pair) ≠ P(ab)*P(abc | ab). If you have ab, it could come acc or bcc on the flop.

13*C(4,2)/C(52,2) * 12*C(4,2)*44/C(50,3) + C(13,2)*4*4/C(52,2) * **(3*3*44 + 6*11*C(4,2))** /C(50,3)

= 4.75%.

Bernoulli Random Variables, ch. 5.1.

If X = 1 with probability p, and X = 0 otherwise, then X = *Bernoulli (p)*.

Probability mass function (pmf):

$P(X = 1) = p$

$P(X = 0) = q$, where $p+q = 100\%$.

If X is Bernoulli (p), then $\mu = E(X) = p$, and $\sigma = \sqrt{(pq)}$.

For example, suppose X = 1 if you have a pocket pair next hand; X = 0 if not.

$p = 5.88\%$. So, $q = 94.12\%$.

[Two ways to figure out p:

(a) Out of choose(52,2) combinations for your two cards, 13 * choose(4,2) are pairs.

13 * choose(4,2) / choose(52,2) = 5.88%.

(b) Imagine *ordering* your 2 cards. No matter what your 1st card is, there are 51 equally likely choices for your 2nd card, and 3 of them give you a pocket pair. 3/51 = 5.88%.]

$\mu = E(X) = .0588$. $SD = \sigma = \sqrt{(.0588 * 0.9412)} = 0.235$.

Binomial Random Variables, ch. 5.2.

Suppose now X = # of times something with prob. p occurs, out of n independent trials

Then X = *Binomial (n , p)*.

e.g. the number of pocket pairs, out of 10 hands.

Now X could = 0, 1, 2, 3, …, or n.

pmf: $P(X = k) = \text{choose}(n, k) * p^k q^{n-k}$.

e.g. say n=10, k=3: $P(X = 3) = \text{choose}(10,3) * p^3 q^7$.

Why? Could have 1 1 1 0 0 0 0 0 0 0, or 1 0 1 1 0 0 0 0 0 0, etc.

choose(10, 3) choices of places to put the 1's, and for each the prob. is $p^3 q^7$.

Key idea: $X = Y_1 + Y_2 + \ldots + Y_n$, where the Y_i are independent and *Bernoulli* (p).

If X is Bernoulli (p), then $\mu = p$, and $\sigma = \sqrt{(pq)}$.

If X is Binomial (n , p), then $\mu = np$, and $\sigma = \sqrt{(npq)}$.

Binomial Random Variables, continued.

Suppose X = the number of pocket pairs you get in the next 100 hands.

What's P(X = 4)? What's E(X)? σ? X = Binomial (100, 5.88%).

$P(X = k) = \text{choose}(n, k) * p^k q^{n-k}$.

So, P(X = 4) = choose(100, 4) * $0.0588^4 * 0.9412^{96}$ = 13.9%, or 1 in **7.2.**

E(X) = np = 100 * 0.0588 = **5.88**. σ = √(100 * 0.0588 * 0.9412) = **2.35**.

So, out of 100 hands, you'd *typically* get about 5.88 pocket pairs, +/- around 2.35.

6. Geometric random variables, ch 5.3.

Suppose now X = # of trials until the *first* occurrence.

(Again, each trial is independent, and each time the probability of an occurrence is p.)

Then X = *Geometric (p)*.

e.g. the number of hands til you get your next pocket pair.

[*Including the hand where you get the pocket pair. If you get it right away, then X = 1.*]

Now X could be 1, 2, 3, …, up to ∞.

pmf: $P(X = k) = p^1 q^{k-1}$.

e.g. say k=5: $P(X = 5) = p^1 q^4$. Why? Must be 0 0 0 0 1. Prob. = q * q * q * q * p.

If X is Geometric (p), then $\mu = 1/p$, and $\sigma = (\sqrt{q}) \div p$.

e.g. Suppose X = the number of hands til your next pocket pair. P(X = 12)? E(X)? σ?

X = Geometric (5.88%).

$P(X = 12) = p^1 q^{11}$ = 0.0588 * 0.9412 ^ 11 = **3.02%.**

E(X) = 1/p = **17.0**. σ = sqrt(0.9412) / 0.0588 = **16.5**.

So, you'd typically *expect* it to take 17 hands til your next pair, +/- around 16.5 hands.

7. Teams, emails, and bruin.
The project is problem 8.2, page 249.
You need to write code to go all in or fold.
For instance, if your function is named "bruin", you might do:

```
bruin = function (numattable, cards, board, round, currentbet, mychips, pot, roundbets,
        blinds, chips, ind, dealer, tablesleft) {
## all in with any pair higher than 7s, or if lower card is J or higher,
## or if you have less than 3 times the big blind
a = 0
if ((cards[1, 1] == cards[2, 1]) && (cards[1, 1] > 6.5)) a = mychips
if (cards[2,1] > 10.5) a = mychips
if(mychips < 3*blinds) a = mychips
a
} ## end of bruin
```

cards[1,1] is your higher card (2-14).
cards[2,1] is your lower card (2-14).
cards[1,2] and cards[2,2] are suits of your higher card & lower card.

Optional. In R, try

```
install.packages("holdem")
library(holdem)
library(help="holdem")
```

gravity, timemachine, tommy, ursula, vera, william, and xena are examples.

```
help(tommy)
tommy
function (numattable, cards, board, round, currentbet, mychips,
    pot, roundbets, blinds, chips, ind, dealer, tablesleft)
{ a = 0
    if (cards[1, 1] == cards[2, 1])
        a = mychips
    a
}
```

```
help(vera)
# All in with a pair, any suited cards, or if the smaller card is at least
    9.
 function (numattable1, crds1, board1, round1, currentbet, mychips1,
    pot1, roundbets, blinds1, chips1, ind1, dealer1, tablesleft) {
 a1 = 0
 if ((crds1[1, 1] == crds1[2, 1]) || (crds1[1, 2] == crds1[2,2]) ||
      (crds1[2, 1] > 8.5)) a1 = mychips1
   a1
}
```

You need to email me your function, to frederic@stat.ucla.edu, by Sat Sep4, 8pm. It should be written (or cut and pasted) simply into the body of the email. If you write it in Word, save as text first, and then paste it into the email. Just submit one email per team.

Here is bruin again.

```
bruin = function (numattable, cards, board, round, currentbet, mychips, pot, roundbets, blinds, chips, ind, dealer, tablesleft) {
## all in with any pair higher than 7s, or if lower card is J or higher, or if you have less than 3 times the big blind
a = 0
if ((cards[1, 1] == cards[2, 1]) && (cards[1, 1] > 6.5)) a = mychips
if (cards[2,1] > 10.5) a = mychips
if(chips < 3*blinds) a = mychips
a
} ## end of bruin
```

The top 3 finishers get 13,8,5 points respectively. 1000 chips. Blinds start at 10/20 and increase by 50% every 10 hands.
numattable = number of players at your table.
cards[1,1] = number of higher card, cards[2,1] = lower card. cards[1,2] and cards[2,2] are their suits.
currentbet = the maximum bet so far this betting round.
mychips is how many chips you have left.
pot = the number of chips in the pot.
blinds = the amount of the big blind. The small blind will be half as much. Both are rounded to integers.
chips is a vector indicating how many chips everyone at your table has left.
ind is your seat number at the table.
dealer is the seat number of the dealer at your table.
tablesleft is how many tables of up to 10 players each are left in the tournament.
Ignore board, round, and roundbets because you have to be all in or fold before the flop.

```
acj = function(numattable, cards, board, round, currentbet, mychips, pot, roundbets, blinds, chips, ind, dealer,
tablesleft){
	# all in with pocket pair of Queens, Kings, or Aces, or AK of any suits
	# all in with AQ, AJ, A10, KQ if same suit and no one else is all in yet
	# all in with any pocket pair if only 1-2 players left to play and nobody is all in yet
	# all in if chip count less than 3 times the big blind with any cards
	a = 0
	if((cards[1,1] == cards[2,1]) && (cards[1,1] >= 12)) a = mychips
	if((cards[1,1] == 14) && (cards[2,1] == 13)) a = mychips
	if(currentbet <= blinds){
		if((cards[1,1] == 14) && (cards[2,1] >= 10) && (cards[1,2] == cards[2,2])) a = mychips
		if((cards[1,1] == 13) && (cards[2,1] == 12) && (cards[1,2] == cards[2,2]))a = mychips
		}
	big.blind = dealer + 2
	if(big.blind > numattable) big.blind <- big.blind - numattable
	players.left = big.blind - ind
	if(players.left < 0) players.left = players.left + numattable
			if(currentbet <= blinds){
			if((players.left <= 2) && (cards[1,1] == cards[2,1])) a = mychips
		}
		if(mychips < 3 * blinds) a = mychips
	a
}
```

I will sort you into teams randomly now using breakout rooms.