1. No class Fri May 28 or Mon May 31.
2. Point processes.
3. Examples.
4. Basic point process concepts.
5. Integration.

2) Definitions of point processes.

a) 0-1 valued stochastic process, or random function, on S.

1 where there's a point, 0 everywhere else.

b) Random measure on S, taking values in Z^+ or ∞.

For any subset B of S, N(B) is the number of points in B.

Sometimes one of the dimensions is time, and sometimes this can be treated like another spatial dimension.

Usually we require in the definition of a point process that it be locally finite, i.e. that $N(B) < \infty$ for any compact set B.

Often there is additional information associated with each point. This additional info is called a mark and the process is then called a marked point process.

3) Examples.
Occurrances of events, such as plane crashes, outbreaks of disease, locations of animals or plants, or environmental disturbances like earthquakes or hurricanes.

In such cases the marks might be the amount of destruction caused by the event.

The idea is that the points occur at random times and locations, and one is interested in the configuration of these locations in space and time.

4) Basic point process concepts.

Simple: a point process is simple if all the points τ_i are distinct, i.e. $\tau_i \neq \tau_j$ if $i \neq j$.

Orderly: a point process is orderly
if $\lim_{B_s \to 0} P\{N(B_s) > 1\} / |B_s| = 0$,
for balls B_s around any location s in space and time.

Orderliness means the points aren't bunching up anywhere in a probabilistic sense; that is, the probabilities of there being lots of points near any particular location and time aren't too high.

Note that a point process is allowed to have a time t (or several such times) where it is *sure* to have a point. But it cannot have infinitely many in any compact set.

5) Integration.

$\int_B dN = N(B) =$ the number of points in B.

Similarly, $\int_B f(x,y)\, dN(x,y) = \sum_i f(\tau_i)\, 1_{\tau_i \text{ in } B}$
= the sum of the function f(t,x,y) only over all times and

locations (t,x,y) in B where there happen to be points.

For instance, suppose N is a point process in time and there are points at times 1, 2, 4, 7, and 12. Suppose B = [0,10]. Then
$\int_B t^2 \, dN(t) = 1^2 + 2^2 + 4^2 + 7^2 = 70$.