Stat 13, Intro. to Statistical Methods for the Life and Health Sciences.

1. t test, continued.
2. Significance level, type I and type II errors.
3. Power.
4. Confidence Intervals for a proportion and the dog sniffing cancer example.
5. CIs for a proportion and the Affordable Care Act example.

Start reading chapter 4.
http://www.stat.ucla.edu/~frederic/13/F17 .
HW2 is due Oct 19 and is problems 2.3.15, 3.3.18, and 4.1.23.

t-distribution

- The shape is very much like a normal distribution, but slightly wider in the tails and is called a t-distribution.
- The t-statistic is the standardized statistic we use with a single quantitative variable and can be found using the formula:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

The $s/\sqrt{n}$ (standard deviation of our sample divided by the square root of the sample size) is called the standard error and is an estimate for the standard deviation of the null distribution.

In the song snippet estimating example $t = \frac{13.71 - 10.0}{6.5/\sqrt{48}} = 3.95.$

p-value = 2*(1-pt(3.95,df=47)) = 0.000261.

Validity Conditions

- The observations must be independent.
- The population must be normally distributed.
- The book says you need the sample size to be at least 20 for the t-test, but this is not right. However, it is often hard to have any idea if the population is normal without having at least 20 observations.

Estimating Time

Formulate Conclusions.

- Based on our small p-value, we can conclude that people don't accurately estimate the length of a 10-second song snippet and in fact they significantly overestimate it.

Summary

- When we test a single quantitative variable, our hypothesis has the following form:
  - $\mathbf{H_0}$: μ = some number
  - $\mathbf{H_a}$: $\mu \neq$ some number, $\mu <$ something or $\mu >$ something.
- We will get our data (or mean, sample size, and SD for our data) and use the Theory-Based Inference to determine the p-value.
- The p-value we get with this test has the same general meaning as those from a test for a single proportion.

2. Significance level, Type 1 and Type 2 errors

Section 2.3

Significance Level

- We think of a p-value as telling us something about the strength of evidence from a test of significance.
- The lower the p-value the stronger the evidence.
- Some people think of this in more black and white terms. Either we reject the null or not.

Significance Level

- The value that we use to determine how small a p-value needs to be to provide convincing evidence whether or not to reject the null hypothesis is called the **significance level**.
- We reject the null when the p-value is less than or equal to ($\leq$) the significance level.
- The significance level is often represented by the Greek letter alpha, α.

Significance Level

- Typically we use 0.05 for our significance level. There is nothing magical about 0.05. We could set up our test to make it
  - harder to reject the null (smaller significance level say 0.01) or
  - easier (larger significance level say 0.10).

Type I and Type II errors

- In medical tests:
  - A type I error is a false positive. (They conclude someone has a disease when they don't.)
  - A type II error is a false negative. (They conclude someone does not have a disease then they actually do.)
- These types of errors can have very different consequences.

Type I and Type II Errors

TABLE 2.9 A summary of Type I and Type II errors

		What is true (unknown to us)	
		Null hypothesis is true	Null hypothesis is false
What we decide (based on data)	Reject null hypothesis	Type I error (false alarm)	Correct decision
	Do not reject null hypothesis	Correct decision	Type II error (missed opportunity)

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Type I and Type II errors

TABLE 2.10 Type I and Type II errors summarized in context of jury trial

		What is true (unknown to the jury)	
		Null hypothesis is true (defendant is innocent)	Null hypothesis is false (defendant is guilty)
What jury decides (based on evidence)	Reject null hypothesis (Jury finds defendant guilty)	Type I error (false alarm)	Correct decision
	Do not reject null hypothesis (Jury finds defendant not guilty)	Correct decision	Type II error (missed opportunity)

The probability of a Type I error

- The probability of a type I error is the significance level.
- Suppose the significance level is 0.05. If the null is true we would reject it 5% of the time and thus make a type I error 5% of the time.
- If you make the significance level lower, you have reduced the probability of making a type I error, but have increased the probability of making a type II error.

The probability of a Type II error

- The probability of a type II error is more difficult to calculate.
- In fact, the probability of a type II error is not even a fixed number. It depends on the value of the true parameter.
- The probability of a type II error can be very high if:
  - The true value of the parameter and the value you are testing are close.
  - The sample size is small.

1. Power.

- Power is 1 – P(Type II error). Usually expressed as a function of μ.
- Recall Type I and Type II errors.
  - A type I error is a false positive. Rejecting the null when it is true.
  - A type II error is a false negative. Failing to reject the null when the null is false.

Power

- The probability of rejecting the null hypothesis when it is false is called the **power** of a test.
- Power is 1 minus the probability of type II error.
- We want a test with high power and this is aided by
  - A large effect size, i.e. true μ far from the parameter in the null hypothesis.
  - A large sample size.
  - A small standard deviation.
  - Significance level. A higher sign. level means greater power. The downside is that you increase the chance of making a type I error.

Estimation and confidence intervals.

Chapter 3

Chapter Overview

- So far, we can only say things like

  "We have strong evidence that the more competent face is more likely to win an election."

- We want a method that says

  "Our data suggests that 68 to 75% of all elections can be correctly predicted by the competent face method."

Confidence Intervals

- Interval estimates of a population parameter are called **confidence intervals**.
- We will find confidence intervals three ways.
  - Through a series of tests of significance to see which proportions are plausible values for the parameter.
  - Using the standard deviation of the simulated null distribution to help us determine the width of the interval.
  - Through traditional theory-based methods.

Statistical Inference: Confidence Intervals

Section 3.1

Can Dogs Sniff Out Cancer?

Section 3.1

Can Dogs Sniff Out Cancer?

Sonoda et al. (2011). Marine, a dog originally trained for water rescues, was tested to see if she could detect if a patient had colorectal cancer by smelling a sample of their breath.

- She first smells a bag from a patient with colorectal cancer.
- Then she smells 5 other samples; 4 from normal patients and 1 from a person with colorectal cancer
- She is trained to sit next to the bag that matches the scent of the initial bag (the “cancer scent”) by being rewarded with a tennis ball.

Can Dogs Sniff Out Cancer?

In Sonoda et al. (2011). Marine was tested in 33 trials.

- Null hypothesis: Marine is randomly guessing which bag is the cancer specimen ($\pi = 0.20$)
- Alternative hypothesis: Marine can detect cancer better than guessing ($\pi > 0.20$)

π represents her long-run probability of identifying the cancer specimen.

Can Dogs Sniff Out Cancer?

- 30 out of 33 trials resulted in Marine correctly identifying the bag from the cancer patient
- So our sample proportion is

$$\hat{p} = \frac{30}{33} \approx 0.909$$

- Do you think Marine can detect cancer?
- What sort of p-value will we get?

Can Dogs Sniff Out Cancer?

Our sample proportion lies more than 10 standard errors above the mean and hence our p-value is very clost to zero.

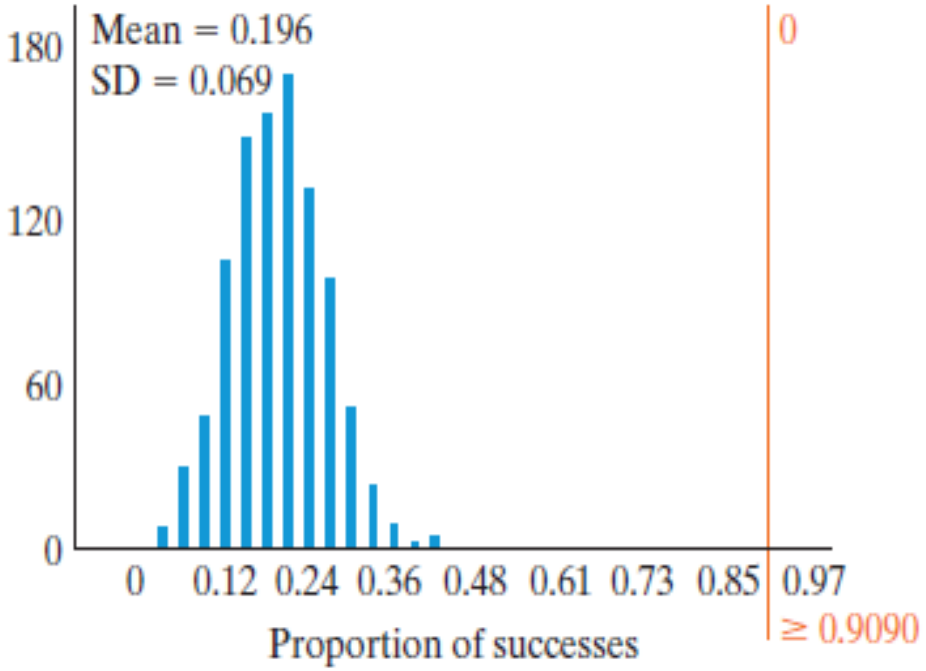
Probability of success (π): 0.20

Sample size (n): 33

Number of samples: 1000

As extreme as ≥ 0.909 Count

Proportion of samples:
0/1000 = 0



Can Dogs Sniff Out Cancer?

- Can we estimate Marine's long run frequency of picking the correct specimen?
- Since our sample proportion is about 0.909, it is plausible that 0.909 is a value for this frequency. What about other values?
- Is it plausible that Marine's frequency is actually 0.70 and she had a lucky day?
- Is a sample proportion of 0.909 unlikely if

  $\pi = 0.70$?

Can Dogs Sniff Out Cancer?

- H_0: $\pi = 0.70$ H_a: $\pi \neq 0.70$
- We get a small p-value (0.0090) so we can essentially rule out 0.70 as her long run frequency.

Probability of success (π): 0.70

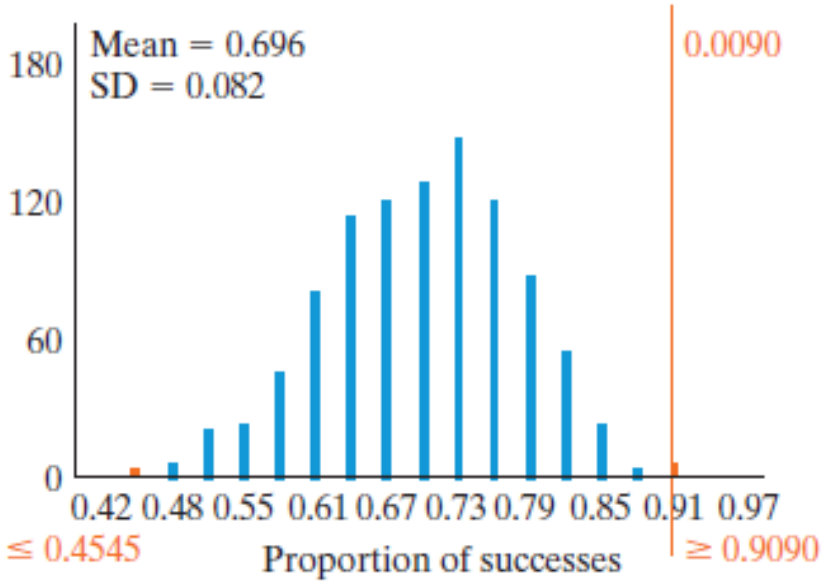
Sample size (n): 33

Number of samples: 1000

As extreme as ≥ 0.909 Count

Proportion of samples:
(3 + 6)/1000 = 0.0090

☑ Two-sided



Can Dogs Sniff Out Cancer?

- What about 0.80?
- Is 0.909 unlikely if π = 0.80?

Can Dogs Sniff Out Cancer?

- H_0: $\pi = 0.80$ H_a: $\pi \neq 0.80$
- We get a large p-value (0.1470) so 0.80 is a ***plausible*** value for Marine's long-run frequency.

Probability of success (π): 0.80

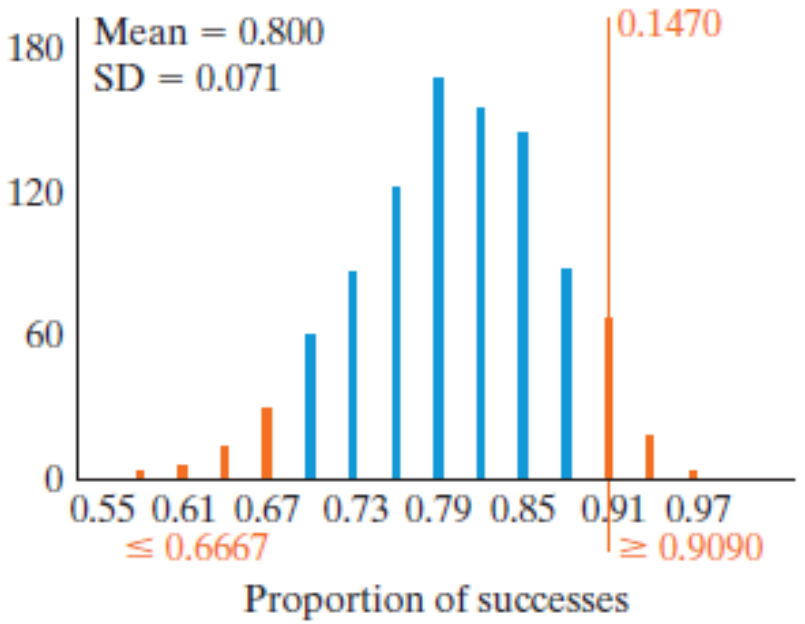
Sample size (n): 33

Number of samples: 1000

As extreme as ≥ 0.909 Count

Proportion of samples:
(52 + 95)/1000 = 0.1470

☑ Two-sided



Developing a range of plausible values

- If we get a small p-value (like we did with 0.70) we will conclude that the value under the null is not plausible. This is when we reject the null hypothesis.
- If we get a large p-value (like we did with 0.80) we will conclude the value under the null is plausible. This is when we can't reject the null.

Developing a range of plausible values

- One could use software (like the one-proportion applet the book recommends) to find a range of plausible values for Marine's long term probability of choosing the correct specimen.
- We will keep the sample proportion the same and change the possible values of π.
- We will use 0.05 as our cutoff value for if a p-value is small or large. (Recall that this is called the **significance level**.)

Can Dogs Sniff Out Cancer?

- It turns out values between 0.761 and 0.974 are plausible values for Marine's probability of picking the correct specimen.

Probability under null	0.759	0.760	0.761	0.762	.....	0.973	0.974	0.975	0.976
p-value	0.042	0.043	**0.063**	**0.063**		**0.059**	**0.054**	0.049	0.044
Plausible?	No	No	Yes	Yes	..... Yes	Yes	Yes	No	No

Can Dogs Sniff Out Cancer?

- (0.761, 0.974) is called a ***confidence interval.***
- Since we used 5% as our significance level, this is a 95% confidence interval. (100% – 5%)
- 95% is the ***confidence level*** associated with the interval of plausible values.

Can Dogs Sniff Out Cancer?

- We would say we are 95% confident that Marine's probability of correctly picking the bag with breath from the cancer patient from among 5 bags is between 0.761 and 0.974.
- This is a more precise statement than our initial significance test which concluded Marine's probability was more than 0.20.
- Sidenote: We do not say $P\{\pi \text{ is in } (.761, .974)\} = 95\%$, because π is not random. The *interval* is random, and would change with a different sample. If we calculate an interval this way, then $P(\text{interval contains } \pi) = 95\%$.

Confidence Level

- If we increase the confidence level from 95% to 99%, what will happen to the width of the confidence interval?

Can Dogs Sniff Out Cancer?

- Since the confidence level gives an indication of how sure we are that we captured the actual value of the parameter in our interval, to be more sure our interval should be wider.
- How would we obtain a wider interval of plausible values to represent a 99% confidence level?
  - Use a 1% significance level in the tests.
  - Values that correspond to 2-sided p-values larger than 0.01 should now be in our interval.

1.96 SE and Theory-Based Confidence Intervals for a Single Proportion

Section 3.2

Introduction

- Section 3.1 found confidence intervals by doing repeated tests of significance (changing the value in the null hypothesis) to find a range of values that were plausible for the population parameter (long run probability or population proportion).
- This is a very tedious way to construct a confidence interval.
- We will now look at two other ways to construct confidence intervals [1.96 SE and Theory-Based].

The Affordable Care Act

Example 3.2

The Affordable Care Act

- A November 2013 Gallup poll based on a random sample of 1,034 adults asked whether the Affordable Care Act had affected the respondents or their family.
- 69% of the sample responded that the act had no effect. (This number went down to 59% in May 2014 and 54% in Oct 2014.)
- What can we say about the proportion of all adult Americans that would say the act had no effect?

The Affordable Care Act

- We could construct a confidence interval just like we did last time.
- We find we are 95% confident that the proportion of all adult Americans that felt unaffected by the ACA is between 0.661 and 0.717.

Probability under null	0.659	0.660	0.661	.....	0.717	0.718	0.719
Two-sided p-value	0.0388	0.0453	0.0514	.....	0.0517	0.0458	0.0365
Plausible value (0.05)?	No	No	Yes	.....	Yes	No	No

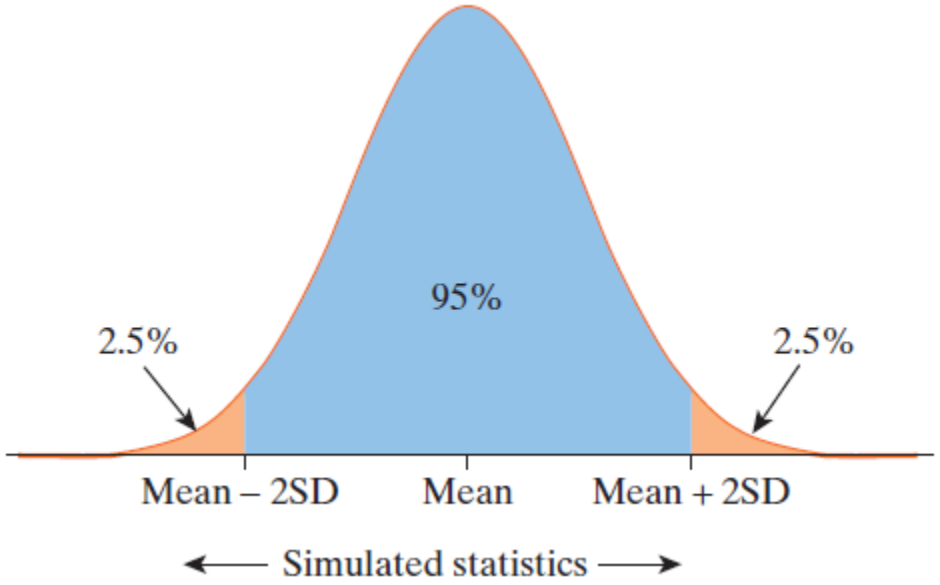
Short cut?

- The method we used last time to find our interval of plausible values for the parameter is tedious and time consuming.
- Might there be a short cut?
- Our sample proportion should be the middle of our confidence interval.
- We just need a way to find out how wide it should be.

1.96SE method

The book calls it the 2 SD method but we will use 1.96 instead of 2 and call it the SE instead of SD.

- When a statistic is normally distributed, about 95% of the values fall within 1.96 standard deviations of its mean with the other 5% outside this region



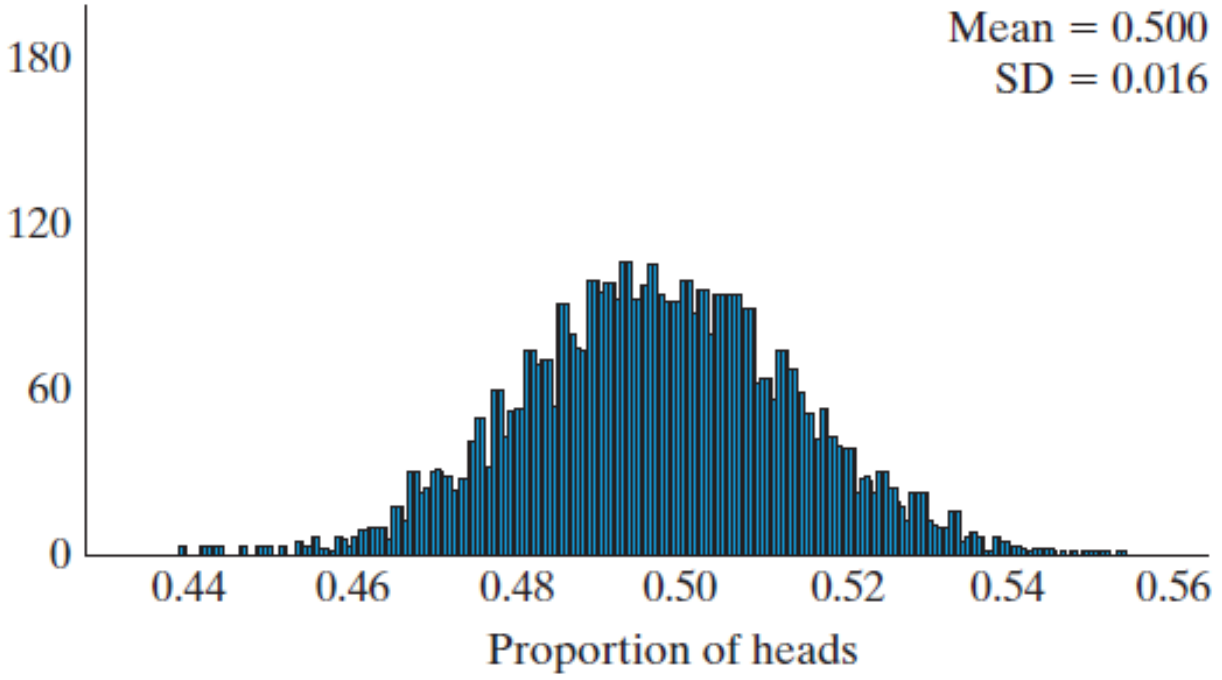
1.96 SD method

- So we could say that a parameter value is plausible if it is within 1.96 standard errors from our estimate of the parameter, our observed sample statistic.
- This gives us the simple formula for a 95% confidence interval of

$$\hat{p} \pm 1.96\,SE$$

Where do we get the SE?

- Null distribution for ACA with $\pi = 0.5$.



1.96 SE method

- Using the 1.96 SE method on our ACA data we get a 95% confidence interval

$$0.69 \pm 1.96(0.016)$$
$$0.69 \pm 0.031$$

- The $\pm$ part, like 0.031 in the above, is called the **margin of error.**
- The interval can also be written as we did before using just the endpoints; (0.659, 0.721)
- This is approximately what we got with our range of plausible values method.

Theory-Based Methods

- The 1.96 SE method only gives us a 95% confidence interval
- If we want a different level of confidence, we can use the range of plausible values (hard) or theory-based methods (easy).
- The theory-based method is valid provided there are at least 10 successes and 10 failures in your sample.

Theory-Based Methods

- With the theory-based method we use the normal distribution to approximate our simulated null distribution.
- This gives us a formula for confidence intervals.

$$\hat{p} \pm \textit{multiplier} \times \sqrt{\hat{p}(1-\hat{p})/n}.$$

For a 95% CI, the book suggests a multiplier of 2. Actually it should be 1.96, not 2.

qnorm(.975) = 1.96.

qnorm(.995) = 2.58.

- Let's check out this example using the theory-based method.
- Remember 69% of 1034 respondents were not affected.

$\hat{p} \pm$ *multiplier* $\times \sqrt{\hat{p}(1-\hat{p})/n}$

= 69% $\pm$ 1.96 x $\sqrt{.69(1-.69)/1034}$

= 69% $\pm$ 2.82%.

With 2 instead of 1.96 it would be 69% $\pm$ 2.88%.