Stat 13, Intro. to Statistical Methods for the Life and Health Sciences.

1. When to use which formula.
2. Multiple testing and publication bias.
3. Two quantitative variables, correlation.
4. Linear regression.

1. When to use which formula.

a. 1 sample numerical data, iid observations, want a 95% CI for μ.

- If n is large and σ is known, use $\bar{x}$ +/- 1.96 $\sigma/\sqrt{n}$.
- If n is small, draws are normal, and σ is known, use $\bar{x}$ +/- 1.96 $\sigma/\sqrt{n}$.
- If n is small, draws are normal, and σ is unknown, use $\bar{x}$ +/- t_{mult} $s/\sqrt{n}$.
- If n is large and σ is unknown, t_{mult} ~ 1.96, so we can use $\bar{x}$ +/- 1.96 $s/\sqrt{n}$.

$n \geq 30$ is often considered large enough to use 1.96.

In practice, we typically do not know the draws are normal, but if the distribution looks roughly symmetrical without enormous outliers, the t formula may be reasonable.

b. 1 sample binary data, iid observations, want a 95% CI for π.

View the data as 0 or 1, so sample percentage $p = \bar{x}$, and

$s = \sqrt{[p(1-p)]}$, $\sigma = \sqrt{[\pi(1-\pi)]}$.

1. When to use which formula.

a. 1 sample numerical data, iid observations, want a 95% CI for μ.

- If n is large and σ is known, use $\bar{x}$ +/- 1.96 σ/√n.
- If n is small, draws are normal, and σ is known, use $\bar{x}$ +/- 1.96 σ/√n.
- If n is small, draws ~ normal, and σ is unknown, use $\bar{x}$ +/- t_{mult} s/√n.
- If n is large and σ is unknown, t_{mult} ~ 1.96, so we can use $\bar{x}$ +/- 1.96 s/√n.

b. 1 sample binary data, iid observations, want a 95% CI for π.

View the data as 0 or 1, so sample percentage p = $\bar{x}$, and

s = √[p(1-p)], σ = √[π(1–π)].

If n is large and π is unknown, use $\bar{x}$ +/- 1.96 s/√n.

Here large n means ≥ 10 of each type in the sample.

What if n is small and the draws are not normal?

That is a situation outside the scope of this course, but some techniques have been developed, such as the bootstrap, which are sometimes useful in these situations.

1. When to use which formula.

c. Numerical data from 2 samples, iid observations, want a 95% CI for μ_1 - μ_2.

If n is large and σ is unknown, use $\bar{x}_1$ - $\bar{x}_2$+/- 1.96 $\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$.

As with one sample, if σ_1 is known, replace s_1 with σ_1, and the same for σ_2. And as with one sample, if σ_1 and σ_2 are unknown, the sample sizes are small, and the distributions are roughly normal, then use t_{mult} instead of 1.96. If the sample sizes are small, the distributions are normal, and σ_1 and σ_2 are known, then use 1.96.

d. Binary data from 2 samples, iid observations, want a 95% CI for π_1 - π_2.

same as in c above, with $p_1 = \bar{x}_1$, s_1 = √[p_1 (1-p_1)], σ_1 = √[π_1 (1–π_1)].

Large for binary data means sample has ≥ 10 of each type.

1. When to use which formula.

e. Matched pairs data, iid observations, want a 95% CI for μ.

Look at differences (score with treatment minus score with control) and treat differences as ordinary numerical data according to parts a or b.

- If n is large and σ is known, use $\bar{x}$ +/- 1.96 σ/√n.
- If n is small, draws are normal, and σ is known, use $\bar{x}$ +/- 1.96 σ/√n.
- If n is small, draws are normal, and σ is unknown, use $\bar{x}$ +/- t_{mult} s/√n.
- If n is large and σ is unknown, t_{mult} ~ 1.96, so we can use $\bar{x}$ +/- 1.96 s/√n.

n ≥ 30 is often considered large enough to use 1.96.

In practice, we typically do not know the draws are normal, but if the distribution looks roughly symmetrical without enormous outliers, the t formula may be reasonable.

2. Multiple testing and publication bias.

A p-value is the probability, assuming the null hypothesis of no relationship is true, that you will see a difference as extreme as, or more extreme than, you observed.

So, 5% of the time you are looking at unrelated things, you will find a statistically significant relationship.

This underscores the need for followup confirmation studies.

If testing many explanatory variables simultaneously, it can become very likely to find something significant even if nothing is actually related to the response variable.

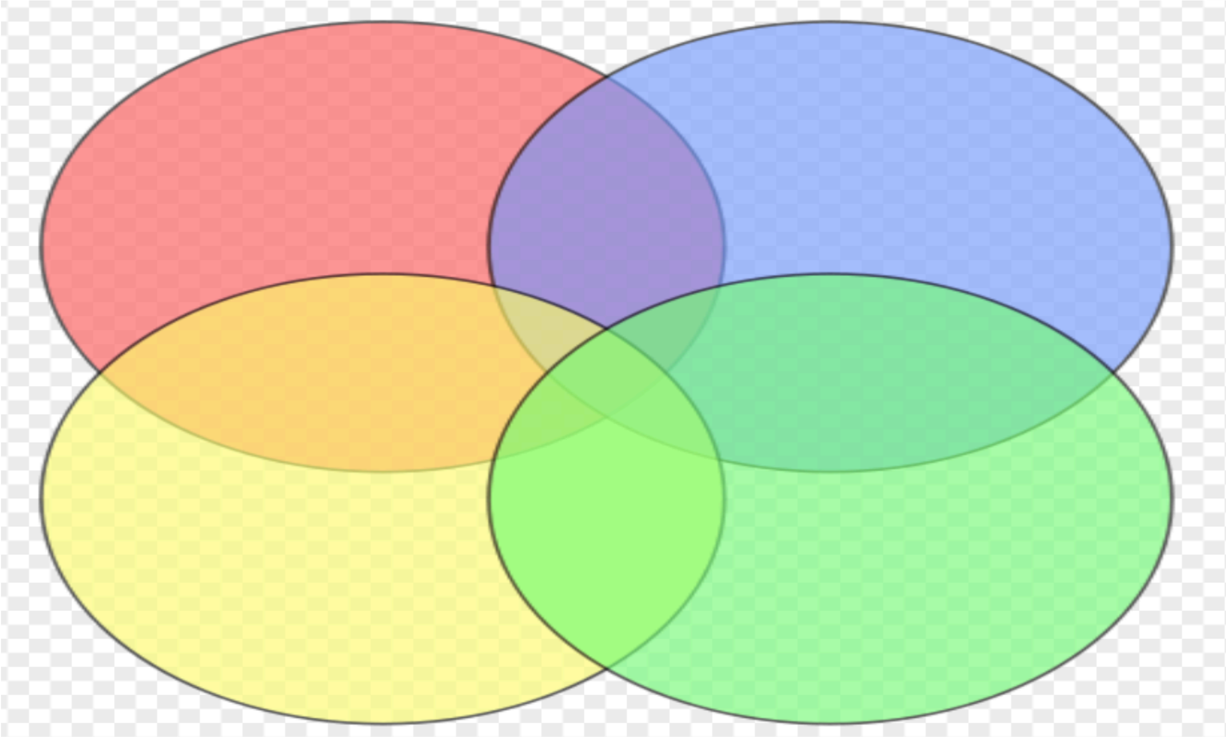
Multiple testing and publication bias.

* For example, if the significance level is 5%, then for 100 tests where all null hypotheses are true, the expected number of incorrect rejections (Type I errors) is 5. If the tests are independent, the probability of at least one Type I error would be 99.4%.

* To address this problem, scientists sometimes change the significance level so that, under the null hypothesis that none of the explanatory variables is related to the response variable, the probability of rejecting *any* of them is 5%.

* One way is to use Bonferroni's correction: with *m* explanatory variables, use significance level 5%/m.

P(at least 1 Type I error) will be $\leq$ m (5%/m) = 5%.



P(Type I error on explanatory 1) = 5%/m.

P(Type I error on explanatory 2) = 5%/m.

P(Type 1 error on at least one explanatory) ≤

P(error on 1) + P(error on 2) + ... + P(error on m) = m x 5%/m.

Multiple testing and publication bias.

Imagine a scenario where a drug is tested many times to see if it reduces the incidence of some response variable. If the drug is testes 100 times by 100 different researchers, the results will be stat. sig. about 5 times.

If only the stat. sig. results are published, then the published record will be very misleading.

Multiple testing and publication bias.

A drug called Reboxetine made by Pfizer was approved as a treatment for depression in Europe and the UK in 2001, based on positive trials.

A meta-analysis in 2010 found that it was not only ineffective but also potentially harmful. The report found that 74% of the data on patients who took part in the trials of Reboxetine were not published because the findings were negative. Published data about reboxetine overestimated its benefits and underestimated its harm.

A subsequent 2011 analysis indicated Reboxetine might be effective for severe depression though.

3. Two Quantitative Variables

Chapter 10

Two Quantitative Variables: Scatterplots and Correlation

Section 10.1

Scatterplots and Correlation

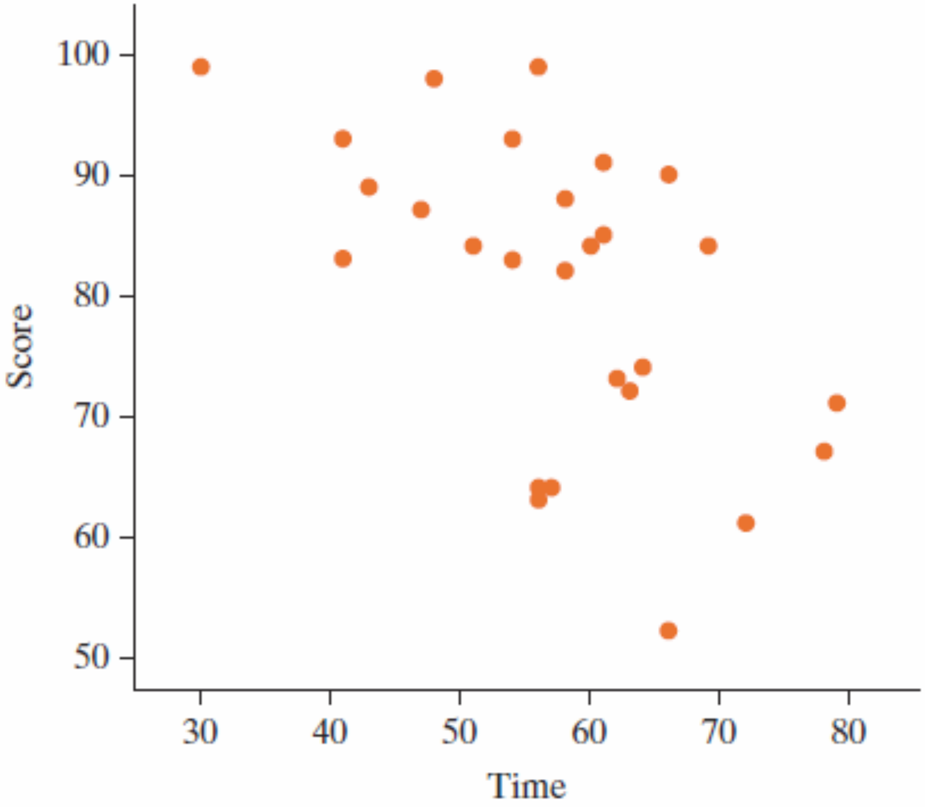
Suppose we collected data on the relationship between the time it takes a student to take a test and the resulting score.

Time	30	41	41	43	47	48	51	54	54	56	56	56	57	58
Score	100	84	94	90	88	99	85	84	94	100	65	64	65	89
Time	58	60	61	61	62	63	64	66	66	69	72	78	79	
Score	83	85	86	92	74	73	75	53	91	85	62	68	72	

Scatterplot

Put explanatory variable on the horizontal axis.

Put response variable on the vertical axis.



Describing Scatterplots

- When we describe data in a scatterplot, we describe the
  - Direction (positive or negative)
  - Form (linear or not)
  - Strength (strong-moderate-weak, we will let correlation help us decide)
  - Unusual Observations
- How would you describe the time and test scatterplot?

Correlation

- **Correlation** measures the strength and direction of a linear association between two quantitative variables.
- Correlation is a number between -1 and 1.
- With positive correlation one variable increases, on average, as the other increases.
- With negative correlation one variable decreases, on average, as the other increases.
- The closer it is to either -1 or 1 the closer the points fit to a line.
- The correlation for the test data is -0.56.

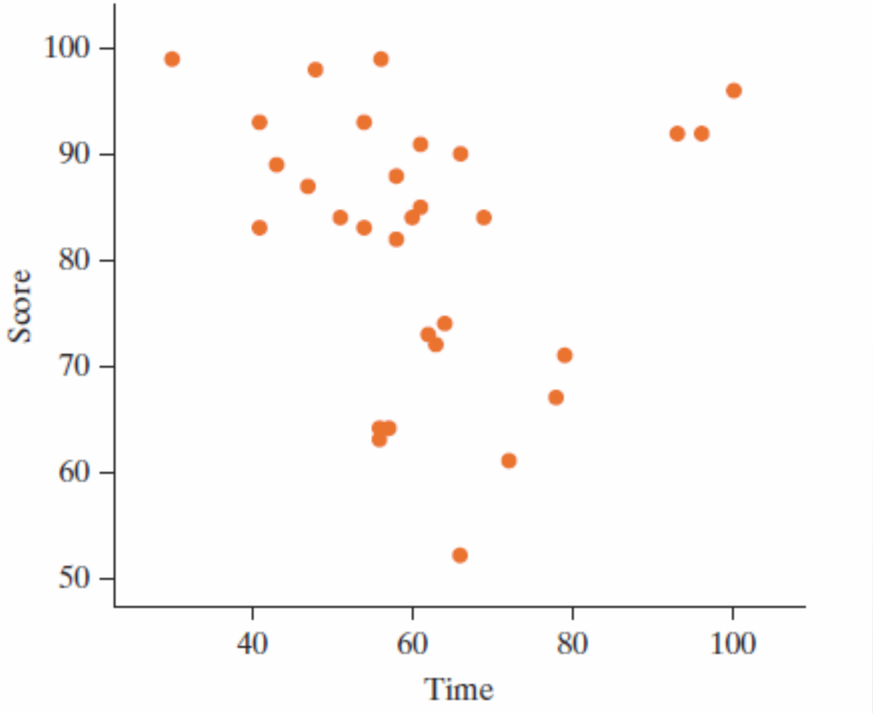
Correlation Guidelines

Correlation Value	Strength of Association	What this means
0.7 to 1.0	Strong	The points will appear to be nearly a straight line
0.3 to 0.7	Moderate	When looking at the graph the increasing/decreasing pattern will be clear, but there is considerable scatter.
0.1 to 0.3	Weak	With some effort you will be able to see a slightly increasing/decreasing pattern
0 to 0.1	None	No discernible increasing/decreasing pattern
Same Strength Results with Negative Correlations		

Back to the test data

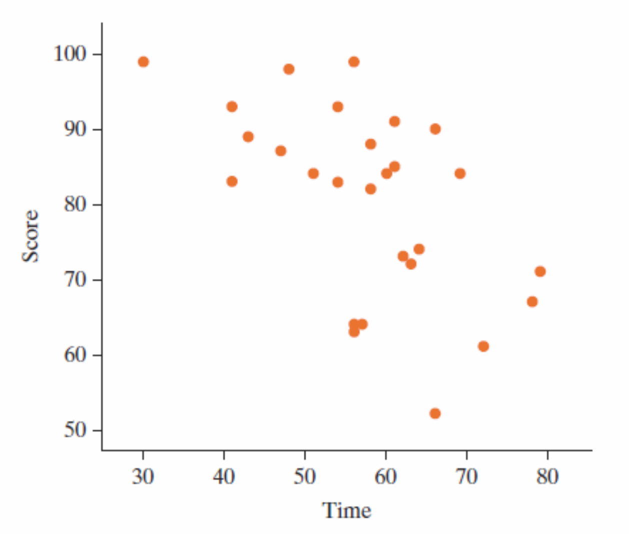
Actually the last three people to finish the test had scores of 93, 93, and 97.

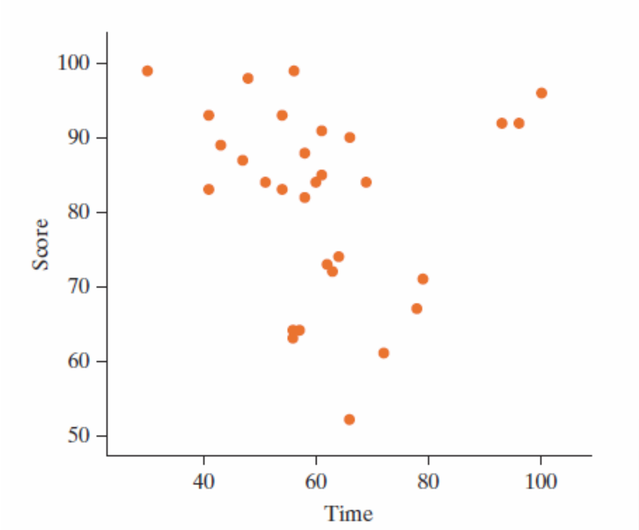
What does this do to the correlation?



Influential Observations

- The correlation changed from -0.56 (a fairly moderate negative correlation) to -0.12 (a weak negative correlation).
- Points that are far to the left or right and not in the overall direction of the scatterplot can greatly change the correlation. (influential observations)





Correlation

- **Correlation** measures the strength and direction of a linear association between two quantitative variables.
  - $-1 \leq r \leq 1$
  - Correlation makes no distinction between explanatory and response variables.
  - Correlation has no units.
  - Correlation is not resistant to outliers. It is sensitive.

Learning Objectives for Section 10.1

- Summarize the characteristics of a scatterplot by describing its direction, form, strength and whether there are any unusual observations.
- Recognize that the correlation coefficient is appropriate only for summarizing the strength and direction of a scatterplot that has linear form.
- Recognize that a scatterplot is the appropriate graph for displaying the relationship between two quantitative variables and create a scatterplot from raw data.
- Recognize that a correlation coefficient of 0 means there is no linear association between the two variables and that a correlation coefficient of -1 or 1 means that the scatterplot is exactly a straight line.
- Understand that the correlation coefficient is influenced by extreme observations.

Inference for the Correlation Coefficient: Simulation-Based Approach

Section 10.2

We will look at a small sample example to see if body temperature is associated with heart rate.

Temperature and Heart Rate

Hypotheses

- Null: There is no association between heart rate and body temperature. ($\rho = 0$)
- Alternative: There is a positive linear association between heart rate and body temperature. ($\rho > 0$)

ρ = rho

Inference for Correlation with Simulation

(Section 10.2)

1. Compute the observed statistic. (Correlation)
2. Scramble the response variable, compute the simulated statistic, and repeat this process many times.
3. Reject the null hypothesis if the observed statistic is in the tail of the null distribution.

Temperature and Heart Rate

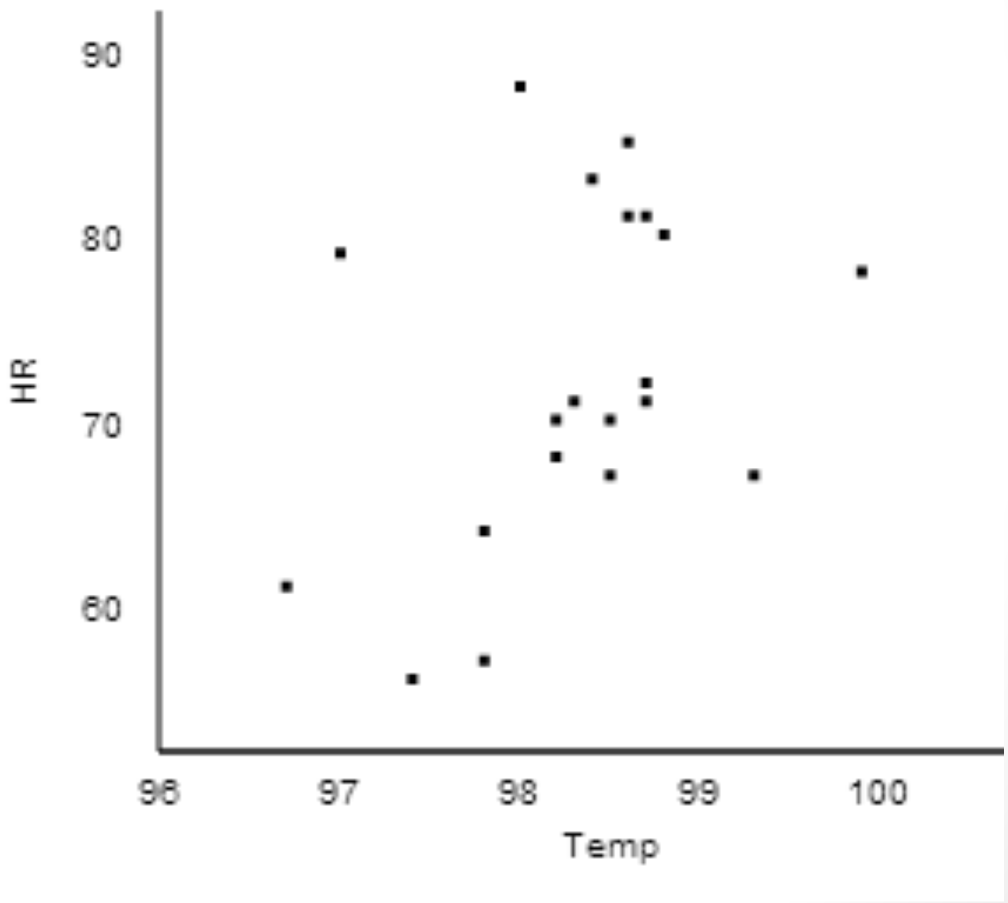
Collect the Data

Tmp	98.3	98.2	98.7	98.5	97.0	98.8	98.5	98.7	99.3	97.8
HR	72	69	72	71	80	81	68	82	68	65
Tmp	98.2	99.9	98.6	98.6	97.8	98.4	98.7	97.4	96.7	98.0
HR	71	79	86	82	58	84	73	57	62	89

Temperature and Heart Rate

Explore the Data

r = 0.378



Temperature and Heart Rate

- If there was no association between heart rate and body temperature, what is the probability we would get a correlation as high as 0.378 just by chance?

- If there is no association, we can break apart the temperatures and their corresponding heart rates. We will do this by shuffling one of the variables.

Shuffling Cards

- Let’s remind ourselves what we did with cards to find our simulated statistics.
- With two proportions, we wrote the response on the cards, shuffled the cards and placed them into two piles corresponding to the two categories of the explanatory variable.
- With two means we did the same thing except this time the responses were numbers instead of words.

Dolphin Therapy

Non-improver	Improver	Improver
Non-improver	Improver	Improver
Non-improver	Improver	Improver
Non-improver	Improver	Improver
Non-improver	Improver	Improver

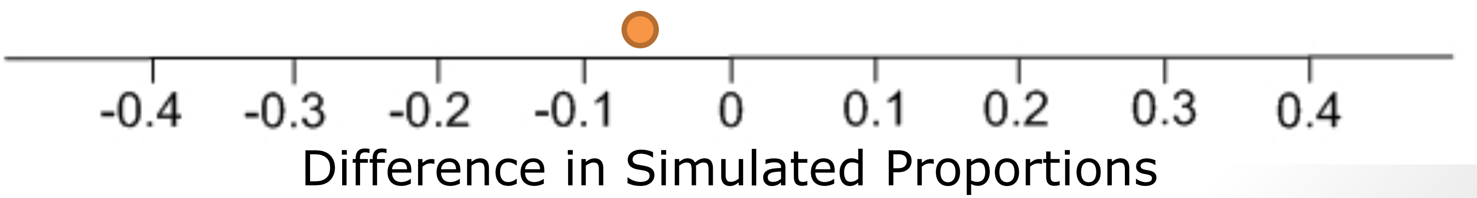
Control

Non-improver	Non-improver	Non-improver
Non-improver	Non-improver	Non-improver
Non-improver	Non-improver	Improver
Non-improver	Non-improver	Improver
Non-improver	Non-improver	Improver

66.7%
40.0%
Improvers

26.7%
46.7%
Improvers

0.400 – 0.467 = -0.067



Music

25.2	45.6
14.5	11.6
-7.0	18.6
12.6	12.1
34.5	30.5

mean = 19.82

mean = 6.38

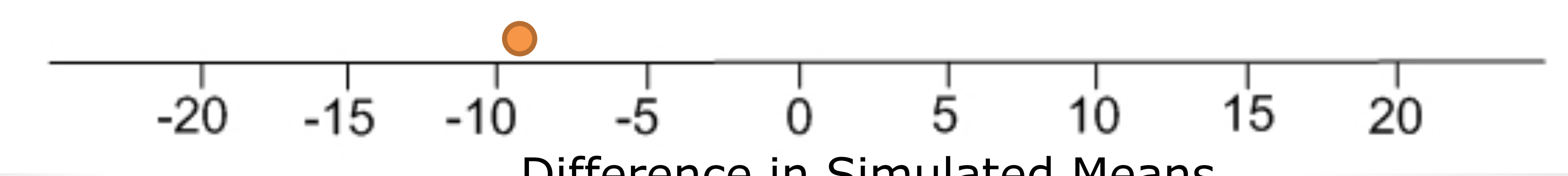
No music

-10.7	-10.7	10.0
4.5	9.6	
2.2	2.4	
21.3	21.8	
-14.7	7.2	

mean = 3.90

mean = 16.12

6.38 – 16.12 = -9.74



Difference in Simulated Means

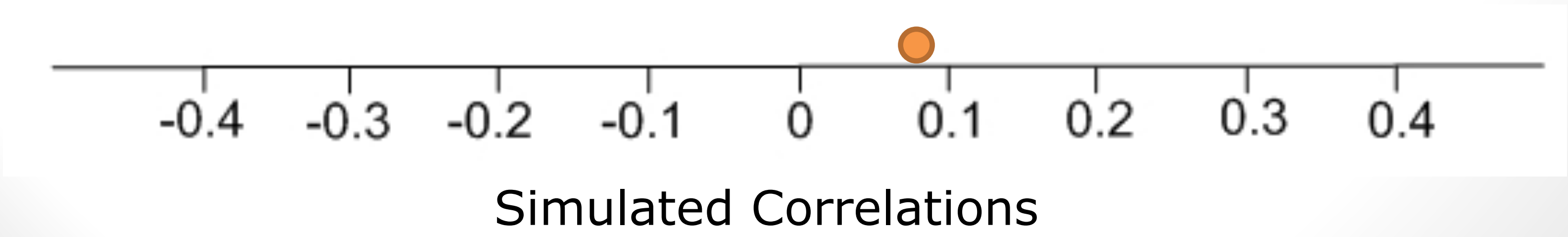
Shuffling Cards

- Now how will this shuffling be different when both the response and the explanatory variable are quantitative?
- We can’t put things in two piles anymore.
- We still shuffle values of the response variable, but this time place them next to two values of the explanatory variable.

Body Temperature and Heart Rate

98.3°	98.2°	97.7°	98.5°	97.0°	98.8°	98.5°	98.7°	99.3°	97.8°
72	69	72	71	80	81	68	82	68	65
98.2°	99.9°	98.6°	98.6°	97.8°	98.4°	98.7°	97.4°	96.7°	98.0°
71	79	86	82	58	84	73	57	62	89

r = 0.078



More Simulations

0.097 0.054 0.062 0.259 0.034

-0.253 -0.345 0.314 0.339

-0.034 0.200 0.212 -0.029 -0.008

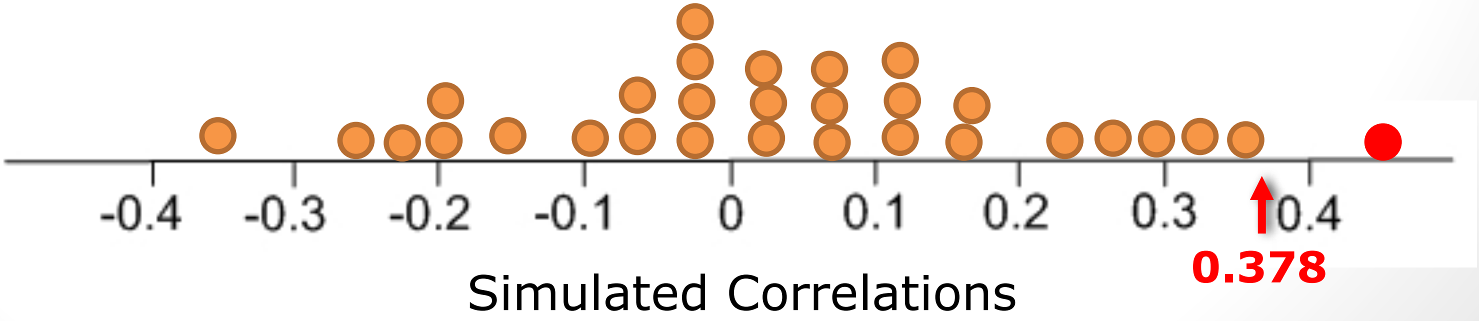
0.059 -0.042 0.006 0.447 0.447 0.167

0.067 0.020

0.232 -0.327 0.100 0.329

-0.220

Only one simulated statistic out of 30 was as large or larger than our observed correlation of 0.378, hence our p-value for this null distribution is 1/30 ≈ 0.03.

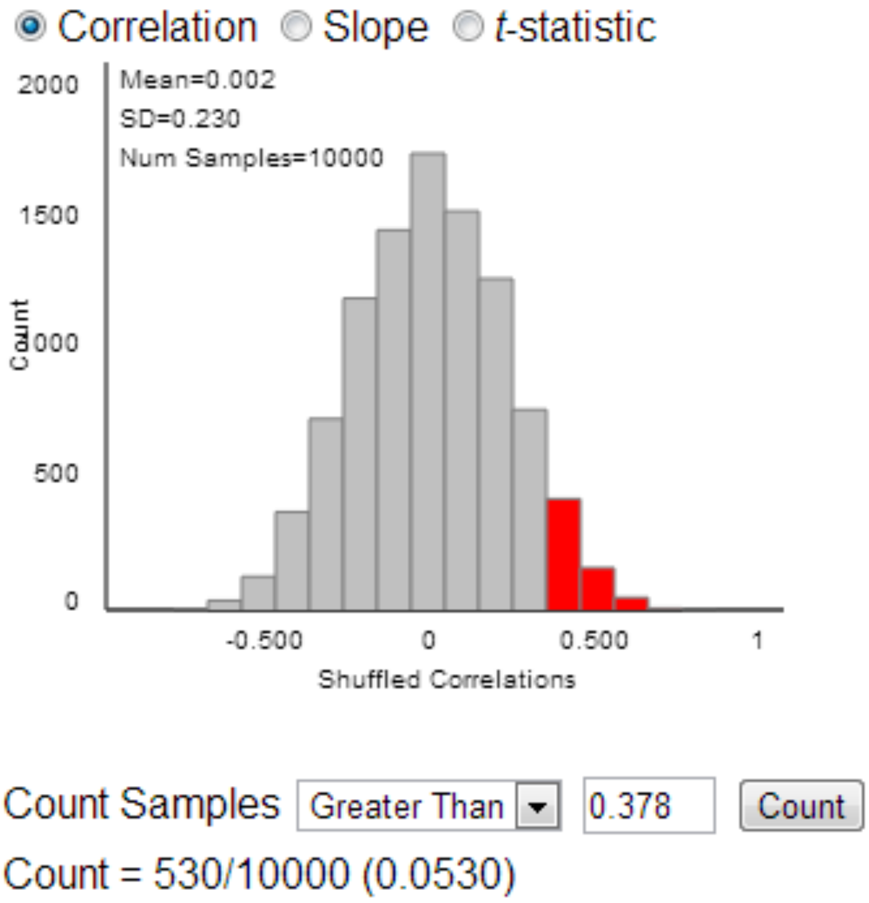


Temperature and Heart Rate

- We can look at the output of 1000 shuffles with a distribution of 1000 simulated correlations.

Temperature and Heart Rate

- Notice our null distribution is centered at 0 and somewhat symmetric.
- We found that 530/10000 times we had a simulated correlation greater than or equal to 0.378.



● Correlation ○ Slope ○ *t*-statistic

Mean=0.002

SD=0.230

Num Samples=10000

Count Samples Greater Than ▾ 0.378

Count = 530/10000 (0.0530)

Temperature and Heart Rate

- With a p-value of 0.053 = 5.3%, we almost but do not quite have statistical significance. This is moderate evidence of a positive linear association between body temperature and heart rate. Perhaps a larger sample would give a smaller p-value.

4. Least Squares Regression

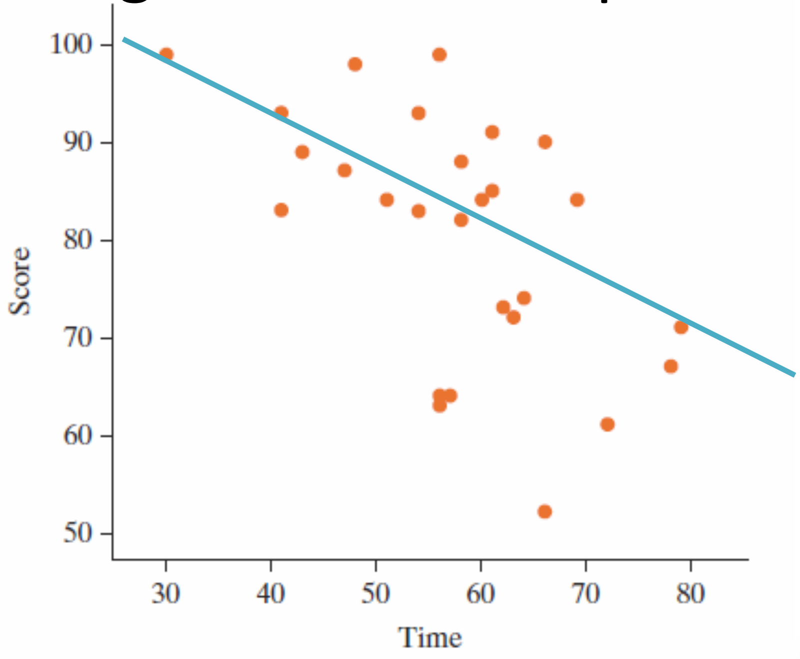
Section 10.3

Introduction

- If we decide an association is linear, it is helpful to develop a mathematical model of that association.
- Helps make predictions about the response variable.
- The ***least-squares regression line*** is the most common way of doing this.

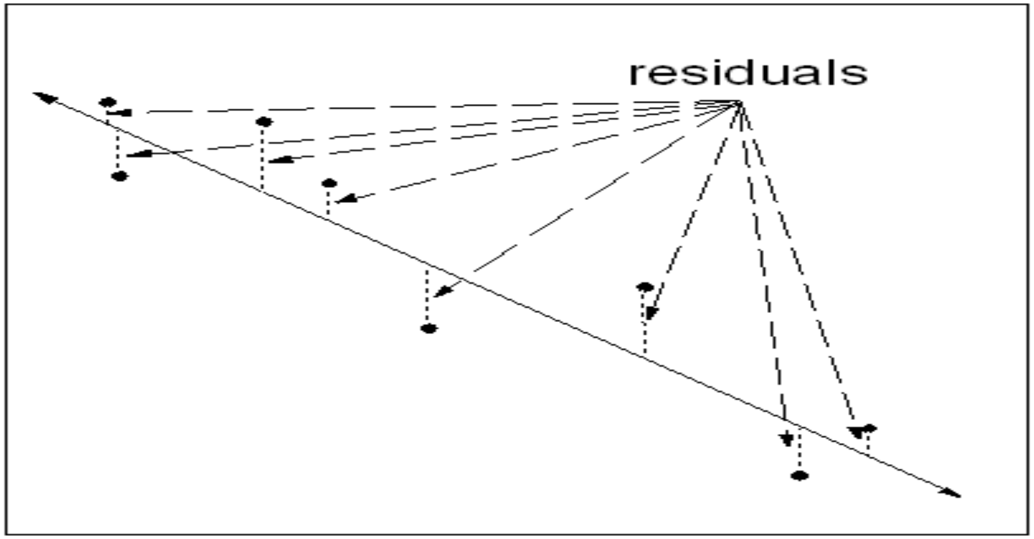
Introduction

- Unless the points are perfectly linearly alligned, there will not be a single line that goes through every point.
- We want a line that gets as close as possible to all the points.



Introduction

- We want a line that minimizes the vertical distances between the line and the points
  - These distances are called **residuals.**
  - The line we will find actually minimizes the sum of the squares of the residuals.
  - This is called a **least-squares regression line**.

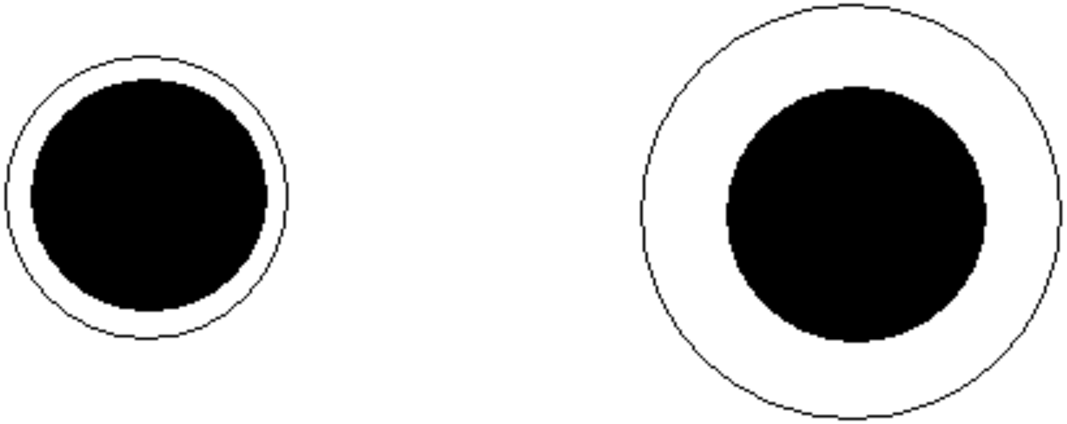


Are Dinner Plates Getting Larger?

Example 10.3

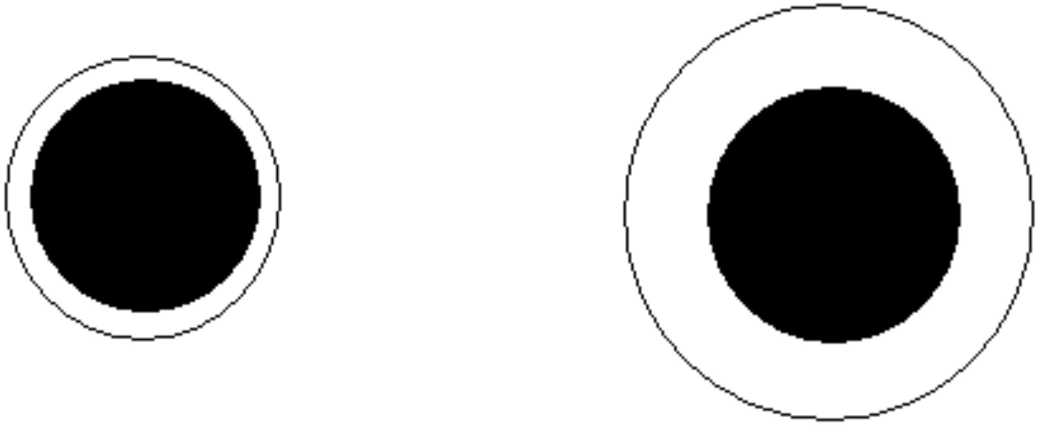
Growing Plates?

- There are many recent articles and TV reports about the obesity problem.
- One reason some have given is that the size of dinner plates are increasing.
- Are these black circles the same size, or is one larger than the other?



Growing Plates?

- They appear to be the same size for many, but the one on the right is about 20% larger than the left.



- This suggests that people will put more food on larger dinner plates without knowing it.
- There is name for this phenomenon: *Delboeuf illusion*

Growing Plates?

- Researchers gathered data to investigate the claim that dinner plates are growing
- American dinner plates sold on ebay on March 30, 2010 (Van Ittersum and Wansink, 2011)
- Year manufactured and diameter are given.

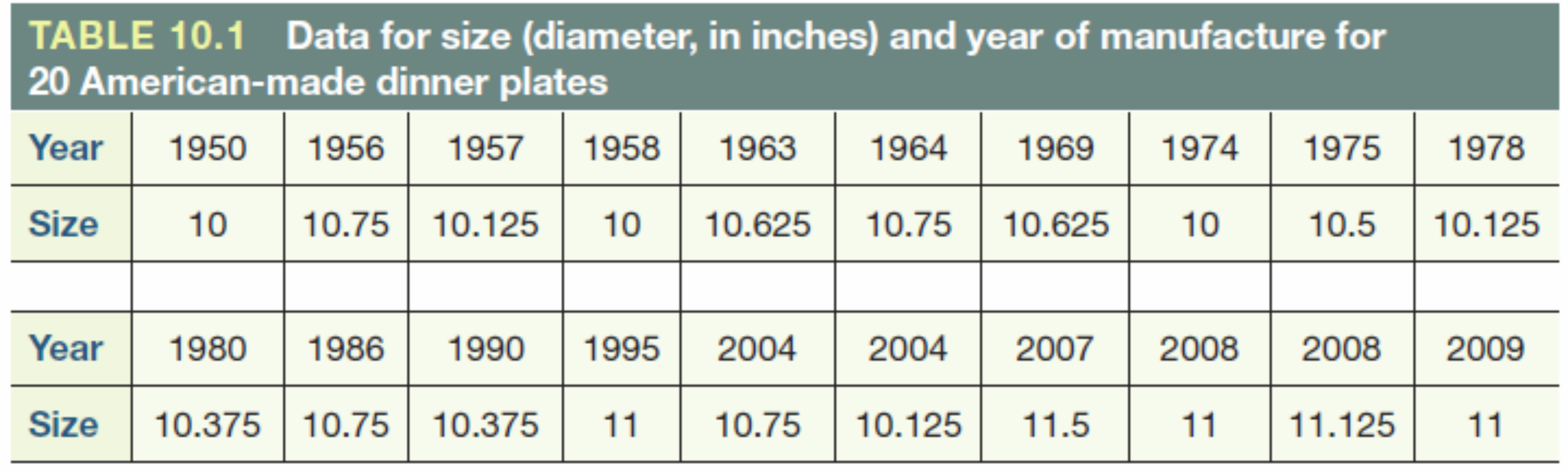
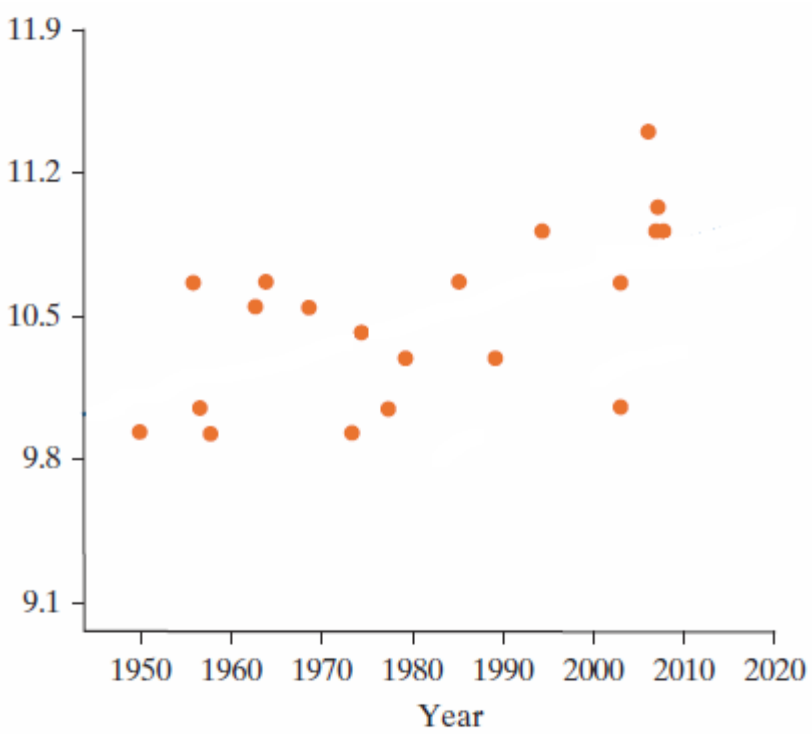


TABLE 10.1 Data for size (diameter, in inches) and year of manufacture for 20 American-made dinner plates

Year	1950	1956	1957	1958	1963	1964	1969	1974	1975	1978
Size	10	10.75	10.125	10	10.625	10.75	10.625	10	10.5	10.125
Year	1980	1986	1990	1995	2004	2004	2007	2008	2008	2009
Size	10.375	10.75	10.375	11	10.75	10.125	11.5	11	11.125	11

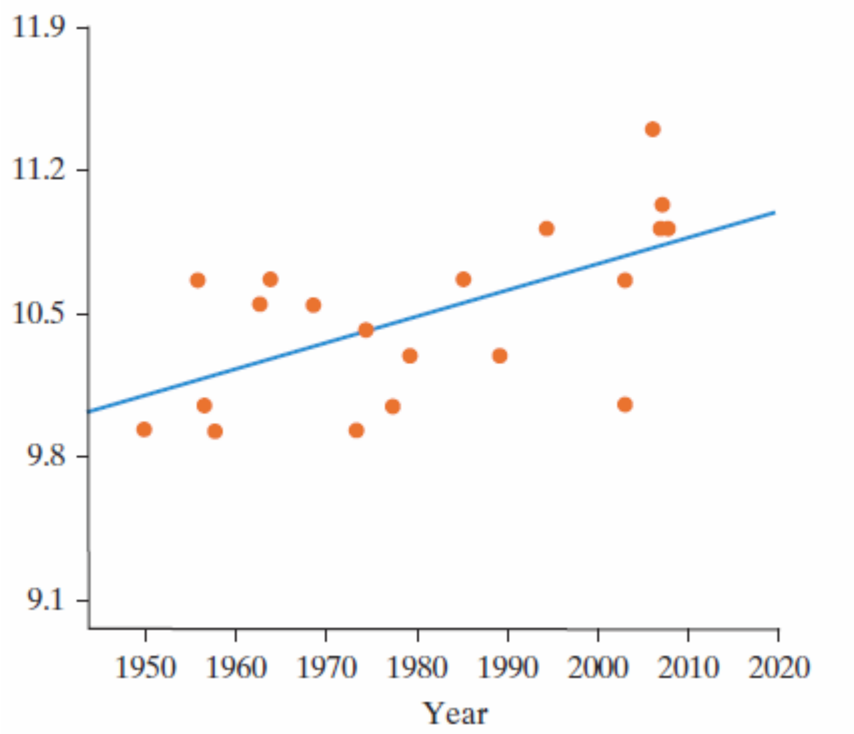
Growing Plates?

- Both year (explanatory variable) and diameter in inches (response variable) are quantitative.
- Each dot represents one plate in this scatterplot.
- Describe the association here.



Growing Plates?

- The association appears to be roughly linear
- The least squares regression line is added
- How can we describe this line?



Regression Line

The regression equation is $\hat{y} = a + bx$:

- *a* is the *y*-intercept
- *b* is the slope
- *x* is a value of the explanatory variable
- *ŷ* is the predicted value for the response variable

- For a specific value of *x*, the corresponding distance $\mathrm{y} - \hat{y}$ (or actual – predicted) is a residual

Regression Line

- The least squares line for the dinner plate data is $\hat{y} = -14.8 + 0.0128x$
- Or $\widehat{\text{diameter}} = -14.8 + 0.0128(\text{year})$
- This allows us to predict plate diameter for a particular year.

Slope

$$\hat{y} = -14.8 + 0.0128x$$

- What is the predicted diameter for a plate manufactured in 2000?
  - -14.8 + 0.0128(2000) = 10.8 in.
- What is the predicted diameter for a plate manufactured in 2001?
  - -14.8 + 0.0128(2001) = 10.8128 in.
- How does this compare to our prediction for the year 2000?
  - 0.0128 larger
- Slope *b* = 0.0128 means that diameters are predicted to increase by 0.0128 inches per year on average

Slope

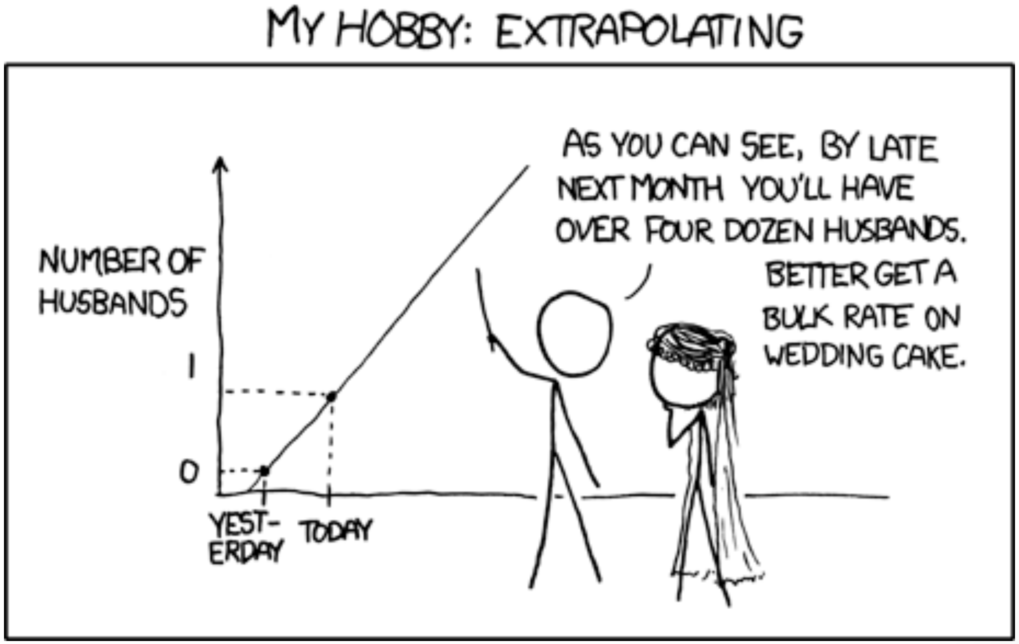
- Slope is the predicted change in the response variable for one-unit change in the explanatory variable.
- Both the slope and the correlation coefficient for this study were positive.
  - The slope is 0.0128
  - The correlation is 0.604
- The slope and correlation coefficient will always have the same sign.

y-intercept

- The *y*-intercept is where the regression line crosses the *y*-axis or the predicted response when the explanatory variable equals 0.
- We had a *y*-intercept of -14.8 in the dinner plate equation. What does this tell us about our dinner plate example?
  - Dinner plates in year 0 were -14.8 inches.
- How can it be negative?
  - The equation works well within the range of values given for the explanatory variable, but fails outside that range.
- Our equation should only be used to predict the size of dinner plates from about 1950 to 2010.

Extrapolation

- Predicting values for the response variable for values of the explanatory variable that are outside of the range of the original data is called ***extrapolation***.


MY HOBBY: EXTRAPOLATING
NUMBER OF HUSBANDS
1
0
YEST-ERDAY
TODAY
AS YOU CAN SEE, BY LATE NEXT MONTH YOU'LL HAVE OVER FOUR DOZEN HUSBANDS.
BETTER GET A BULK RATE ON WEDDING CAKE.

Coefficient of Determination

- While the intercept and slope have meaning in the context of year and diameter, remember that the correlation does not. It is just 0.604.
- However, the square of the correlation (coefficient of determination or r^2) does have meaning.
- $r^2 = 0.604^2 = 0.365$ or 36.5%
- 36.5% of the variation in plate size (the response variable) can be explained by its linear association with the year (the explanatory variable).

Learning Objectives for Section 10.3

- Understand that one way a scatterplot can be summarized is by fitting the best-fit (least squares regression) line.
- Be able to interpret both the slope and intercept of a best-fit line in the context of the two variables on the scatterplot.
- Find the predicted value of the response variable for a given value of the explanatory variable.
- Understand the concept of residual and find and interpret the residual for an observational unit given the raw data and the equation of the best fit (regression) line.
- Understand the relationship between residuals and strength of association and that the best-fit (regression) line this minimizes the sum of the squared residuals.

Learning Objectives for Section 10.3

- Find and interpret the coefficient of determination (r^2) as the squared correlation and as the percent of total variation in the response variable that is accounted for by the linear association with the explanatory variable.
- Understand that extrapolation is when a regression line is used to predict values outside of the range of observed values for the explanatory variable.
- Understand that when slope = 0 means no association, slope < 0 means negative association, slope > 0 means positive association, and that the sign of the slope will be the same as the sign of the correlation coefficient.
- Understand that influential points can substantially change the equation of the best-fit line.