Stat 13, Thu 4/19/12.
0. Hand in HW2!
1. Resistance.
2. n-1 in sample sd formula, and parameters and statistics.
3. Probability basic terminology.
4. Probability axioms.
5. Multiplication rules.
6. Probability trees.
7. Combinations.

Read chapter 3.
Hw3 is due Thur, 4/26, and Midterm 1 is Thur, 4/26.
You can use calculators, pen or pencil, and one 8.5x11 page of notes, double sided, for the exam.

1. Resistance to outliers.
Sample mean, sd, variance, range, and CV → sensitive to outliers.
Sample median and IQR → resistant to outliers.

{1,2,1,3,5,0}. mean is 2, median is 1.5, sd ~ 1.79, IQR = 2.
{1,2,1,3,5,0, 1000}. mean ~ 145, median is 2, sd ~ 377, IQR = 3.

2. n-1, parameters, and statistics.

In the formula for the sample sd, $s = \sqrt{\sum[(x_i - \bar{x})^2 / (n-1)]}$
If we replace the n-1 by n, then s is the *RMS* (root-mean-square) of deviations from $\bar{x}$.
In other words, s is the RMS of these deviations times a correction factor $\sqrt{(\frac{n}{n-1})}$
Why this correction factor?
Parameters are properties of the population. Typically unknown. Represented by Greek letters (like μ, or σ).
Statistics (also called *random variables* or *estimates*) are properties of the sample. Represented by Roman letters (like $\bar{x}$ or s).
Typically, you're interested in a value of a parameter. But you can't know it. So you *estimate* it with a statistic, based on the sample.

There are two means and two standard deviations!
The sample mean $\bar{x}$ and sample std deviation s are statistics.
Define the population average μ as the sum of all values in the population ÷ the number of subjects in the population. (parameter).
It turns out $\bar{x}$ is an unbiased estimate of μ.
That is, $\bar{x}$ is neither higher nor lower, on average, than μ.

Define the population std deviation σ as the RMS of the deviations in the whole population. (a parameter.)

It turns out that if you estimate σ with the RMS of the sample deviations, the estimate you get tends to be a little SMALLER on average than σ.
We multiply by the correction factor $\sqrt{\left(\frac{n}{n-1}\right)}$, so that the estimate s is unbiased.

Note that this correction factor is very nearly 1 for large n, so it doesn't matter much when the sample size n is large.

3. Basic terminology of probability.

a) Notation. P(event) = #. P(X = 5) means *the probability that X is 5*.

b) Conditional probability. P(A | B) means the probability of A <u>GIVEN</u> B.

c) Disjoint. Events A and B are *disjoint* if P(A and B) = 0.

d) Independent. A and B are independent if knowing A doesn't effect the probability that B will happen, and vice versa.

That is, if P(A | B) = P(A), and P(B | A) = P(B).

e.g. A = event first die roll is a 5, B = event second die is a 5.

e) Or. In math, A <u>OR</u> B always means one or the other or both! If you mean but not both, must say ``but not both''.

f) E^c means not E.

g) P(AB) means P(A and B).

4. Probability axioms.

(i) For any event E, P(E) ≥ 0.

(ii) For any event E, $P(E) + P(E^c) = 1$.

(iii) For any disjoint events $E_1, E_2, ..., E_n$,

$$P(E_1 \text{ or } E_2 \text{ or } ... \text{ or } E_n) = P(E_1) + P(E_2) + ... P(E_n).$$

(iii) is sometimes called the addition rule for disjoint events.

Note connection with Venn diagrams.

For *any* events E_1 and E_2, $P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) – P(E_1E_2)$.

5. Multiplication rules.

We define P(A | B) as P(AB) / P(B).

Independence means P(A | B) = P(A), which means P(AB)/P(B) = P(A), i.e.

P(AB) = P(A) P(B).

If this true, then P(B | A) = P(AB)/P(A) = P(A)P(B)/P(A) = P(B), as well.

This is sometimes called the multiplication rule, because it means if A and B are independent, then P(AB) = P(A) x P(B).

Outcomes of different die rolls, spins of a spinner, flips of a coin, etc. can be assumed independent. Similarly, observations sampled *with replacement* are independent, or without replacement from large population are nearly independent.

e.g. roll 2 dice. P(add up to 12) = P(1st is 6 and 2nd is 6) = P(1st is 6)P(2nd is 6)=1/6 x1/6.

P(at least one 6) = 1 - P(1st isn't 6 and 2nd isn't 6) = 1 – (5/6 x 5/6) = 1 – 25/36 = 11/36.

In general, if A and B might not be independent, P(AB) = P(A) x P(B | A).

because the right side = P(A) P(AB)/P(A) = P(AB).

e.g. pick a card. A = the event that it's a king. B = the event that it's a spade.

P(AB) = P(it's the king of spades) = 1/52.

P(A) = 4/52, P(B) = ¼, so here P(AB) = P(A)P(B), so A and B are independent.

6. Probability trees.

Sometimes you can use probability trees for multiplication rule problems.

e.g. deal two cards. P(two red kings) = P(K of hearts -> K of diamonds or K of diamonds -> K of hearts)
= P(K of hearts -> K of diamonds) + P(K of diamonds -> K of hearts)
= 1/52 x 1/51 + 1/52 x 1/51.

Suppose 1% of the population has a disease, and a test is 80% accurate at detection, i.e. P(+ l you have it) = 80%, and P(- l you don't have it) = 80%. Choose someone at random. What is P(test +)?

P(+) = P(has it and test +) + P(doesn't have it and test +)
= P(has it) P(test + l has it) + P(doesn't) P(+ l doesn't)
= 1% (80%) + 99% (20%)
= 20.6%.

What is P(has it l test +)?

P(has it l +) = P(has it and +) / P(+)
= 1% (80%) / 20.6%
~ 38.8%.

7. Combinations.

The number of ways of choosing k distinct objects from a group of n different objects, where the order doesn't matter, is C(n,k) = n!/[k! (n-k)!], with the convention that 0! = 1. Your book writes this ${}_nC_k$.

k! means 1 x 2 x .... x k.

These are called combinations.

For instance, pick 2 cards from a deck. P(Ace and King, in either order)?

There are n = 52 cards in the deck.

Thus there are C(52,2) = 52! / (2! 50!) = 1326 different possible combinations. If each combination is equally likely, then each combination has a probability of 1/1326.

How many combinations are AK? 4 aces, and for each choice of ace, there are 4 kings to go with it, so there are 4 x 4 = 16 such combinations. Each has a probability of 1/1326, so P(AK) = 16/1326.

8. Expected value.

The *expected value* or *mean* of a random variable is the long-term average, if we observe it over and over.

e.g. Mean of a die roll. 1/6 (1) + 1/6 (2) + ... + 1/6 (6) = 3.5.