

Stat 13, Tue 5/15/12.

1. Hand in HW5
2. Review list.
3. Assumptions and CLT again.
4. Examples.

Hand in Hw5. Midterm 2 is Thur, 5/17. Hw6 is due Thu, 5/24.

On Thur, 5/17, I won't be able to have my usual office hour from 230 to 3:30, so it will be instead from 1:30 to 2:15pm.

The midterm is primarily on chapters 4-6, though it has a bit of probability also.

The std normal and t tables will be provided.

Again, you can have 1 page, double-sided, of notes, plus a calculator and a pen or pencil.

## 2. Review list, of stuff since the first midterm.

### a) More probability

- (i) Expected value.
- (ii) Expected value of sums of random variables.
- (iii) Bernoulli random variables.
- (iv) Binomial random variables.
- (v) Independence.

### b) Normal calculations.

- (i) Calculating the area under the normal curve between a and b.
- (ii) Normal percentiles.
- (iii) Normal probability plots.

### c) CLT and CIs.

- (i) CLT.
- (ii) Construction of CIs, for the mean and for proportions.
- (iii) Interpretation of CIs.
- (iii) SE versus  $\sigma$ .
- (iv) Margin of error.
- (v) Sample size calculations.
- (vi) CIs using the t table.
- (vii) Assumptions behind CIs.

### 3. Assumptions and Central Limit Theorem (CLT), again.

If you have a SRS (or observations are iid with mean  $\mu$ ), and  $n$  is large (or the population is normally distributed), then  $\bar{x}$  is approximately normally distributed with mean  $\mu$  and std deviation  $\frac{\sigma}{\sqrt{n}}$ , where  $\sigma$  is the std deviation of the population and  $n$  is the sample size.

Equivalently, we can say  $[(\bar{x} - \mu) \div \frac{\sigma}{\sqrt{n}}]$  is approximately standard normally distributed.

When to use  $z^*$  and  $t^*$ .

a) If it's a simple random sample (SRS) and the population is normal,  $\sigma$  is unknown, and  $n$  is small ( $< 25$ ), then use  $t^*$ .

b) If it's a SRS and the population is normal,  $\sigma$  is known, and  $n$  is small ( $< 25$ ), then use  $z^*$ .

c) If it's a SRS and  $n$  is large, then  $t^*$  and  $z^*$  are very close together, so it doesn't really matter which you use. Use  $z^*$ , though the book recommends  $t^*$ .

d) If the population might NOT be normal and  $n$  is NOT large, then neither  $t^*$  nor  $z^*$  is appropriate.

#### 4. Examples.

In order to see what tv shows Americans watch, the Nielsen corporation surveys a sample of approximately 50,000 Americans. They recently (May 20, 2009) reported that the average American watches approximately 5.1 hours of tv a day. Suppose it's a SRS and that the sample standard deviation is  $s = 2$  hours per day. Find an 70%-CI for the population mean.

Answer: It's a SRS and  $n = 50,000$  is large, so the standard formulas apply, but we don't know  $\sigma$  so we will plug in  $s$ . For a 70%-CI, we look for a value close to 15% (or 85%) in the table, and find the closest is -1.04 (or 1.04), so  $z^* = 1.04$ . The 70% CI is  $\bar{x} \pm (z^*)s/\sqrt{n} = 5.1 \pm (1.04)(2) \div \sqrt{50,000} = 5.1 \pm 0.0093$ .

What does this  $5.1 \pm 0.0093$  mean? It's a range likely to contain  $\mu$ . Here,  $n = 50,000$  is so large that we can be confident that  $\mu$  is likely close to 5.1.

Q. A typical American watches \_\_\_\_  $\pm$  \_\_\_\_ hours of tv per day?

Q. If we took another SRS of 50,000 Americans, we'd expect to get a sample mean of around \_\_\_\_  $\pm$  \_\_\_\_ hours of tv per day?

Q. If we change this 70% CI to a 95% CI, will the margin of error increase or decrease?

#### 4. Examples.

In order to see what tv shows Americans watch, the Nielsen corporation surveys a sample of approximately 50,000 Americans. They recently (May 20, 2009) reported that the average American watches approximately 5.1 hours of tv a day. Suppose it's a SRS and that the sample standard deviation is  $s = 2$  hours per day. Find an 70%-CI for the population mean.

Answer: It's a SRS and  $n = 50,000$  is large, so the standard formulas apply, but we don't know  $\sigma$  so we will plug in  $s$ . For a 70%-CI, we look for a value close to 15% (or 85%) in the table, and find the closest is -1.04 (or 1.04), so  $z^* = 1.04$ . The 70% CI is  $\bar{x} \pm (z^*)s/\sqrt{n} = 5.1 \pm (1.04)(2) \div \sqrt{50,000} = 5.1 \pm 0.0093$ .

What does this  $5.1 \pm 0.0093$  mean? It's a range likely to contain  $\mu$ . Here,  $n = 50,000$  is so large that we can be confident that  $\mu$  is likely close to 5.1.

Q. A typical American watches 5.1  $\pm$  2 hours of tv per day?

Q. If we took another SRS of 50,000 Americans, we'd expect to get a sample mean of around \_\_\_\_\_  $\pm$  \_\_\_\_\_ hours of tv per day?

Q. If we change this 70% CI to a 95% CI, will the margin of error increase or decrease?

#### 4. Examples.

In order to see what tv shows Americans watch, the Nielsen corporation surveys a sample of approximately 50,000 Americans. They recently (May 20, 2009) reported that the average American watches approximately 5.1 hours of tv a day. Suppose it's a SRS and that the sample standard deviation is  $s = 2$  hours per day. Find an 70%-CI for the population mean.

Answer: It's a SRS and  $n = 50,000$  is large, so the standard formulas apply, but we don't know  $\sigma$  so we will plug in  $s$ . For a 70%-CI, we look for a value close to 15% (or 85%) in the table, and find the closest is -1.04 (or 1.04), so  $z^* = 1.04$ . The 70% CI is  $\bar{x} \pm (z^*)s/\sqrt{n} = 5.1 \pm (1.04)(2) \div \sqrt{50,000} = 5.1 \pm 0.0093$ .

What does this  $5.1 \pm 0.0093$  mean? It's a range likely to contain  $\mu$ . Here,  $n = 50,000$  is so large that we can be confident that  $\mu$  is likely close to 5.1.

Q. A typical American watches 5.1  $\pm$  2 hours of tv per day?

Q. If we took another SRS of 50,000 Americans, we'd expect to get a sample mean of around 5.1  $\pm$  0.009 hours of tv per day?

Q. If we change this 70% CI to a 95% CI, will the margin of error increase or decrease?

#### 4. Examples.

In order to see what tv shows Americans watch, the Nielsen corporation surveys a sample of approximately 50,000 Americans. They recently (May 20, 2009) reported that the average American watches approximately 5.1 hours of tv a day. Suppose it's a SRS and that the sample standard deviation is  $s = 2$  hours per day. Find an 70%-CI for the population mean.

Answer: It's a SRS and  $n = 50,000$  is large, so the standard formulas apply, but we don't know  $\sigma$  so we will plug in  $s$ . For a 70%-CI, we look for a value close to 15% (or 85%) in the table, and find the closest is -1.04 (or 1.04), so  $z^* = 1.04$ . The 70% CI is  $\bar{x} \pm (z^*)s/\sqrt{n} = 5.1 \pm (1.04)(2) \div \sqrt{50,000} = 5.1 \pm 0.0093$ .

What does this  $5.1 \pm 0.0093$  mean? It's a range likely to contain  $\mu$ . Here,  $n = 50,000$  is so large that we can be confident that  $\mu$  is likely close to 5.1.

Q. A typical American watches 5.1  $\pm$  2 hours of tv per day?

Q. If we took another SRS of 50,000 Americans, we'd expect to get a sample mean of around 5.1  $\pm$  0.009 hours of tv per day?

Q. If we change this 70% CI to a 95% CI, will the margin of error increase or decrease? Increase.  $z^*$  will go from 1.04 to 1.96. Margin of error will almost double.

According to the CDC, the largest number of reported cases in the U.S. for any condition is chlamydia, a sexually transmitted disease reported each year in about 0.4% of people overall. In 2009-2010, the National Health and Nutrition Examination Survey (NHANES) took a SRS of 10,253 Americans, and among the (roughly) 510 females age 15-24, they found the prevalence of chlamydia to be 8.0%.

Find a 95%-CI for the population percentage of females age 15-24 with chlamydia.

Answer: This is a 0-1 question. It's a SRS and  $n$  is large because the number of females with chlamydia in the sample =  $8\% \times 510 = 41 \geq 10$ , and the number without =  $92\% \times 510 = 469 \geq 10$ .

For a 95%-CI,  $z^* = 1.96$  from the bottom row of Table 4.  
The formula for the 95%-CI is  $\bar{x} \pm z^* \sigma/\sqrt{n}$ .

We don't know  $\sigma$  so use  $s = \sqrt{\hat{p}\hat{q}} = \sqrt{(8.0\% \times 92.0\%)} \sim 0.271$ .

Our 95%-CI is  $8.0\% \pm (1.96) (0.271) / \sqrt{510}$   
which is  $8.0\% \pm 2.35\%$ .



Suppose you take a SRS of 17 UCLA students and find their IQs. You find that  $\bar{x}$  is 120 and  $s = 20$ . Find a 95% CI for  $\mu$ , the mean IQ for UCLA, assuming these IQs are normally distributed.

Here we have a SRS, the pop. is normal, and  $\sigma$  is unknown, so use the t table.  
 $df = n-1 = 17-1 = 16$ . From Table 4, for a 95% CI, with  $df = 16$ ,  $t^* = 2.12$ .  
So, the 95% CI is  $\bar{x} \pm t^* s/\sqrt{n} = 120 \pm 2.12 (20)/\sqrt{17} = 120 \pm 10.28$ .

What is the standard error?  
 $s/\sqrt{n} = 20/\sqrt{17} = 4.85$ .

A typical UCLA student has an IQ of about \_\_\_\_\_  $\pm$  \_\_\_\_\_ .

If you take other samples each of 17 UCLA students, you'd expect your samples typically to have a mean of about \_\_\_\_\_  $\pm$  \_\_\_\_\_ .

Suppose you take a SRS of 17 UCLA students and find their IQs. You find that  $\bar{x}$  is 120 and  $s = 20$ . Find a 95% CI for  $\mu$ , the mean IQ for UCLA, assuming these IQs are normally distributed.

Here we have a SRS, the pop. is normal, and  $\sigma$  is unknown, so use the t table.  
 $df = n-1 = 17-1 = 16$ . From Table 4, for a 95% CI, with  $df = 16$ ,  $t^* = 2.12$ .  
So, the 95% CI is  $\bar{x} \pm t^* s/\sqrt{n} = 120 \pm 2.12 (20)/\sqrt{17} = 120 \pm 10.28$ .

What is the standard error?  
 $s/\sqrt{n} = 20/\sqrt{17} = 4.85$ .

A typical UCLA student has an IQ of about 120  $\pm$  20 .

If you take other samples each of 17 UCLA students, you'd expect your samples typically to have a mean of about \_\_\_\_\_  $\pm$  \_\_\_\_\_ .

Suppose you take a SRS of 17 UCLA students and find their IQs. You find that  $\bar{x}$  is 120 and  $s = 20$ . Find a 95% CI for  $\mu$ , the mean IQ for UCLA, assuming these IQs are normally distributed.

Here we have a SRS, the pop. is normal, and  $\sigma$  is unknown, so use the t table.  
 $df = n-1 = 17-1 = 16$ . From Table 4, for a 95% CI, with  $df = 16$ ,  $t^* = 2.12$ .  
So, the 95% CI is  $\bar{x} \pm t^* s/\sqrt{n} = 120 \pm 2.12 (20)/\sqrt{17} = 120 \pm 10.28$ .

What is the standard error?  
 $s/\sqrt{n} = 20/\sqrt{17} = 4.85$ .

A typical UCLA student has an IQ of about 120  $\pm$  20 .

If you take other samples each of 17 UCLA students, you'd expect your samples typically to have a mean of about 120  $\pm$  4.85 .