Stat 13, Intro. to Statistical Methods for the Life and Health Sciences.

1. 1.96 SE method for CIs for a proportion.
2. Formulas for CIs for a proportion.
3. Formulas for CIs for a quantitative variable and used car example.

Read chapter 4.

HW2 is due Wed, Feb12, 1159pm. 2.3.15, 3.3.18, and 4.1.23.

Midterm is Mon Feb24 in class.

On both Mon Feb3 and Wed Feb5, lecture will be recorded, rather than in person, and you can watch it whenever you want. The links will be on the course website at recordedlectures.html on Mon Feb 3.

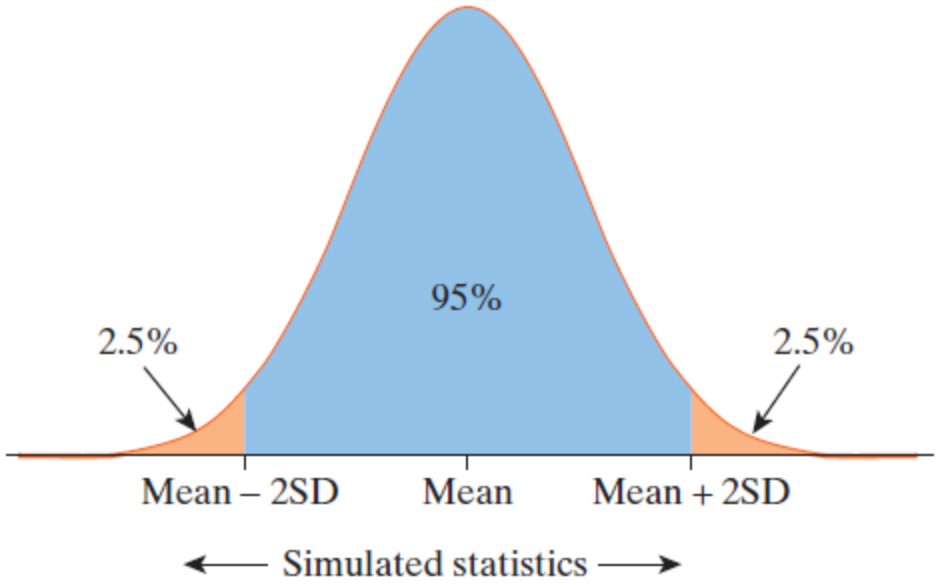
The course website is http://www.stat.ucla.edu/~frederic/13/W25 .

Short cut?

- The method we used last time to find our interval of plausible values for the parameter is tedious and time consuming.
- Might there be a short cut?
- Our sample proportion should be the middle of our confidence interval.
- We just need a way to find out how wide it should be.

1.96SE method

- When a statistic is normally distributed, about 95% of the values fall within 1.96 standard errors of its mean with the other 5% outside this region



1.96SE method

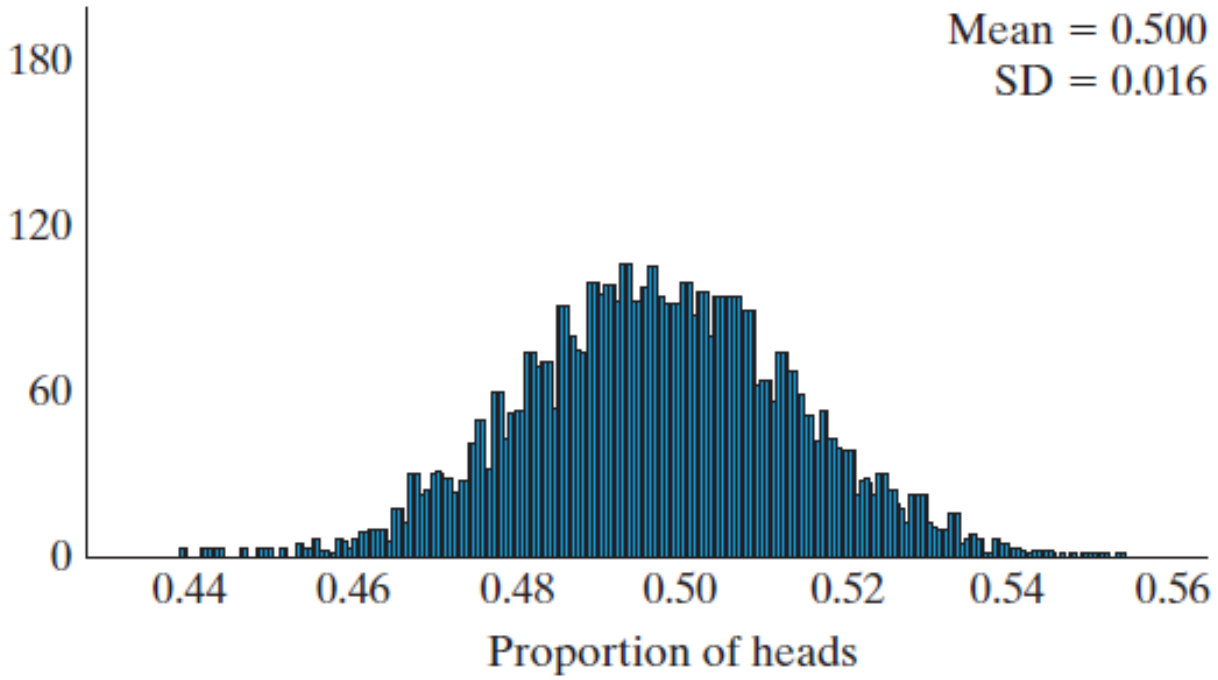
- So we could say that a parameter value is plausible if it is within 1.96 standard errors from our best estimate of the parameter, which is our observed sample statistic.
- This gives us the simple formula for a 95% confidence interval of

$$\hat{p} \pm 1.96SE$$

Note that your book calls this the 2SD method but it really should be called the 1.96SE method.

Where do we get the SE?

- One way is via simulation.
- When the null hypothesis is $\pi = 0.5$, the SE = 0.016.



1.96SE method

- Using the 1.96SE method on our ACA data we get a 95% confidence interval

$$0.69 \pm 1.96(0.016)$$
$$0.69 \pm 0.031$$

- The $\pm$ part, like the 0.031 in the above, is called the **margin of error.**
- The interval can also be written as we did before using just the endpoints; (0.659, 0.721)
- This is approximately what we got using simulations, with our range of plausible values method. We had (0.661, 0.717).

2. Formula or Theory-Based Method

- The 1.96SE method is for a 95% confidence interval.
- If we want a different level of confidence, we can use the range of plausible values (hard) or theory-based methods (easy).
- The theory-based method is valid for CIs for a proportion, provided it's a Simple Random Sample (SRS) and there are at least 10 successes and 10 failures in your sample.

FORMULA FOR CIs FOR A PROPORTION.

- On the previous slides, we relied on simulations to tell us that the SE was 0.016. But we don't need this. In general for testing a proportion, under the null hypothesis, SE = $\sqrt{\pi(1-\pi)/n}$.
- For confidence intervals, we do not assume the null hypothesis, and since π is unknown, use $\hat{p}$ in its place:

$$\hat{p} \pm \textit{multiplier} \times \sqrt{\hat{p}\,(1-\hat{p}\,)/n}.$$

For a 95% CI, the book suggests a multiplier of 2. Actually people use 1.96, not 2. This comes from a property of the normal distribution.
qnorm(.975) = 1.96.

qnorm(.995) = 2.58, the multiplier for a 99% CI.

- Going back to the ACA example, recall

69% of 1034 respondents were not affected.

With no default value of π, to get a 95% CI for $\hat{p}$, use

$\hat{p} \pm multiplier \times \sqrt{\hat{p}(1-\hat{p})/n}$

$= 69\% \pm 1.96 \times \sqrt{.69(1-.69)/1034}$

$= 69\% \pm 2.82\%.$

With 2 instead of 1.96 it would be 69% $\pm$ 2.88%.

This is the formula we actually use for CIs for a proportion.

$$\hat{p} \pm multiplier \times \sqrt{\hat{p}(1-\hat{p})/n}\ .$$

To review, the book first explains how to get a CI by repeated testing, then using the "2 SE" method where the SE is found via simulation, then gives you this formula. But the formula is actually the correct answer. The others are approximations and require simulation.

3. 1.96SE and formula based Confidence Intervals for a mean of a quantitative variable and used car example.

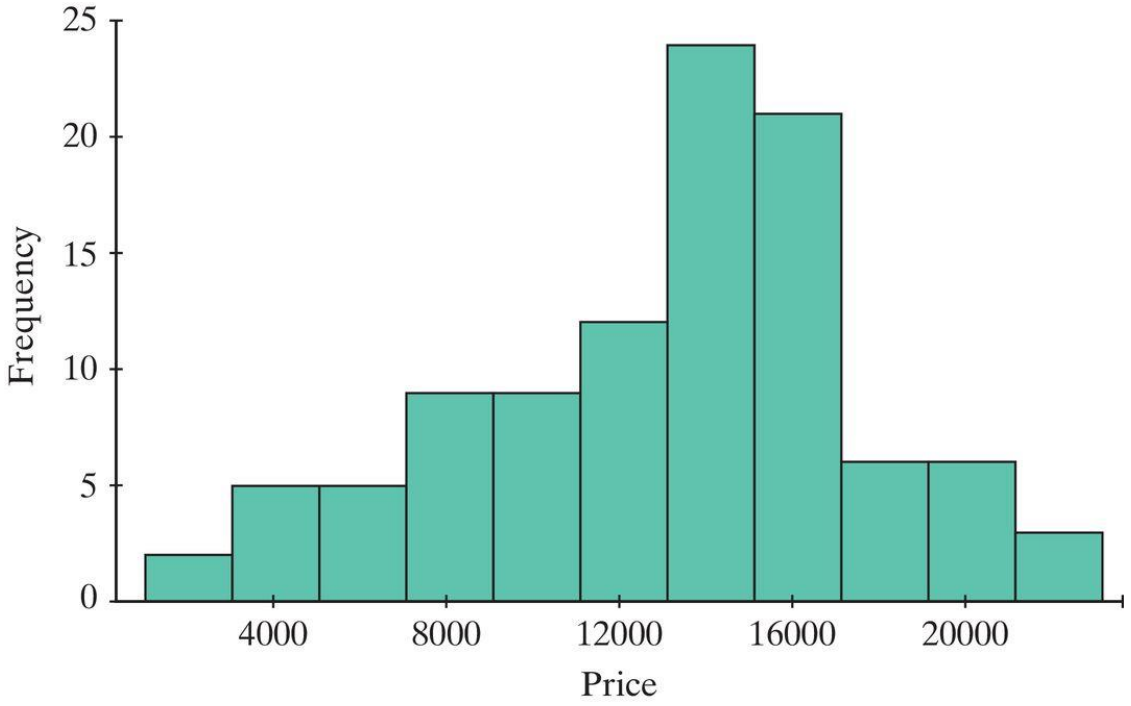
Section 3.3

Used Cars

Example 3.3

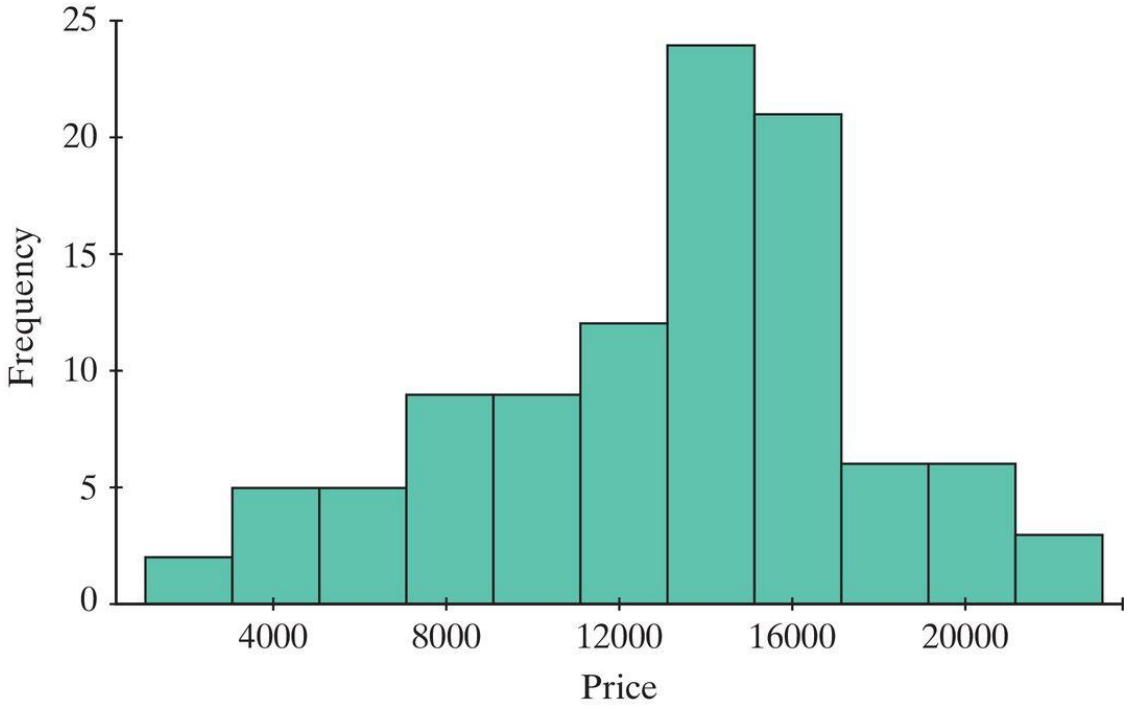
Used Cars

The following histogram displays data for the selling price of 102 Honda Civics that were listed for sale on the Internet in July 2006.



Used Cars

- The average of this sample is $\bar{x}$ = \$13,292 with a standard deviation of *s* = \$4,535.
- What can we say about μ, the average price of all used Honda Civics?



Used Cars

- While we should be cautious about our sample being representative of the population, let's treat it as such.
- μ might not equal $13,292 (the sample mean), but it should be close.
- To determine how close, we can construct a confidence interval.

Confidence Intervals

- Remember the basic form of a confidence interval is:

$$\text{statistic} \pm \text{multiplier} \times \text{SE}$$

SE is called by the book "SD of statistic".

- In our case, the statistic is $\bar{x}$ and for large n, for a 95% CI our multiplier is 1.96, so we can write our 1.96SE confidence interval as:

$$\bar{x} \pm 1.96(\text{SE})$$

Confidence Intervals

- It is important to note that the SE, which is the SD of $\bar{x}$, is not the same as the SD of our sample, s = \$4,535.
- There is more variability in the data (the car-to-car variability) than in sample means.
- The SE is $s/\sqrt{n}$. Which means in general we can write a 1.96SE confidence interval for the mean as

$$\bar{x} \pm 1.96 \frac{s}{\sqrt{n}}.$$

This 1.96 multiplier may be valid when n is large.

Summary Statistics

- When n is small and the population is approximately normal, we will use a multiplier that is based on a *t*-distribution, instead of 1.96. The t multiplier is dependent on the sample size and confidence level.
- For a theory-based confidence interval for a population mean (called a one-sample t-interval) to be valid, the observations should be approximately iid (independent and identically distributed), and either the population should be normal or n should be large. Check the sample distribution for skew and asymmetry.

Confidence Intervals

- We find our 95% CI for the mean price of all used Honda Civics is from $12,401.20 to $14,182.80.
- Notice that this is a much narrower range than the prices of all used Civics.
- For a 99% confidence interval, it would be wider. The multiplier would be 2.58 instead of 1.96.