Stat 13, Intro. to Statistical Methods for the Life and Health Sciences.

1. Two quantitative variables, scatterplots and correlation.
2. Inference for correlation, temperature and heart rate example.
3. Calculating correlation.
4. Regression line, dinner plates.

Read chapters 7 and 10.

HW4 is due Wed, Mar12, 1159pm. 10.1.8, 10.3.14, 10.3.21, and 10.4.11.
The problems are on the next 5 slides.

The course website is http://www.stat.ucla.edu/~frederic/13/W25 .

If I haven't given your midterm back to you yet, I can do so after class.

10.1.8.

10.1.8 Which of the following statements is correct?

A. Changing the units of measurements of the explanatory or response variable does not change the value of the correlation.

B. A negative value for the correlation indicates that there is no relationship between the two variables.

C. The correlation has the same units (e.g., feet or minutes) as the explanatory variable.

D. Correlation between *y* and *x* has the same number but opposite sign as the correlation between *x* and *y*.

10.3.12 Reconsider the previous five exercises and the **Legos** data file. The last product listed in the data file has 415 pieces and a price of $49.99.

a. Determine the predicted price for such a product.

b. Determine the residual value for this product.

c. Interpret what this residual value means.

d. Does the product fall above or below the least squares line in the graph? Explain how you can tell, based on its residual value.

10.3.13 Reconsider the previous six exercises and the **Legos** data file. This is very unrealistic, but suppose that one of the products were to be offered at a price of $0.

a. Would you expect this change to affect the least squares line very much? Explain.

b. For which one product would you expect this change to have the greatest impact on the least squares line? Explain how you choose this product.

c. Change the price to $0 for the product that you identified in part (b). Report the (new) equation of the least squares line and the (new) value of r^2. Have these values changed considerably?

Crickets

10.3.14 Consider the following two scatterplots based on data gathered in a study of 30 crickets, with temperature measured in degrees Fahrenheit and chirp frequency measured in chirps per minute.

a. If the goal is to predict temperature based on a cricket's chirps per minute, which is the appropriate scatterplot to examine—the one on the left or the one on the right? Explain briefly.

One of the following is the correct equation of the least squares line for predicting temperature from chirps per minute:

A. predicted temperature = 35.78 + 0.25 chirps per minute

B. predicted temperature = −131.23 + 3.81 chirps per minute

C. predicted temperature = 83.54 − 0.25 chirps per minute

b. Which is the correct equation? Circle your answer and explain briefly.

c. Use the correct equation to predict the temperature when the cricket is chirping at 100 chirps per minute.

d. Interpret the value of the slope coefficient, in this context, for whichever equation you think is the correct one.

Cat jumping*

10.3.15 Harris and Steudel (2002) studied factors that might be associated with the jumping performance of domestic cats. They studied 18 cats, using takeoff velocity (in centimeters per second) as the response variable. They used body mass (in grams), hind limb length (in centimeters), muscle mass (in grams), and percent body fat in addition to sex as potential explanatory variables. The data can be found in the **CatJumping** data file. A scatterplot of takeoff velocity vs. body mass is shown in the figure for Exercise 10.3.15.

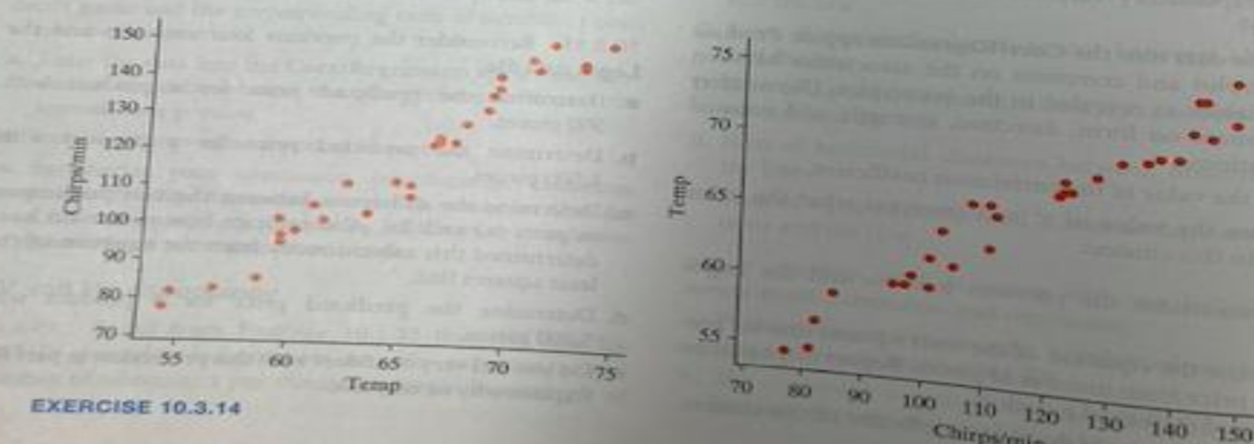
a. Describe the association between these variables.

b. Use the **Corr/Regression** applet to determine the equation of the least squares line for predicting a cat's takeoff velocity from its mass.

c. Interpret the value of the slope coefficient in this context.

d. Interpret the value of the intercept coefficient. Is this a context in which the intercept coefficient is meaningful?

e. Determine the proportion of variability in takeoff velocity that is explained by the least squares line with mass.

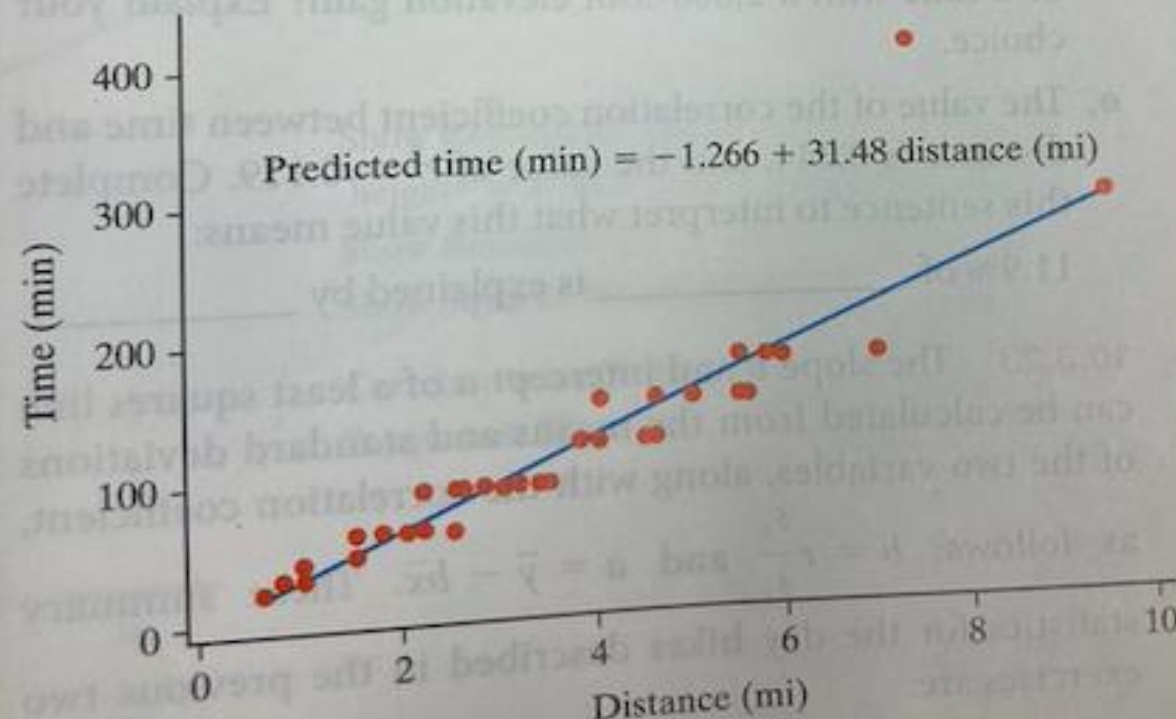


EXERCISE 10.3.14

10.3.21

Day hikes

10.3.21 The book *Day Hikes in San Luis Obispo County* lists information about 72 hikes, including the distance of the hike (in miles), the elevation gain of the hike (in feet), and the time that the hike is expected to take (in minutes). Consider the scatterplot below, with least squares regression line superimposed:



10.3.21.

a. Report the value of the slope coefficient for predicting time from distance.

b. Write a sentence interpreting the value of the slope coefficient for predicting time from distance.

c. Use the line to predict how long a 4-mile hike will take.

d. Would you feel more comfortable using the line predict the time for a 4-mile hike or for a 12-mile hike? Explain your choice.

e. The value of the correlation coefficient between time and distance is 0.916, and the value of $r^2 = 0.839$. Complete this sentence to interpret what this value means:

83.9% of ____________ is explained by ____________.

10.3.22 Reconsider the previous exercise. The following

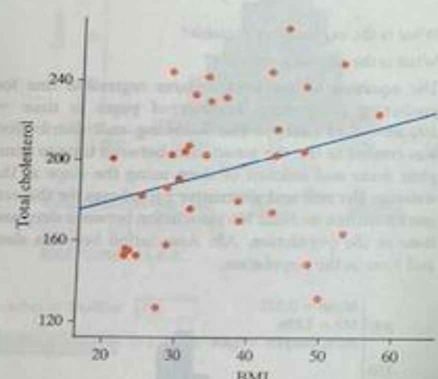
10.4.10 Reconsider the previous exercise about the amount of sleep (in hours) obtained in the previous night and time to complete a paper and pencil maze (in seconds). The equation of the least squares regression line for predicting price from number of pages is $\widehat{\text{time}}$ = 190.33 − 7.76 (sleep).

a. Interpret what the slope coefficient means in the context of sleep and time to complete the maze.

b. Interpret the intercept. Is this an example of extrapolation? Why or why not?

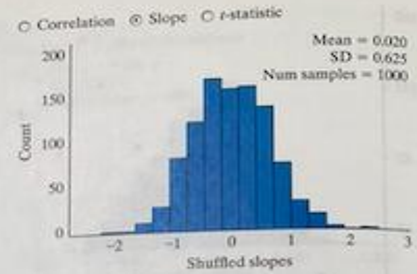
Weight loss and protein

10.4.11 In a study to see if there was an association between weight loss and the amount of a certain protein in a person's body fat, the researchers measured a number of different attributes in their 39 subjects at the beginning of the study. The article reported, "These subjects were clinically and ethnically heterogeneous." Two of the variables they measured were body mass index (BMI) and total cholesterol. The results are shown in the scatterplot along with the regression line.



a. What are the observational units in the study?

b. The equation of the least squares regression line for predicting total cholesterol from BMI is $\widehat{\text{cholesterol}}$ = 162.56 − 0.9658 (BMI). The following null distribution was created to test the association between people's total cholesterol number and their BMI using the slope as the statistic. The null and alternative hypotheses for this test can be written as: Null: No association between cholesterol and BMI in the population. Alt: Association between cholesterol and BMI in the population.



i. Based on information shown in the null distribution, how many standard deviations is our observed statistic below the mean of the null distribution? (That is, what is the standardized statistic?)

ii. Based on your standardized statistic, do you have strong evidence of an association between a people's total cholesterol and their BMI? Explain.

10.4.12 Reconsider the previous exercise about the cholesterol and BMI. The equation of the least squares regression line obtained was $\widehat{\text{cholesterol}}$ = 162.56 − 0.9658 (BMI).

a. Interpret what the slope coefficient means in the context of cholesterol and BMI.

b. Interpret the intercept. Is this an example of extrapolation? Why or why not?

Honda Civic prices*

10.4.13 The data in the file **UsedHondaCivics** come from a sample of used Honda Civics listed for sale online in July 2006. The variables recorded are the car's age (calculated as 2006 minus year of manufacture) and price. Consider conducting a simulation analysis to test whether the sample data provide strong evidence of an association between a car's price and age in the population in terms of the population slope.

a. State the appropriate null and alternative hypotheses.

b. Conduct a simulation analysis with 1,000 repetitions. Describe how to find your p-value from your simulation results and report this p-value.

c. Summarize your conclusion from this simulation analysis. Also describe the reasoning process by which your conclusion follows from your simulation results.

10.4.14 Reconsider the previous exercise on prices of Honda Civics.

a. Find the regression equation that predicts the price of the car given its age.

b. Interpret the slope and intercept of the regression line.

Correlation

- **Correlation** measures the strength and direction of a <u>linear</u> association between two <u>quantitative</u> variables.
- Correlation is a number between -1 and 1.
- With positive correlation one variable increases, on average, as the other increases.
- With negative correlation one variable decreases, on average, as the other increases.
- The closer it is to either -1 or 1 the closer the points fit to a line.
- The correlation for the test data is -0.56.

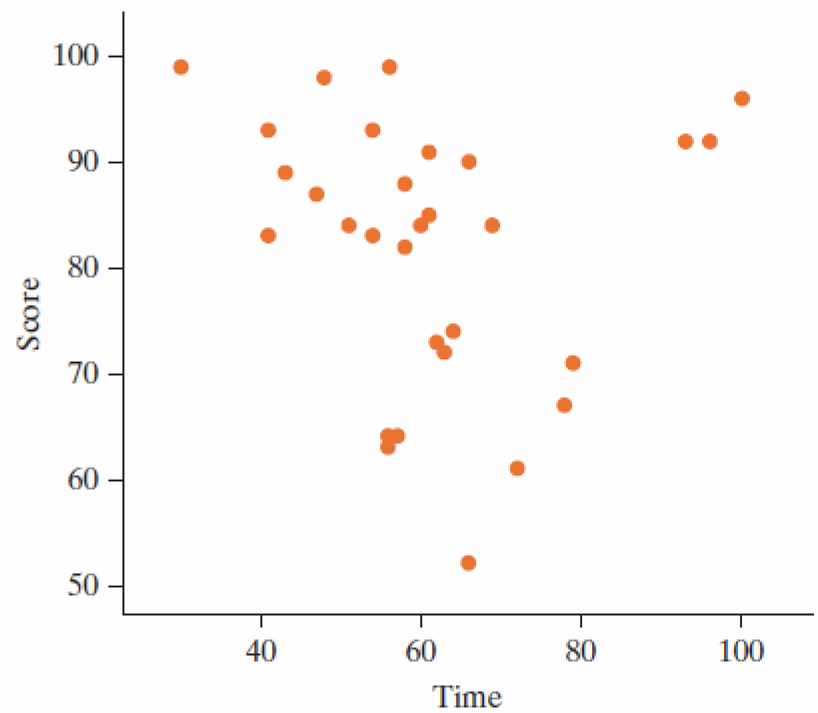
Correlation Guidelines

Correlation Value	Strength of Association	What this means
0.7 to 1.0	Strong	The points will appear to be nearly a straight line
0.3 to 0.7	Moderate	When looking at the graph the increasing/decreasing pattern will be clear, but there is considerable scatter.
0.1 to 0.3	Weak	With some effort you will be able to see a slightly increasing/decreasing pattern
0 to 0.1	None	No discernible increasing/decreasing pattern
Same Strength Results with Negative Correlations		

Back to the test data

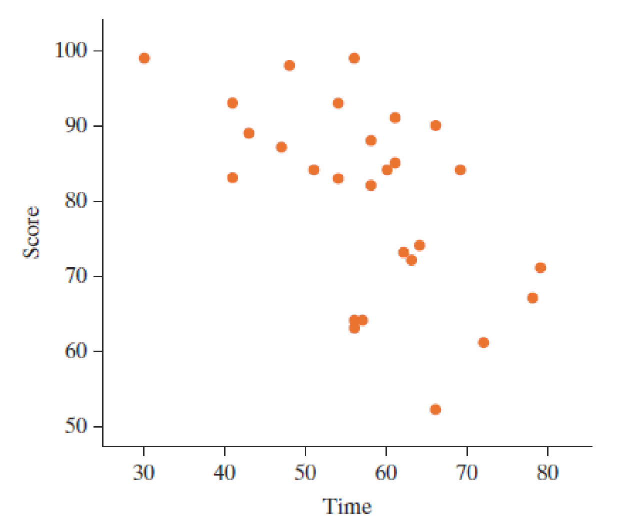
Actually the last three people to finish the test had scores of 93, 93, and 97.

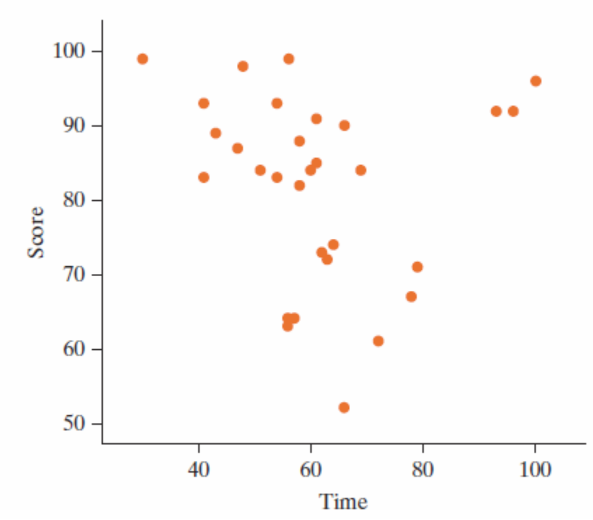
What does this do to the correlation?



Influential Observations

- The correlation changed from -0.56 (a fairly moderate negative correlation) to -0.12 (a weak negative correlation).
- Points that are far to the left or right and not in the overall direction of the scatterplot can greatly change the correlation. (influential observations)





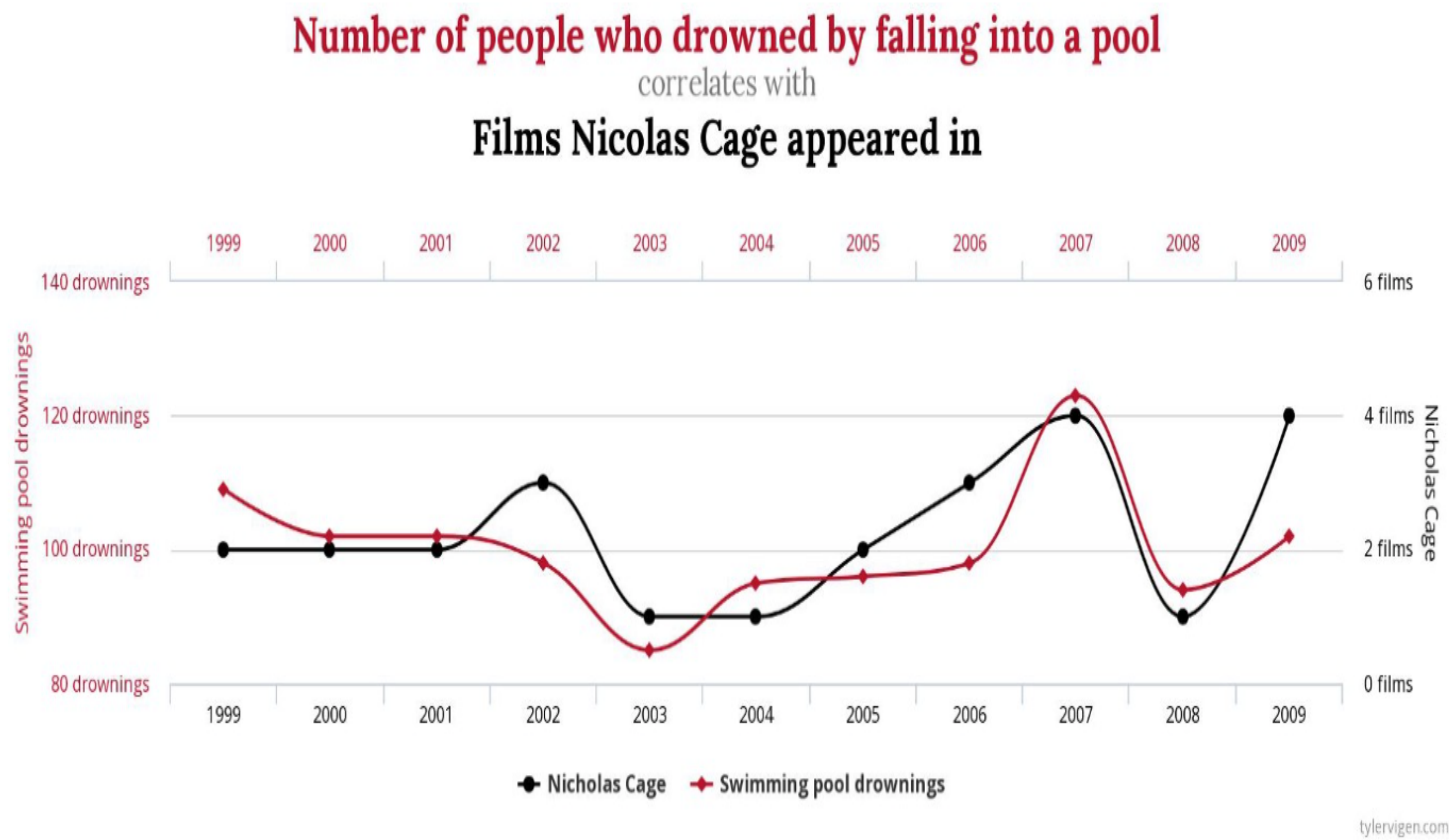
Correlation

- **Correlation** measures the strength and direction of a linear association between two quantitative variables.
  - $-1 \leq r \leq 1$
  - Correlation makes no distinction between explanatory and response variables.
  - Correlation has no units.
  - Correlation is not resistant to outliers. It is sensitive.

Learning Objectives for Section 10.1

- Summarize the characteristics of a scatterplot by describing its direction, form, strength and whether there are any unusual observations.
- Recognize that the correlation coefficient is appropriate only for summarizing the strength and direction of a scatterplot that has linear form.
- Recognize that a scatterplot is the appropriate graph for displaying the relationship between two quantitative variables and create a scatterplot from raw data.
- Recognize that a correlation coefficient of 0 means there is no linear association between the two variables and that a correlation coefficient of -1 or 1 means that the scatterplot is exactly a straight line.
- Understand that the correlation coefficient is influenced by extreme observations.

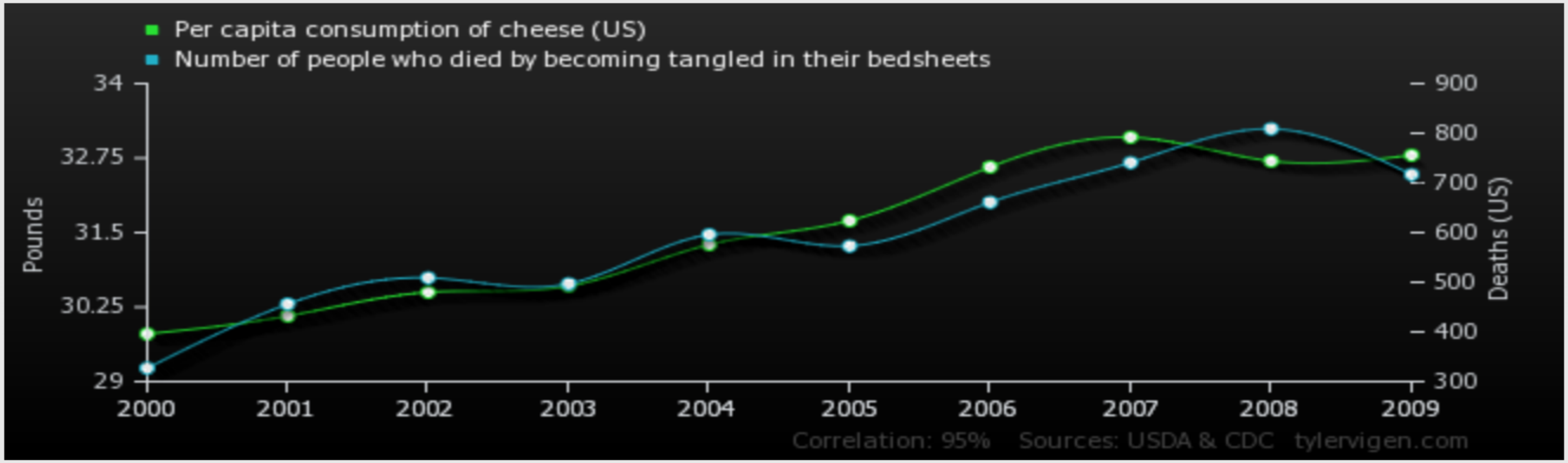
Note that correlation ≠ causation.



from: http://tylervigen.com

Note that correlation ≠ causation.

Per capita consumption of cheese (US)
correlates with
Number of people who died by becoming tangled in their bedsheets



	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Per capita consumption of cheese (US) *Pounds (USDA)*	29.8	30.1	30.5	30.6	31.3	31.7	32.6	33.1	32.7	32.8
Number of people who died by becoming tangled in their bedsheets *Deaths (US) (CDC)*	327	456	509	497	596	573	661	741	809	717

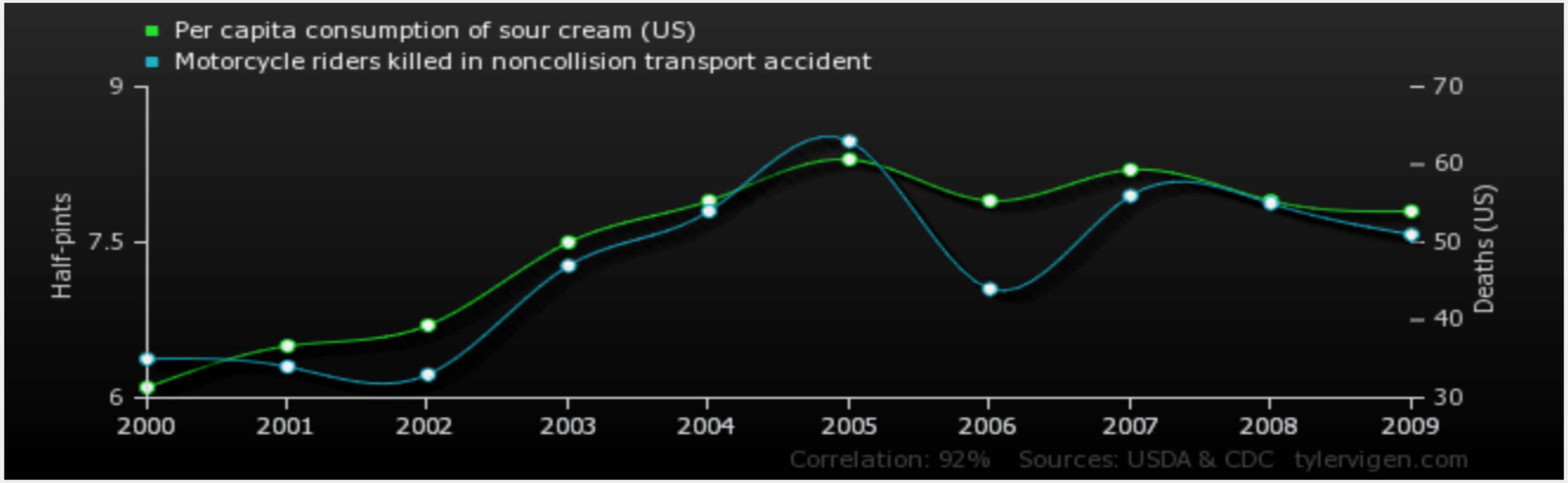
Correlation: 0.947091

Note that correlation ≠ causation.

Per capita consumption of sour cream (US)

correlates with

Motorcycle riders killed in noncollision transport accident



	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Per capita consumption of sour cream (US) *Half-pints (USDA)*	6.1	6.5	6.7	7.5	7.9	8.3	7.9	8.2	7.9	7.8
Motorcycle riders killed in noncollision transport accident *Deaths (US) (CDC)*	35	34	33	47	54	63	44	56	55	51

Correlation: 0.916391

Inference for the Correlation Coefficient: Simulation-Based Approach

Section 10.2

We will look at a small sample example to see if body temperature is associated with heart rate.

Temperature and Heart Rate

Hypotheses

- Null: There is no association between heart rate and body temperature. ($\rho = 0$)
- Alternative: There is a positive linear association between heart rate and body temperature. ($\rho > 0$)

ρ = rho

Inference for Correlation with Simulation

(Section 10.2)

1. Compute the observed statistic. (Correlation)
2. Scramble the response variable, compute the simulated statistic, and repeat this process many times.
3. Reject the null hypothesis if the observed statistic is in the tail of the null distribution.

Temperature and Heart Rate

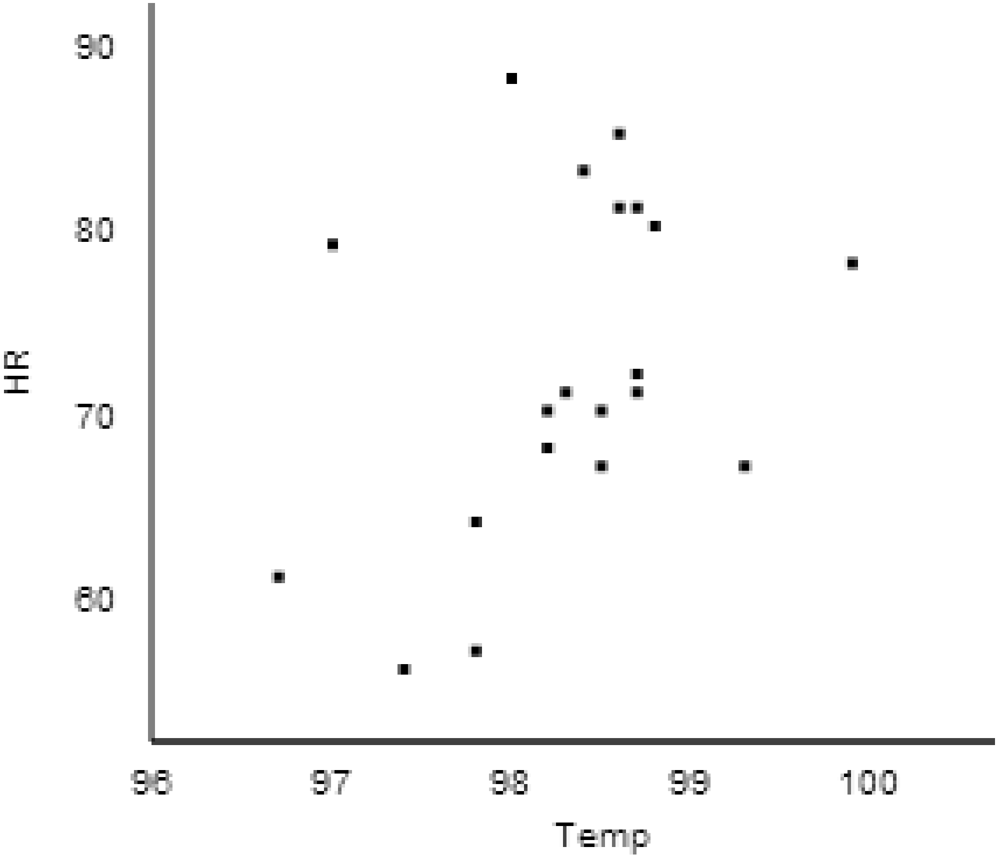
Collect the Data

Tmp	98.3	98.2	98.7	98.5	97.0	98.8	98.5	98.7	99.3	97.8
HR	72	69	72	71	80	81	68	82	68	65
Tmp	98.2	99.9	98.6	98.6	97.8	98.4	98.7	97.4	96.7	98.0
HR	71	79	86	82	58	84	73	57	62	89

Temperature and Heart Rate

Explore the Data

r = 0.378



Temperature and Heart Rate

- If there was no association between heart rate and body temperature, what is the probability we would get a correlation as high as 0.378 just by chance?

- If there is no association, we can break apart the temperatures and their corresponding heart rates. We will do this by shuffling one of the variables.

Shuffling Cards

- Let’s remind ourselves what we did with cards to find our simulated statistics.
- With two proportions, we wrote the response on the cards, shuffled the cards and placed them into two piles corresponding to the two categories of the explanatory variable.
- With two means we did the same thing except this time the responses were numbers instead of words.

Dolphin Therapy

Non-improver	Improver	Improver
Non-improver	Improver	Improver
Non-improver	Improver	Improver
Non-improver	Improver	Improver
Non-improver	Improver	Improver

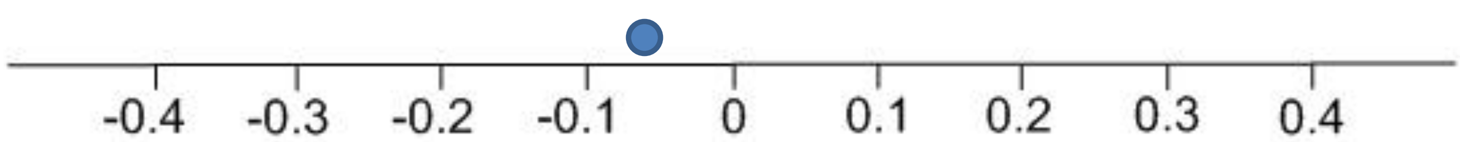
40.0% Improvers

Control

Non-improver	Non-improver	Non-improver
Non-improver	Non-improver	Non-improver
Non-improver	Non-improver	Improver
Non-improver	Non-improver	Improver
Non-improver	Non-improver	Improver

46.7% Improvers

0.400 – 0.467 = -0.067



Music

25.2	45.6
14.5	11.6
-7.0	18.6
12.6	12.1
34.5	30.5

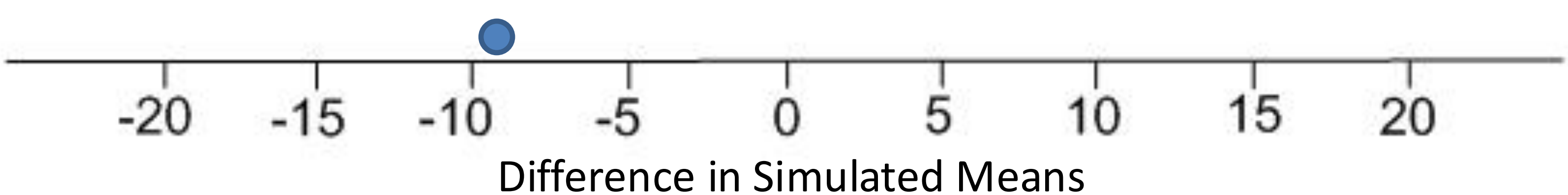
mean = 6.38

No music

-10.7	-10.7	10.0
4.5	9.6	
2.2	2.4	
21.3	21.8	
-14.7	7.2	

mean = 16.12

6.38 – 16.12 = -9.74



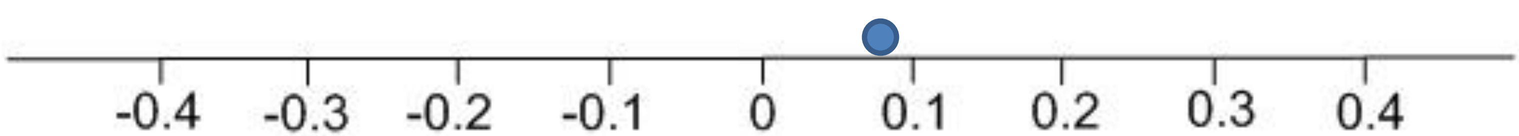
Shuffling Cards

- Now how will this shuffling be different when both the response and the explanatory variable are quantitative?
- We can't put things in two piles anymore.
- We still shuffle values of the response variable, but this time place them next to two values of the explanatory variable.

Body Temperature and Heart Rate

98.3°	98.2°	97.7°	98.5°	97.0°	98.8°	98.5°	98.7°	99.3°	97.8°
72	69	72	71	80	81	68	82	68	65
98.2°	99.9°	98.6°	98.6°	97.8°	98.4°	98.7°	97.4°	96.7°	98.0°
71	79	86	82	58	84	73	57	62	89

r = 0.078

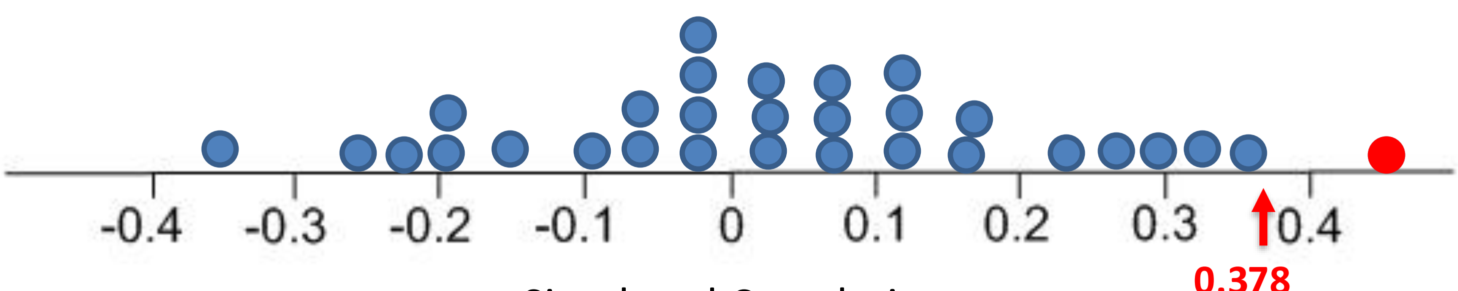


Simulated Correlations

More Simulations

0.097 0.054 0.062 0.259 0.034
-0.253 -0.345 0.314 0.339
-0.034 0.212 -0.029 -0.008
0.200 0.059 -0.006 0.447
-0.042 0.447 0.167
0.067 0.020
0.232 -0.327 0.100 0.329
-0.229

Only one simulated statistic out of 30 was as large or larger than our observed correlation of 0.378, hence our p-value for this null distribution is 1/30 ≈ 0.03.



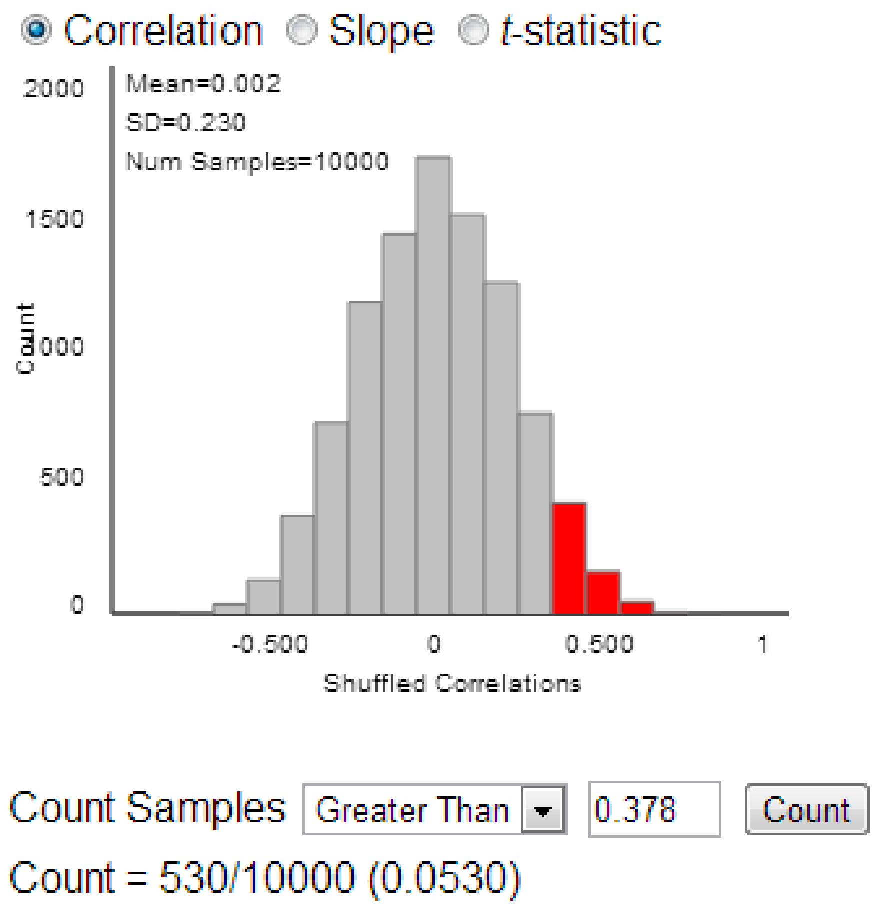
Simulated Correlations

Temperature and Heart Rate

- We can look at the output of 1000 shuffles with a distribution of 1000 simulated correlations.

Temperature and Heart Rate

- Notice our null distribution is centered at 0 and somewhat symmetric.
- We found that 530/10000 times we had a simulated correlation greater than or equal to 0.378.



Temperature and Heart Rate

- With a p-value of 0.053 = 5.3%, we almost but do not quite have statistical significance. We observe a positive linear association between body temperature and heart rate but this association is not statistically significant. Perhaps a larger sample should be investigated to get a better idea if the two variables are related or not.

3. Calculating correlation, r.

ρ = rho = correlation of the population.
Suppose there are N people in the population,
X = temperature, Y = heart rate,
the mean and sd of temp in the pop. are μ_x and σ_x,
and the pop. mean and sd of heart rate are μ_y and σ_y.

$$\rho = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{x_i-\mu_x}{\sigma_x}\right)\left(\frac{y_i-\mu_y}{\sigma_y}\right).$$

Given a sample of size n, we estimate ρ using

$$r = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i-\overline{x}}{s_x}\right)\left(\frac{y_i-\overline{y}}{s_y}\right).$$

This is in Appendix A.

4. Linear Regression

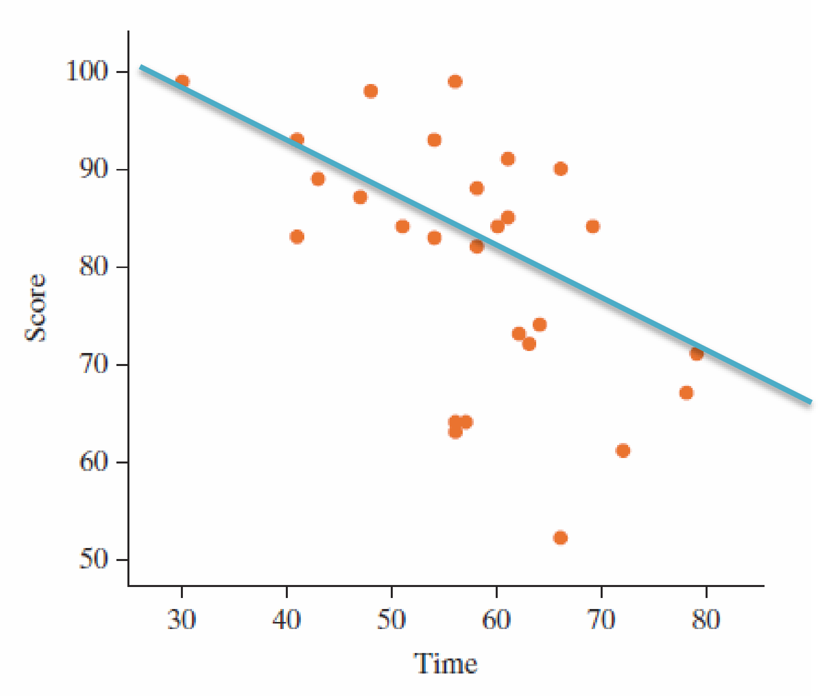
Section 10.3

Introduction

- If we decide an association is linear, it is helpful to develop a mathematical model of that association.
- Helps make predictions about the response variable.
- The ***least-squares regression line*** is the most common way of doing this.

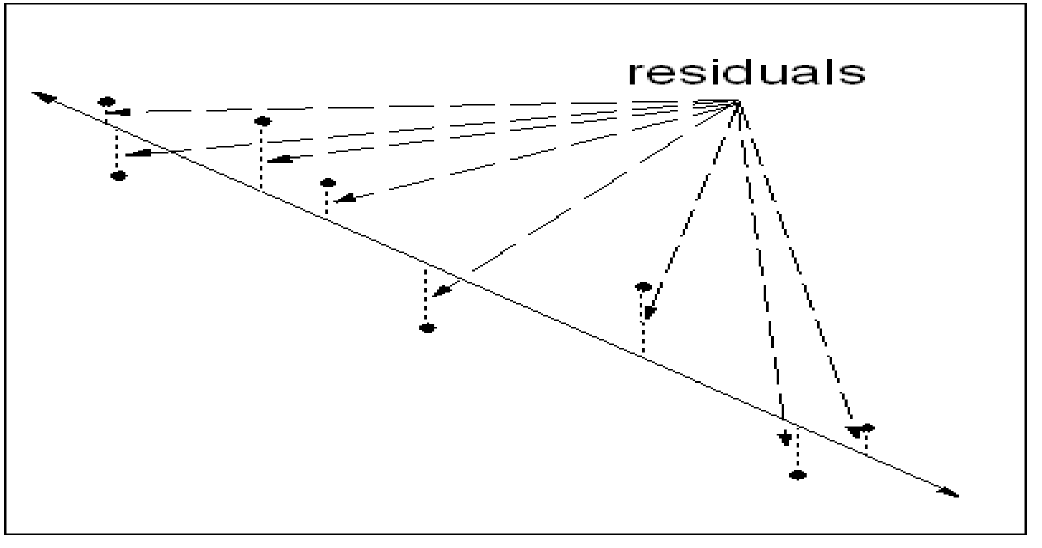
Introduction

- Unless the points are perfectly linearly alligned, there will not be a single line that goes through every point.



Introduction

- We want a line that minimizes the vertical distances between the line and the points
  - These distances are called **residuals.**
  - The line we will find actually minimizes the sum of the squares of the residuals.
  - This is called a **least-squares regression line**.

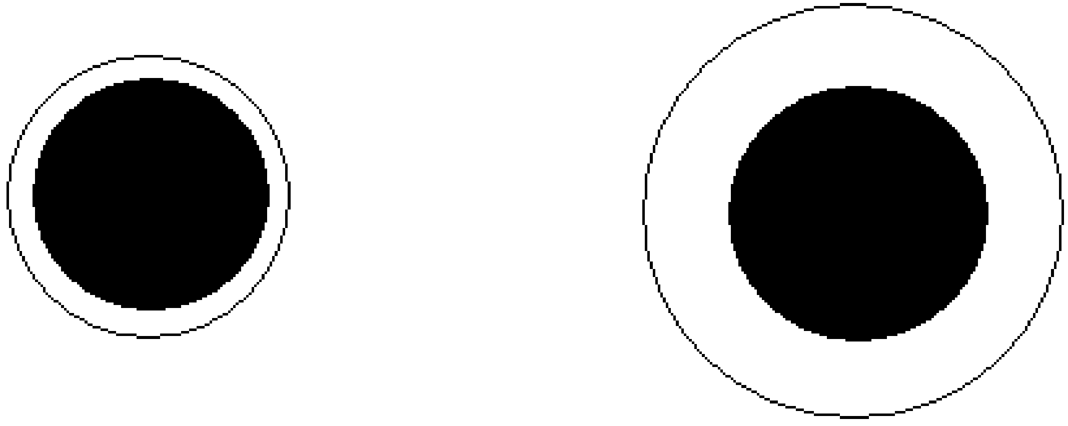


Are Dinner Plates Getting Larger?

Example 10.3

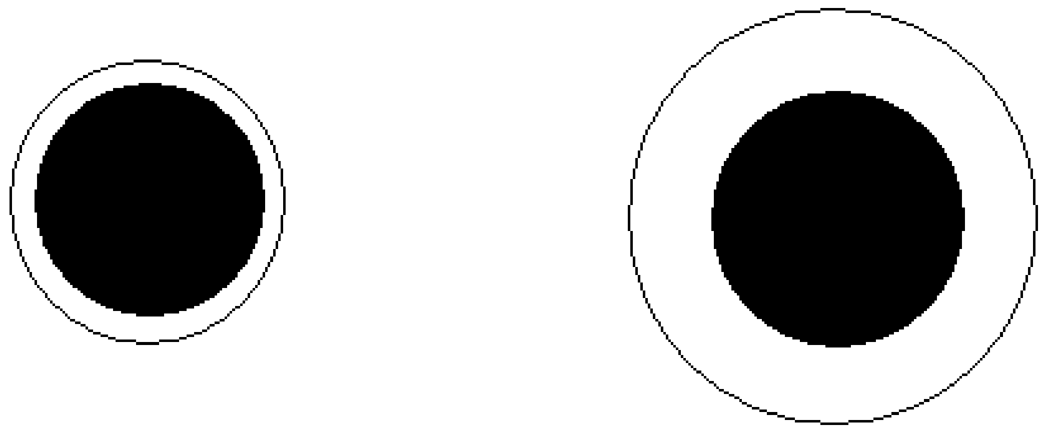
Growing Plates?

- There are many recent articles and TV reports about the obesity problem.
- One reason some have given is that the size of dinner plates are increasing.
- Are these black circles the same size, or is one larger than the other?



Growing Plates?

- They appear to be the same size for many, but the one on the right is about 20% larger than the left.



- This suggests that people will put more food on larger dinner plates without knowing it.
- There is name for this phenomenon: *Delboeuf illusion.*

Growing Plates?

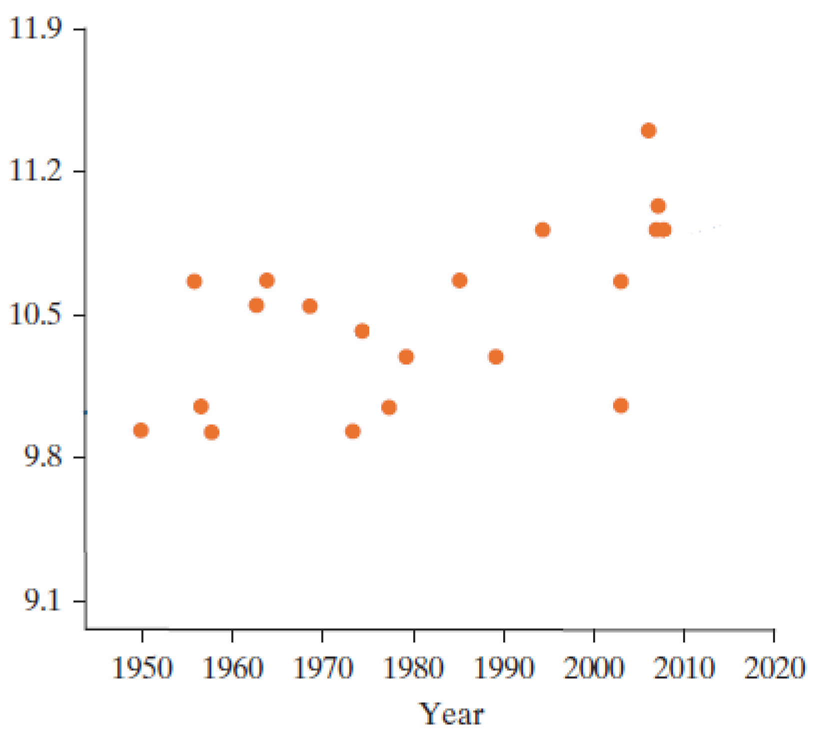
- Researchers gathered data to investigate the claim that dinner plates are growing
- American dinner plates sold on ebay on March 30, 2010 (Van Ittersum and Wansink, 2011)
- Year manufactured and diameter are given.

TABLE 10.1 **Data for size (diameter, in inches) and year of manufacture for 20 American-made dinner plates**

Year	1950	1956	1957	1958	1963	1964	1969	1974	1975	1978
Size	10	10.75	10.125	10	10.625	10.75	10.625	10	10.5	10.125
Year	1980	1986	1990	1995	2004	2004	2007	2008	2008	2009
Size	10.375	10.75	10.375	11	10.75	10.125	11.5	11	11.125	11

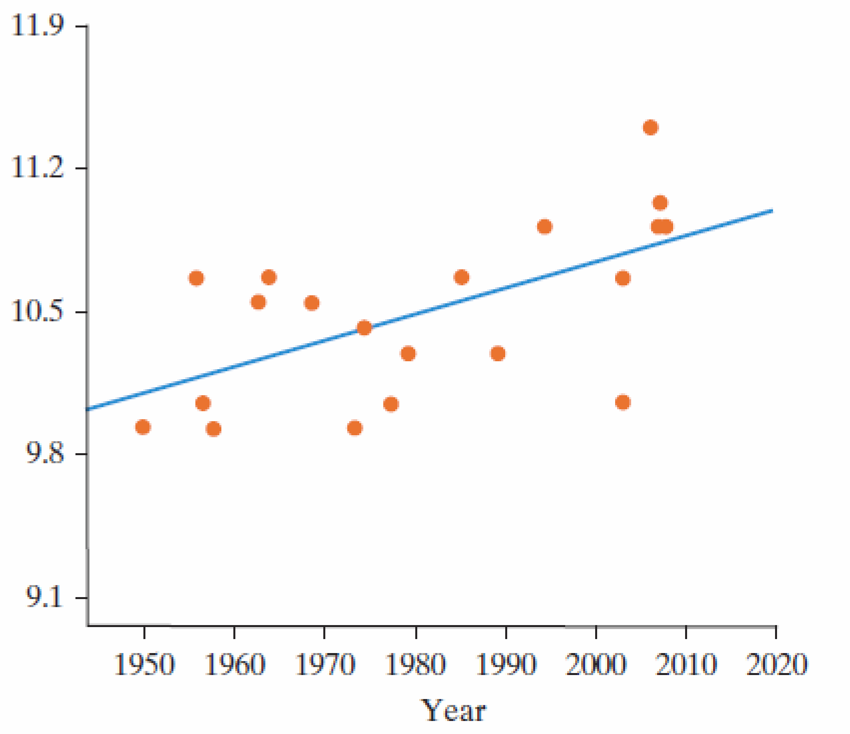
Growing Plates?

- Both year (explanatory variable) and diameter in inches (response variable) are quantitative.
- Each dot in this scatterplot represents one plate.



Growing Plates?

- The association appears to be roughly linear.
- The least squares regression line is added.
- The line slopes upward, but is the slope significant?



Regression Line

The regression equation is $\hat{y} = a + bx$:

– *a* is the *y*-intercept

– *b* is the slope

– *x* is a value of the explanatory variable

– $\hat{y}$ is the predicted value for the response variable

- For a specific value of *x*, the corresponding distance $\mathrm{y} - \hat{y}$ (or actual – predicted) is a residual

Regression Line

- The least squares line for the dinner plate data is $\hat{y} = -14.8 + 0.0128x$
- Or $\widehat{\text{diameter}} = -14.8 + 0.0128(\text{year})$
- This allows us to predict plate diameter for a particular year.

Slope

$$\hat{y} = -14.8 + 0.0128x$$

- What is the predicted diameter for a plate manufactured in 2000?
  - -14.8 + 0.0128(2000) = 10.8 in.
- What is the predicted diameter for a plate manufactured in 2001?
  - -14.8 + 0.0128(2001) = 10.8128 in.
- How does this compare to our prediction for the year 2000?
  - 0.0128 larger
- Slope b = 0.0128 means that diameters are predicted to increase by 0.0128 inches per year on average

Slope

- Slope is the predicted change in the response variable for one-unit change in the explanatory variable.
- Both the slope and the correlation coefficient for this study were positive.
    - The slope is 0.0128
    - The correlation is 0.604
- The slope and correlation coefficient will always have the same sign.

Slope of regression line.

- Suppose $\hat{y}$ = a + bx is the regression line.
- The slope b of the regression line is b = r $\frac{s_y}{s_x}$.

This is usually the thing of primary interest to interpret, as the predicted increase in y for every unit increase in x.

- Beware of assuming causation though, esp. with observational studies. Be wary of extrapolation too.
- The intercept a = $\bar{y}$ - b $\bar{x}$.
- The SD of the residuals is $\sqrt{1-r^2}\ s_y$.

This is a good estimate of how much the regression predictions will typically be off by.

y-intercept

- The *y*-intercept is where the regression line crosses the *y*-axis. It is the predicted response when the explanatory variable equals 0.
- We had a *y*-intercept of -14.8 in the dinner plate equation. What does this tell us about our dinner plate example?
  - Dinner plates in year 0 would be predicted to be -14.8 inches???
- How can it be negative?
  - The equation works well within the range of values given for the explanatory variable, but fails outside that range.
- Our equation should only be used to predict the size of dinner